ESAT Mock Papers

An extract: the first 10 questions of each module, in full

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50 questions across 5 modules (Mathematics 1, Mathematics 2, Physics, Chemistry, Biology), with a question paper, an answer sheet, a mark scheme and full worked solutions for each. 301 pages. The full paper carries 27 questions per module.

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To any school or college that downloads this pack.

Dear colleague,

The ESAT mock papers are enclosed, ready to print. There are five module papers enclosed, each with a question paper, the worked solutions in full, a keys only mark scheme and a separate answer sheet. There is also a reference setting out what the ESAT assesses, module by module, for a candidate to revise from.

Print as many copies as you need.

On the questions. This is the free sample pack, so these questions ARE published: they live on lucasrey.com and anyone may download them. Your students may already have seen them. That is the one way this pack differs from a commissioned one, and it is the whole reason a commissioned one exists: its questions are drawn fresh, recorded against your school, and issued to nobody else and published nowhere. Every question here is also now retired from that pool, so a pack you order cannot contain one of them.

Sitting on paper. Run it on paper rather than on screen. Printed papers are more reliable for a room of any size and they avoid every device and login problem on the day.

Notes for the exam room. The format, the syllabus coverage and the marking all follow the published ESAT specification: 27 questions, 40 minutes, one mark each, no penalty for a wrong answer, no calculator. A few things are worth knowing when you run it and when you read the marks.

- Run this on paper, then send the cohort to the official on-screen practice at least once before the window of 12 to 16 October 2026. The real ESAT is sat on screen at a Pearson test centre, so a printed mock trains the mathematics and the pacing while the official platform trains the interface. Doing both is what a full rehearsal looks like.
- Reading the scores: the option count varies question by question as it does on the real paper, 57 at 5 options, 28 at 6 options, 12 at 7 options, 38 at 8 options. A blind guess therefore scores about 0.16 here, against roughly 0.14 across the published ENGAA and NSAA questions and 0.20 on a paper that offered five throughout.
- Questions built around a diagram, by module: Mathematics 1 2, Physics 6, Chemistry 2, Biology 9.
- Difficulty is set on what a question demands: the number of worked steps, the length of the working, and the time a fast solver needs, rather than on a label somebody typed. Each module carries a spread from one-step to five-step questions rather than a uniform level, because a paper with no answerable opening questions does not discriminate at the bottom of the range. One honest caveat: the official papers publish an answer key and no worked solutions, so that spread is matched to our own reference papers. Stem length and diagram share, above, are measured against the official papers themselves.
- How it was checked: every question was independently re-derived from its own stem, its key confirmed as uniquely correct, and each wrong option traced to a specific error that produces that exact value. The pack is then checked mechanically against the specification. That is a question-by-question review rather than one read through end to end, and it is the stronger of the two.

Scope. Five unseen module papers with full worked solutions and mark schemes. Marking and cohort analysis are not included in this engagement. The mark scheme lets your department mark in house and see which topics the cohort is losing marks on. If you would like the marking and the analysis as a separate piece of work, tell me and I will quote.

Cost. Nothing. This pack is free to download and free to use with your own students. It is not for resale or republication.

If anything needs reissuing, a module renumbered or a question you would rather swap out, tell me and I will send a corrected version at no charge.

Yours sincerely,

Lucas Rey

What the ESAT assesses, and what to watch

This is the content of the test, module by module, written so that a student can revise from it: each line says what you must be able to do. What the test covers is not hard to find out; it is simply scattered, and phrased for the people who administer the test rather than for the person sitting it. This puts it in one place, in order, in the words a candidate would use. Where a line carries a note in smaller type, that is the slip candidates most often make on it.

How the test is put together. Mathematics 1 is compulsory. You then sit further modules chosen from Mathematics 2, Physics, Chemistry and Biology according to what your course asks for, and every module is 27 multiple-choice questions in 40 minutes. Questions offer between four and eight options, one of which is correct. There is no negative marking and no calculator.

Deadlines, and what to watch. The dates change every year and are published only by the test provider, so read them there and put them in a diary the day you do. The shape does not change: you register yourself, on the provider's site, some weeks before the sitting; registration closes well before test day and there is no late entry; the test sits once in the autumn, before university decisions are made; your result goes to the universities you name at registration. If you need access arrangements, they are requested at registration with evidence, not on the day. On the day you need photographic identification that matches the name you registered under. The three things that cost candidates a sitting are registering late, registering under a name that does not match their identification, and assuming a school will register for them.

Mathematics 1

M1 · Units and compound measures

- Work with quantities in the standard units for mass, length, time and money, and in the standard units used for other kinds of measure.
- Calculate and interpret compound measures: speed, pay rates, price per unit, density and pressure. A compound unit tells you the calculation: km/h means kilometres divided by hours.
- Carry decimal values through these unit calculations whenever the situation calls for them. Decimal hours are not minutes: 2.25 hours is 2 h 15 min.
- Convert between related units of time, length, area, volume or capacity and mass, and between compound units such as speed, pay rates, prices, density and pressure, whether the quantities are plain numbers or algebraic expressions. Area conversions square the factor and volume conversions cube it.

M2 · Number

- Put a mixed list of whole numbers, decimals and fractions, negatives included, into size order. Negative numbers reverse the order: minus seven is less than minus three.
- Read and use the comparison symbols correctly: equals, not equal to, less than, greater than, less than or equal to, and greater than or equal to, each the right way round. The open end of the inequality sign faces the larger number.
- Add, subtract, multiply and divide whole numbers, decimals, proper and improper fractions and mixed numbers, with negatives in the mix as well as positives. Convert mixed numbers to improper fractions before multiplying or dividing.
- Know and use the language of primes, factors (also called divisors), multiples, common factors and common multiples. Find a highest common factor and a lowest common multiple, and write a number as a product of its prime factors. HCF uses the lowest power of each shared prime, LCM the highest.
- Strip out common factors before you multiply or divide, so that a calculation or an algebraic expression becomes shorter to handle. Cancel only factors of the whole top and bottom, never single terms.
- Apply the agreed order of operations: brackets first, then powers, then multiplication and division, then addition and subtraction. Multiplication and division share a rank, so work them left to right.
- List possibilities in an organised way so that none is missed or counted twice, and use the multiplication principle: m ways for one choice and n ways for a second give m times n ways for the pair. Multiply the counts when choices are made together, add when they are alternatives.
- Know what square, cube, square root and cube root mean, and remember that every positive number has two square roots, one positive and one negative. The root sign on its own means the positive root only
- Apply the rules of indices to tidy up numerical expressions, multiplying and dividing powers whose exponents may be whole, fractional or negative. A negative index means take the reciprocal, not make the answer negative
- Read a number given in standard form, put several such numbers in order of size, and do arithmetic with them. Compare the powers of ten first, then the leading digits
- Know the shape of standard form: a number from 1 up to but not including 10, multiplied by a power of ten whose exponent is an integer, negative ones included. Adjust the power whenever the leading part drifts outside 1 to 10
- Turn a terminating decimal into a percentage or a fraction and back again, in any direction between the three forms. Simplify the fraction fully: 0.35 is 7/20, not 35/100
- Write a recurring decimal as an exact fraction, and turn a fraction back into the recurring decimal it produces. Multiply by the power of ten that matches the repeating block
- Switch freely between fraction, decimal and percentage forms in the middle of a calculation, choosing whichever makes the arithmetic easiest. A percentage is out of 100, so 12.5% is 0.125
- Recognise when two fractions stand for the same value, and produce an equivalent fraction by scaling numerator and denominator by the same factor. Adding the same number to top and bottom is not equivalence
- Calculate exactly, leaving answers in terms of fractions, surds or multiples of pi rather than decimals. Simplify a surd by taking out any square factor, so root 12 becomes 2 root 3, and clear a surd from a denominator. Rationalise by multiplying top and bottom, never the denominator alone
- Find the lower and upper bounds between which a rounded measurement must lie, and carry those bounds through a calculation, including in a word problem. For a quotient, the upper bound divides by the lower bound
- Round a number or a measurement to a sensible precision, whether that is a stated count of decimal places or of significant figures. Leading zeros are not significant: 0.0305 has three significant figures

- Write the interval a value could have lain in before it was rounded or truncated, using inequality signs, for instance 2.45 up to but not including 2.55 for a value that rounded to 2.5. The top end takes a strict less-than, the bottom end a less-than-or-equal
- Round the numbers in a calculation to convenient values and work out a rough answer for it. Round each number before calculating, not the final answer
- Estimate the value of an expression containing pi or a surd by swapping each for a nearby simple number. Take pi as about 3 and root 2 as about 1.4

M3 · Ratio and proportion

- Use a scale factor to move between lengths on a scale drawing or map and the real lengths they represent, in either direction. Match units before applying the scale: cm on the map, km on the ground
- Write one amount as a fraction of a second amount, whether the result comes out below 1 or above it. Put both amounts in the same units before forming the fraction
- Read and write a ratio in the colon form, such as 3 : 5, and know what each part of it stands for. Order matters: 3 : 5 is not 5 : 3
- Split an amount into two or more shares in a stated ratio, finding the size of each share. Divide by the total of the ratio parts, not by their count
- Given the two pieces an amount has been split into, write the split as a ratio. Give the ratio of part to part, not part to whole
- Use ratio in practical settings: converting between units or currencies, comparing amounts, scaling up or down, mixing components, and describing concentrations. Mixing 1 : 4 gives five parts in total, not four
- When one quantity is a fixed multiple of another, capture that link as a ratio or as a fraction. A ratio of 2 : 5 means multiply by 2.5, not add 3
- Turn a ratio into the fraction of the whole that each part takes, and recognise that two quantities kept in a fixed ratio are joined by a straight line through the origin, y = kx. In 2:3 the first part is 2/5 of the whole, not 2/3
- Spot the fraction hidden in a ratio problem, whether one part as a fraction of another or as a fraction of the whole, and switch between ratio and fraction to reach the answer. A fraction compares part to whole; a ratio compares part to part
- Know that a percentage means so many parts out of a hundred. Write a percentage, or a percentage change, as a decimal or a fraction and apply it as a multiplier: up 15% means multiply by 1.15, down 15% means multiply by 0.85. A 20% fall followed by a 20% rise does not restore the original
- Give one amount as a percentage of another: divide the first by the second, then multiply by a hundred. Divide by the quantity you are comparing against, not the other way round
- Recognise when one quantity is a fixed multiple of another, y = kx, and when their product is fixed, y = k/x, and write the matching equation from the values you are given. Inverse proportion keeps the product constant, not the sum
- Tell direct from inverse proportion by the shape of a graph, a straight line through the origin for one and a curve that never touches either axis for the other, and read the constant off it. A straight line that misses the origin is not direct proportion
- Build the proportion equation from the information in a question, including one where a quantity varies with a power of another such as its square or its square root, then use it to find missing values and say what its constant means in context. Find the constant from the given pair before answering anything else
- Write how two lengths, two areas or two volumes compare as a ratio in its simplest form. Convert to the same units before you write the ratio
- Connect ratio to similar shapes: matching sides of similar figures are all in one ratio, so a ratio found from one pair gives every other pair. Pair corresponding sides, not any two sides that happen to be given
- Treat sine, cosine and tangent as side ratios shared by every similar right-angled triangle, and use scale factors between similar shapes: k for lengths, k squared for areas, k cubed for volumes. Area scales by the square of the length factor, volume by the cube
- Model something that grows or shrinks by a fixed percentage each step as a multiplier raised to a power, solve for the unknown, and say what the answer means in the situation. Repeated percentage change multiplies each step; do not add the percentages
- Handle compound interest, where each period's interest is added before the next period's is worked out, and follow any rule that feeds each result back in to produce the next one. Compound interest builds on the new total, not the starting sum

M4 · Algebra

- Read and write algebra in its standard shorthand, so that ab stands for a multiplied by b and every compressed form is understood before you work with it. Letters written side by side are multiplied, not added

- Know that 3y is the shorthand for adding three lots of y together, or for three multiplied by y. 3y means 3 times y, not 3 plus y
- Know that a squared is the shorthand for a multiplied by itself. a squared is a times a, not 2a
- Know that a cubed is the shorthand for three copies of a multiplied together. a cubed is not 3a: the power counts factors, not terms
- Know that a squared b is the shorthand for a times a times b: the square belongs to a alone. The index applies only to the letter it sits on, not to b
- Apply the rules for powers to algebraic terms: add the indices when you multiply, subtract them when you divide, and do this with whole-number, fractional and negative indices alike. Index laws need the same base; x squared times y cubed does not combine
- Put given numbers into an expression or a formula, including one taken from science, and evaluate it correctly. Substituting a negative number needs brackets: the square of minus 3 is 9
- Know the difference between an expression, an equation, a formula, an identity, an inequality, a term and a factor, and use each word correctly. An identity holds for every value; an equation only for some
- Gather terms that match, multiply out a bracket by the single term in front of it, pull a shared factor out of an expression, and multiply out a product of two or more brackets such as $(x + 2)(x - 3)$. A minus sign before a bracket flips every sign inside it
- Turn a quadratic whose x^2 coefficient is 1, so $x^2 + bx + c$, back into a product of two brackets. The two numbers must multiply to c and add to b, signs included
- Spot one square subtracted from another, whether $x^2 - 9$ or $4x^2 - 25$, and write it as $(a + b)(a - b)$. A sum of two squares does not factorise this way
- Factorise a quadratic whose x^2 coefficient is not 1, so $ax^2 + bx + c$, into two brackets whose x terms may carry coefficients of their own. The number pair now multiplies to a times c, not to c
- Tidy an expression that mixes added terms, multiplied terms and powers into its simplest form. x times x is x^2, but x plus x is 2x
- Apply the index laws to algebraic terms: add the powers when multiplying like bases, subtract them when dividing, and multiply them when raising a power to a power. Same base only: x^2 times y^3 does not combine
- Reduce an algebraic fraction by cancelling a factor common to top and bottom, factorising first when the shared factor is not yet visible. Cancel whole factors only, never a single term inside a sum
- Add, subtract, multiply and divide algebraic fractions. Adding needs a common denominator; multiplying does not
- Rewrite a formula so that a different letter stands alone on one side. If the new subject appears twice, gather its terms and factorise it out
- Tell apart an equation, true only for particular values of the letter, from an identity, true whatever value the letter takes. The three-bar sign ≡ marks an identity: it holds for every value
- Prove two expressions are the same by rewriting one step by step until it matches the other. Checking one or two values is not a proof
- Plot and read off points with any mix of positive and negative x and y values. The x coordinate comes first: along before up
- Read off the gradient and the intercepts of a straight line $y = mx + c$, from a sketch or from the equation, and explain what each one means. Get y on its own first, or the gradient you read is wrong
- Spot when two lines are parallel, sharing one gradient, and when they are perpendicular, their gradients multiplying to give -1. A perpendicular gradient is the negative reciprocal, not just the negative
- From the graph of a quadratic, read off where it crosses the x axis, where it cuts the y axis and where it turns, and say what each of those points means. The turning point sits midway between the roots, not at the y intercept
- Find the roots by solving the quadratic, and find the turning point by writing it in completed square form and reading off the vertex. From $(x + p)^2 + q$ the vertex is $(-p, q)$, sign flipped
- Recognise a straight line graph, sketch one from its equation, and read what it shows. $x = 3$ is a vertical line, not a horizontal one
- Recognise the parabola of a quadratic, sketch it from its equation, and interpret its features. A negative x^2 coefficient turns the parabola upside down
- Recognise the S-shaped curve of a simple cubic such as $y = x^3$, sketch it, and interpret its graph. A cubic's two ends point opposite ways, one up and one down
- Recognise and sketch $y = 1/x$, two branches in opposite quadrants that approach the axes without touching them, and read what the graph shows. There is no point at $x = 0$. The curve never meets either axis: no root, no y intercept
- Recognise the exponential curve y = k^x, where k is a positive constant, sketch it, and interpret its graph. Every y = k^x graph passes through (0, 1), since $k^0 = 1$

- Read a graph set in a real situation, whether reciprocal, exponential or of an unfamiliar shape, and use it to get an approximate answer, for instance to a simple problem about motion. Check what each axis measures before reading a value off the curve.
- Find, exactly or by estimating, the gradient of a graph and the area between the graph and the horizontal axis. For a curve, the gradient at a point is that of the tangent there.
- Do the same for quadratic and other curved graphs, and say what the gradient or area means in context: speed on a distance-time graph, distance covered on a speed-time graph, or a rate or total on a graph about money. Area under a speed-time graph is distance; its gradient is acceleration, not speed.
- Turn a problem into a simple equation, then solve it either by algebra or by drawing and reading a graph. Say which coordinate of the crossing point the question actually wants.
- Solve a pair of simultaneous equations in two unknowns, by algebra or by finding where their graphs cross. Substitute your pair back into both equations, not just one.
- Be ready for a pair in which one equation is linear and the other quadratic: substitute the linear one into the quadratic and expect up to two solution pairs. Each x-value needs its own y-value; pair them, never mix them.
- Find the values of the unknown that make a quadratic equation true. A quadratic can have two, one or no real solutions: give them all.
- When the equation is not already set equal to zero, rearrange it so it is, then solve by whichever method fits: factorising, completing the square, or the quadratic formula. Only factorise once one side is zero: $(x+1)(x+2)=6$ tells you nothing yet.
- Memorise the quadratic formula: x equals minus b, plus or minus the square root of b squared minus 4ac, all divided by 2a. The whole of minus b plus or minus the root sits over 2a.
- Solve a linear inequality, whether it involves one variable or two. Multiplying or dividing by a negative flips the inequality sign.
- Show the set of values that satisfy an inequality in three ways: marked on a number line, shaded on a graph, or described in words. An open circle means the endpoint is excluded; a filled one includes it.
- Write out the terms of a sequence from either kind of rule: one that tells you how to get each term from the one before, or one that gives any term straight from its position. The first term has position 1, not 0, when you substitute.
- Find a formula for the nth term of a sequence whose terms change by the same amount each step, and of one whose second differences are constant. Constant second difference d gives a leading coefficient of d over 2.

M5 · Geometry

- Know and use the standard words and symbols of geometry: point and line, line segment, edge and vertex, plane, parallel and perpendicular lines, right angle, and an angle subtended at a point. Perpendicular lines meet at a right angle; crossing alone is not enough.
- Recall and use the angle facts: angles round a point total 360 degrees, angles along a straight line total 180 degrees, perpendicular lines meet at 90 degrees, and vertically opposite angles are equal. Only vertically opposite angles are equal; the neighbours add to 180 degrees.
- Understand and use the angle facts for a pair of parallel lines cut by another line, for lines that intersect, and for the angles inside a triangle or a quadrilateral. Work out and use the total of the interior angles of any polygon, and the total of its exterior angles. Alternate and corresponding angles are equal; co-interior angles add to 180 degrees.
- Know how each special quadrilateral is defined, derive its properties from that definition, and use them to solve problems.
- Cover all six named shapes: the square, the rectangle, the parallelogram, the trapezium, the kite and the rhombus. Diagonals: equal in a rectangle, perpendicular in a rhombus, both in a square.
- Do the same for the kinds of triangle and for other plane shapes: start from the definition, derive the properties that follow, and use them in problems, naming each shape and property correctly.
- Know the four tests that prove two triangles congruent, SSS, SAS, ASA and RHS, and use them to show or decide that two triangles are identical in shape and size.
- Use what you know about angles, congruent and similar triangles, and the special quadrilaterals to find unknown angles and lengths, or to justify a claim about them. Similar triangles give equal angles and proportional sides, not equal sides.
- Recognise when two shapes are congruent or similar, explain why, and draw a congruent or similar copy of a given shape. Congruent means identical in size as well as shape; similar allows scaling.
- Carry out translations, rotations, reflections and enlargements on a coordinate grid, describe each one fully, and use them to show that two shapes are congruent or similar. A full description of a rotation needs centre, angle and direction.

- Enlarge a shape using a scale factor between 0 and 1, which makes it smaller, or a negative scale factor, which puts the image on the other side of the centre, rotated through a half turn. Do not draw a negative enlargement on the same side as the object.
- Use Pythagoras' theorem, $a^2 + b^2 = c^2$, to find a missing side of a right-angled triangle, and extend it to three dimensions, applying it twice to reach lengths such as the long diagonal of a cuboid. The hypotenuse is opposite the right angle: subtract when finding a shorter side.
- Know what each part of a circle is called and pick it out on a diagram: centre, radius, diameter, circumference, chord, arc, sector, segment and tangent. Sector: between two radii. Segment: cut off by a chord.
- Call the smaller of the two arcs, sectors or segments the minor one and the larger the major one. For a major sector, use 360 minus the given angle.
- Use the standard circle theorems, the ones about angles, tangents, chords and radii, to find unknown angles and lengths in a circle diagram. A tangent and the radius to its point of contact meet at 90 degrees.
- Use the theorems to prove further results, starting from the fact that an arc subtends an angle at the centre equal to twice the angle it subtends at the circumference. Both angles must stand on the same arc before you double.
- Answer questions about shapes drawn on a coordinate grid, working from the coordinates of the vertices to find lengths, midpoints and areas, or the position of a missing vertex. Sketch the points before naming the shape; a tilted square looks like a diamond.
- Know the words face, surface, edge and vertex, and be able to say how many of each a cube, cuboid, prism, pyramid, cylinder, cone, sphere or hemisphere has. Count the base as a face; a square pyramid has five, not four.
- Read a plan, the view from directly above, and the front and side elevations of a solid, and deduce from them the shape of the solid. A plan shows the footprint only; height appears in the elevations.
- Convert between a length measured on a map or scale drawing and the real distance it stands for, using the scale whether it is given as a ratio or in words. A ratio scale gives centimetres on the ground; convert to kilometres afterwards.
- Find the area of a triangle, a parallelogram or a trapezium from its formula, taking the height at right angles to the base. The height is perpendicular to the base, not the slanted side.
- Find the volume of a cuboid or any other right prism as the area of the cross-section multiplied by the length. The cross-section is the face that stays the same along the length.
- Know that a circle's circumference is $2\pi r$, which is the same as πd, and pick whichever form matches the measurement you are given. With the diameter given, use πd; $2\pi d$ doubles the answer.
- Know that a circle's area is πr^2, and halve the diameter first if that is what the question gives you. Square the radius, not the diameter; using d gives four times the area.
- Know that a cylinder's volume is $\pi r^2 h$, the area of the circular end times the height. You need not memorise the formulae for spheres, pyramids or cones: any question needing one supplies it. Learn $\pi r^2 h$; do not rely on the question to supply it.
- Find the length of an arc or the area of a sector as the matching fraction, angle over 360, of the whole circumference or area, and reverse the working to recover the angle. Arc length uses the circumference, $2\pi r$; sector area uses πr^2.
- Spot congruent or similar shapes inside a diagram, match up the corresponding sides and angles, and use the match to find an unknown length or angle. Corresponding sides lie opposite equal angles, whichever way the shapes are turned.
- For similar shapes and solids with length scale factor k, areas scale by k^2 and volumes by k^3; work in either direction, from a given area or volume ratio back to the length ratio. A volume ratio of 8 means a length ratio of 2, not 8.
- Know sine, cosine and tangent as ratios of the sides of a right-angled triangle, and use them to find a missing side or a missing angle. Do the same in a general triangle or a solid shape wherever a right-angled triangle can be found inside it. Opposite and adjacent are named from the angle in use, not fixed
- Add and subtract vectors, multiply a vector by a number, and move between a vector drawn as an arrow and one written as a column. The vector from A to B is b minus a, not a minus b
- Express the lines in a figure as combinations of given vectors and reason from them to prove a geometric result, such as showing two lines are parallel. Collinear needs parallel vectors that also share a point, not parallel alone

M6 · Statistics

- Read and draw the displays used for data in categories: pie charts, bar charts, pictograms, frequency tables and two-way tables. A pie sector shows a share of the total, not a count
- Read and draw a vertical line chart for discrete data that has not been put into groups. Never join the tops of the lines: values between them do not exist
- Read and draw histograms for grouped or continuous data, with classes of equal width and with classes of different widths. With unequal widths the height is frequency density: area, not height, gives frequency

- Read and draw a cumulative frequency graph for grouped or continuous data, and be able to say which of these diagrams suits a given set of data. Plot cumulative frequency at the upper bound of each class, not the midpoint
- Find the mean, the median, the mode and the range of a list of raw values. Order the values first for the median; an even count averages the middle two
- For grouped data, pick out the class with the highest frequency. Answer with the class interval itself, not the frequency it holds
- From a grouped frequency table, estimate the mean with class midpoints, and estimate the median and the range. Be able to say why these are estimates: grouping hides the actual values. Use the midpoint of each class, not its upper bound, for the mean
- Plot and read a scatter graph of paired measurements, and describe what the pattern of points shows about the two variables.
- Say whether a scatter graph shows positive, negative or no correlation, and remember that a correlation on its own does not show that one variable causes the other. Strong correlation still proves nothing about cause; a third factor may drive both
- Draw a straight line by eye through a scatter of points so that it follows the trend, with the points spread evenly on either side. The line need not pass through the origin or through any point
- Use the line of best fit to estimate a value inside the range of the data, and to extend the apparent trend to a value outside it. Know that the second is less reliable, since nothing in the data shows the pattern continues beyond it. Reading beyond the data assumes the trend continues, which it may not

M7 · Probability

- Record how often each outcome of an experiment occurred in a table or a frequency tree, and read counts and proportions from it. A frequency tree holds counts at each branch, not probabilities
- Using the idea that a fair device gives random, equally likely results, work out how many times an outcome should turn up over a set number of trials. Expected frequency is probability times number of trials, not the probability itself
- Know that running the same experiment again may well give a different result, since each outcome is random. An expected frequency of 50 does not mean exactly 50 will occur
- Link the fraction of trials an outcome is expected to take up with its theoretical probability, and describe likelihood in proper terms on the scale from 0, impossible, to 1, certain. Relative frequency approaches the theoretical value only over many trials
- Use the fact that the probabilities of all the possible outcomes of an experiment add up to 1, for example to find a missing probability in a table. The rule needs every outcome listed; a partial list does not sum to 1
- Use the fact that when events cannot happen together and between them cover every possibility, their probabilities add up to 1. Overlapping events break the rule: their probabilities can total more than 1
- List every member of a set, or of a combination of sets, in an orderly way, choosing whichever of a table, a grid, a Venn diagram or a tree diagram fits the problem. The overlap is counted in both sets: adding the totals double counts it.
- You do not need formal set notation: do the work in words and with those diagrams, not with the symbols for union, intersection or complement.
- Set out every outcome of a single trial, or of two or more trials combined, where each outcome is equally likely, for example as a list or a grid. Two dice give 36 ordered outcomes, not 11 totals or 21 pairs.
- Read a probability off that space by counting the outcomes that fit the event and dividing by the total number of outcomes. Only divide by the total when every outcome is equally likely.
- Add the probabilities of two events that cannot both happen to get the chance of one or the other; multiply to get the chance of both, using for the second event its probability once the first has happened. Know what a conditional probability is: the chance of an event given that another has occurred. After drawing without replacement, the second fraction changes: do not reuse the first.
- Turn the probabilities into expected numbers out of a chosen total, lay them out in a two-way table, a tree diagram or a Venn diagram, and read a conditional probability as a fraction of the one group the condition picks out. Be able to say in words what that figure means. Divide by the total of the ’given’ group, not the whole table.

Mathematics 2

MM1 · Algebra and functions

- Use the rules for multiplying and dividing powers and for raising a power to a power when the index is zero, negative or a fraction, not only a positive whole number, and read a fractional index as a root. A negative index means a reciprocal, not a negative answer.
- Handle square roots that are not whole numbers as exact quantities: add and subtract like roots, multiply and divide roots, and split the root of a product into a product of roots. Only products and quotients split into separate roots, never sums.
- Reduce an expression containing roots to its simplest form: take square factors out of a root, collect like roots, and clear a root from the bottom of a fraction by multiplying top and bottom by a suitable surd. For a denominator like $2 - \sqrt{3}$, multiply by $2 + \sqrt{3}$.
- Sketch the graph of a quadratic from its equation, and read from the equation or the graph which way it opens, where it meets the axes and where its turning point is. In $(x + p)^2 + q$ the turning point is at $x = -p$, not p.
- Work out $b^2 - 4ac$ and use its sign to say whether the quadratic has two distinct real roots, one repeated root or none, and so whether its graph crosses, touches or misses the x-axis.
- Solve a quadratic equation by factorising, by completing the square or by the formula, picking the method that suits the numbers in front of you. Rearrange to equal zero before factorising; dividing by x loses a root.
- Solve a pair of simultaneous equations exactly by rearranging one to give an unknown in terms of the other and substituting into the second, in particular when one equation is a straight line and the other is a quadratic. Pair each x with its own y; the answer is two points, not four.
- Find the set of values of x that satisfies an inequality, whether the expression is linear or quadratic, and give the answer as a single range or as two separate ranges. Dividing by a negative reverses the sign; sketch a quadratic before choosing the region.
- Multiply out brackets in a polynomial expression, including products of several brackets, and gather the terms of equal power so that the result is one tidy polynomial. $(x + 3)^2$ has a middle term $6x$; it is not $x^2 + 9$.
- Factorise a polynomial, and divide one by a linear or a quadratic expression, including divisors such as $2x + 1$ or $2x^2 + 3x + 1$ whose leading coefficient is not one, to find the quotient and any remainder. Use the Factor Theorem to test for a factor and the Remainder Theorem to find a remainder without dividing. For factor $2x + 1$, test $x = -\frac{1}{2}$, not $x = -1$.
- Know, without formal definitions, that a function sends each input to exactly one output, so several inputs may share an output but one input never has two. Many-to-one means several x for one y, not several y for one x.
- Recognise the functions that come up most often and know how each one behaves, including the shape of its graph and the values it can and cannot take.
- Know that $\sqrt{x}$ always means the positive square root, so $\sqrt{9}$ is 3 and never -3, and be familiar with the modulus function $|x|$, which makes a negative input positive and leaves a positive one alone.

MM2 · Sequences and series

- Work with a sequence whether it is defined by an expression for its general term or by a rule that produces each term from the one before, such as $x_{n+1} = f(x_n)$, and find terms of either kind. Substitute the previous term, not n, into the recurrence rule.
- Add the whole numbers from 1 to n in one step with n(n + 1)/2, and recognise this as the simplest arithmetic series, first term 1 and common difference 1. It starts at 1: subtract the early terms if the run begins later
- Add up the first n terms of a geometric sequence using a(1 - r^n)/(1 - r), and work backwards from a known total to find n, a or r. Count the terms: the power in the formula is n, not n - 1
- Find the limiting total of a geometric series that goes on for ever, a/(1 - r), when its terms shrink towards zero. Divide by 1 - r, not r - 1, or the sign comes out wrong
- Test whether a never-ending geometric series settles to a total at all: it does only when the ratio r lies strictly between -1 and 1, so state or apply that condition where a question turns on it. Negative ratios count too: it is the size of r, not its sign
- Expand (1 + x) raised to a positive whole-number power, and likewise a bracket of the shape (a + f(x)) to such a power where f(x) is something simple. Read and use n! and the nCr notation for the coefficients. Raise the whole term to the power: (2x)^3 is 8x^3, not 2x^3

MM3 · Coordinate geometry

- Write down a line's equation from its gradient m and any one point (x1, y1) on it: y - y1 equals m(x - x1). Substitute the point's coordinates for x1 and y1, not for x and y

- Work with a line in the general form ax + by + c set equal to zero, including reading its gradient from it. Decide whether two lines are parallel (same gradient) or perpendicular (gradients multiplying to -1), and find a line's equation from whatever you are told about it. In the general form the gradient is -a/b: the minus is easily dropped
- Read a circle's centre (a, b) and radius r straight from (x - a)^2 + (y - b)^2 = r^2, and write that equation down from a given centre and radius. The right-hand side is r squared, so 25 there means radius 5
- Recognise a circle written out in full as x^2 + y^2 plus terms in x, in y and a constant, all equal to zero, and complete the square in x and in y to recover its centre and radius. Halve the x and y coefficients when completing the square, then square that half
- Use the fact that a line from the centre meeting a chord at right angles cuts the chord in half, so the radius, half the chord and the centre's distance from the chord form a right-angled triangle. Use half the chord in Pythagoras with the radius, not the whole chord
- Use the fact that a tangent and the radius to its point of contact meet at a right angle, so the gradient of the tangent is the negative reciprocal of that of the radius. Take the radius gradient from centre to contact point, then flip and negate it

MM4 · Trigonometry

- Use the sine rule and the cosine rule to find missing sides and angles, and find a triangle's area as half ab sin C. Know that the sine rule can give two possible triangles when two sides and a non-included angle are known, and expect these problems in three dimensions as well as two. Sine gives two angles: check whether 180 minus your answer also fits
- Work in radians to find an arc's length as r times theta, a sector's area as half r squared theta, and a segment's area by taking the triangle away from the sector. These formulas need the angle in radians; a degree value gives nonsense
- Know the exact values of sin, cos and tan at 0, 30, 45, 60 and 90 degrees from memory, in surd form where needed. Do not swap the 30 and 60 degree values: sin 30 is one half
- Treat sin, cos and tan as functions that take any angle, negative or beyond 90 degrees included, not only as ratios inside a right-angled triangle. Sin and cos give a value for every angle; tan has none at 90 degrees or at any odd multiple of it. Sine and cosine never exceed 1 in size; tangent can be any value
- Sketch the graphs of sin, cos and tan and use their symmetry and their repeat: sin and cos repeat every 360 degrees and tan every 180, and the shapes give relations such as cos(-x) = cos x and sin(180 - x) = sin x. tan repeats every 180 degrees, not 360, with a break at 90
- Know that tan of an angle is its sine divided by its cosine, and swap between tan and sin over cos wherever that simplifies an equation or an identity. It is sine over cosine, not cosine over sine
- Know that sin squared plus cos squared of one angle equals 1, and use it to trade sin^2 for 1 - cos^2, or the reverse, when solving or simplifying. Replace sin^2 x by 1 - cos^2 x, not by 1 - cos x
- Find every solution of a simple trig equation inside a stated interval, using the identities above where they help, including equations in a multiple or shifted angle such as 2x + pi/3 and ones with sin or cos squared. Change the interval to match the bracket before listing solutions for 2x + pi/3

MM5 · Exponentials and logarithms

- Sketch y = a^x for a simple positive base a: it passes through (0, 1), rises when a is above 1, falls towards zero when a is between 0 and 1, and never crosses the x-axis. Every such graph passes through (0, 1), not the origin
- Convert between index form and log form: a raised to the power b equals c says the same thing as b equals the log of c to base a, so you can go either way. The exponent becomes the logarithm's value, not its base.
- Combine two logs with the same base that are being added into a single log of the product, and split the log of a product back into a sum. There is no rule for the log of a sum.
- Turn the difference of two logs to the same base into one log of a quotient, and split the log of a fraction back into a subtraction. Order matters: the log being subtracted goes in the denominator.
- Move a multiplier in front of a log inside as a power on the argument, and bring a power out as a multiplier. Know the special cases that come with the laws, including that the log of a to base a is one, because a to the power one is a. A power on the whole log, (log x)^k, is not k log x.
- Rewrite the log of a reciprocal as minus the log of the number itself, and rewrite minus the log of a number as the log of its reciprocal. A negative log means a reciprocal argument, not a negative argument.
- Solve an equation with the unknown in the exponent, a fixed base raised to some multiple of x equalling a number, and spot an equation that can be rewritten into that shape. log b divided by log a is not log of b over a.
- Do the algebra first when the equation is not yet in that shape: for instance, write 25 to the x as the square of 5 to the x, so the equation becomes a quadratic in that power. Solving for the substituted power is not the end: still find x.

MM6 · Differentiation

- Understand a derivative as the rate at which one quantity changes with respect to another, which on the graph is the tangent's gradient at that point. The rate of change at a point is f'(a), not f(a).
- Differentiate a derivative again to obtain the second derivative.
- Read and write both notations for a first derivative, dy/dx and f'(x), and for a second derivative, d^2y/dx^2 and f''(x), whichever form a question uses. In d^2y/dx^2 the 2 marks two differentiations, not a power.
- Differentiate any power of x, fractional and negative powers included, and expressions built by adding and subtracting such terms. Subtract one from a negative power too: x^-2 gives -2x^-3.
- Simplify first when an expression is not yet a sum of powers of x, for example by expanding a product or writing roots and fractions as powers, then differentiate term by term. Differentiating top and bottom of a fraction separately is wrong.
- Apply differentiation to find the gradient at a point, the equation of a tangent or a normal, and the stationary points of a curve, which here means maxima and minima only. Decide where a function is strictly increasing or strictly decreasing from the sign of its derivative, positive for increasing and negative for decreasing. A normal's gradient is minus the reciprocal of the tangent's, not just minus.

MM7 · Integration

- Link a definite integral to the region a curve encloses with an axis, and understand why the integral and the area can differ: where the curve dips below the axis the integral counts that part as negative, while an area is always positive. Integrating straight across an axis crossing cancels area; split at the roots.
- Integrate any power of x other than x to the minus one, both with limits and without, and expressions formed by adding or subtracting such terms. Divide by the new power, not the old, and remember the constant.
- Rewrite an expression as a sum of powers of x before integrating, for example by multiplying out a product or splitting a fraction into separate terms. A product cannot be integrated factor by factor: expand it first.
- Evaluate a definite integral between two limits by substituting the upper limit and then the lower limit into an antiderivative and subtracting the second from the first. Upper minus lower, and keep the sign when the lower value is negative.
- Know that the antiderivative you substitute into is any function whose derivative is the integrand, and see why this makes integration the reverse of differentiation. The constant of integration cancels in a definite integral; any antiderivative works.
- Add or subtract integrals that share the same limits by integrating the combined function, and join integrals of one function over ranges that meet end to end into a single integral across the whole range, allowing for a range that runs backwards. Reversing the limits reverses the sign of the integral.
- Estimate an area beneath a curve with the trapezium rule: divide the interval into strips of equal width and treat each strip's top edge as a straight line. With n strips there are n + 1 ordinates; the middle ones are doubled.
- Decide from the way the curve bends whether the trapezium rule gives too much or too little: the straight tops sit above a curve that bends upward, giving an overestimate, and below one that bends downward, giving an underestimate. Judge from how the curve bends, not from whether it rises or falls.
- Solve a differential equation in which dy/dx is given as a function of x alone, by integrating to find y. Include the constant of integration; a given point is what fixes it.

MM8 · Graphs and transformations

- Know the shape of each standard graph and sketch it from its equation: straight lines, quadratics and cubics, the modulus and square root functions, and the trigonometric, logarithmic and exponential ones. Exponential and log curves approach an axis but never touch it.
- Say how the curve y = f(x) shifts, stretches or reflects when it becomes y = af(x), y = f(x) + a, y = f(x + a) or y = f(ax), with a positive or negative, and when two or more of these are applied in turn. Read and use the notation f(g(x)) for one function applied after another. f(x + a) with positive a shifts the curve left, not right.
- Explain what changing m does to the line y = mx + c, and what changing c does. c is the y intercept, not where the line meets the x axis.
- Say how the parabola y = a(x + b)^2 + c changes when a, b or c is altered, including where its vertex ends up. The vertex sits at x = minus b, not at x = b.
- Differentiate, set the derivative equal to zero and solve to locate the maximum and minimum points of a curve; stationary points of inflexion are not required. Solving dy/dx = 0 gives only x; substitute back to get y.
- Use the sign of the derivative to decide over which ranges of x a curve is rising and over which it is falling. A curve is increasing where dy/dx is positive, whatever the sign of y.
- Find where a curve meets the axes by algebra: put x = 0 for the y intercept and solve y = 0 for the x intercepts. Solve y = 0 by factorising or the formula, never by reading a sketch.

- Know which numbers of real roots a polynomial of a given degree can have, for example a quadratic has none, one or two and a cubic has one, two or three. Every cubic has at least one real root; a quadratic may have none.
- Connect the algebra to the picture: a solution of $f(x) = 0$ is an x value where the curve $y = f(x)$ meets the x axis. A repeated root is where the curve touches the axis rather than crossing.
- Know that the points where two graphs cross are exactly the solutions of the pair of equations solved together, so counting crossings counts solutions. Where the curves merely touch, the simultaneous equations have a repeated solution.

Physics

P1 · Electricity

- Explain how rubbing two insulators together leaves each of them charged. Rubbing works on insulators because the charge cannot flow away again.
- State that an object becomes charged only by gaining or losing electrons: negative when it gains them, positive when it loses them. Electrons move, protons stay put: a positive charge is a shortage of electrons.
- State that two charges of the same sign push each other apart and two of opposite sign pull together. Attraction alone does not prove opposite charge; a neutral object is attracted too.
- Recognise the standard symbol for each component in a circuit diagram and draw it yourself: diode, switch, voltmeter, ammeter, variable and fixed resistor, light source, battery and single cell. In a diagram the ammeter sits in series and the voltmeter in parallel.
- Say what distinguishes alternating current from direct current: whether the current keeps one direction or keeps reversing. A cell or battery supplies direct current, never alternating.

P2 · Magnetism and electromagnetism

- Name the two poles of a magnet as north and south, and say which pairs of poles attract and which repel. Two north poles repel each other; a north and a south attract.
- Sketch the field lines round a bar magnet, with arrows leaving the north pole and entering the south. Field lines run north to south outside the magnet and never cross.
- Say what distinguishes a magnetically soft material from a magnetically hard one: how easily each is magnetised and how well it keeps its magnetism afterwards. Soft iron loses its magnetism when the field is removed; steel keeps it.
- State that a current flowing in a wire produces a magnetic field around it, and that a compass needle nearby turns as a result. The field exists only while current flows; stop the current and it vanishes.
- Sketch the field around a long straight wire (concentric circles) and around a coil or solenoid (the pattern of a bar magnet), and give the direction of each from the direction of the current. Reverse the current and every field line reverses; use the right-hand grip.
- Recognise that when a current flows along a wire lying in a magnetic field, the wire can be pushed by a force. No force arises when the current runs parallel to the field lines.
- Work out which way the force on the wire acts from the directions of the current and the field, using the left-hand rule. Reversing the current or the field reverses the force; reversing both does not.
- Say what decides how strong the force on the wire is, and how it changes when the current, the field strength or the length of wire in the field is increased.
- Explain why a voltage appears across a wire whenever it slices through field lines, and also whenever the magnetic field around it changes. No voltage is induced while the field is steady and the wire stays still.
- List the things that decide how big the induced voltage is, and say how changing each one affects it. The rate of cutting field lines matters, not the field strength alone.
- Say what decides the direction of the induced voltage, and show that reversing the wire's motion or the field reverses it.
- Tell a step-up transformer from a step-down one, and say what each does to the voltage. Step-up refers to the voltage, not to the current or the power.
- Relate the voltage on each coil to its number of turns: secondary voltage over primary voltage equals secondary turns over primary turns, and rearrange to find whichever is missing. Keep secondary over primary on both sides; a flipped ratio inverts the answer.

P3 · Mechanics

- Explain what separates a scalar from a vector, a vector having a direction as well as a size, and say which kind a given quantity is.
- Distinguish distance from displacement, and speed from velocity: in each pair the first is a scalar, the second is a vector with a stated direction. A body back at its start has zero displacement but non-zero distance.
- Use speed = distance divided by time, and velocity = change in displacement divided by time, in calculations, finding whichever quantity the question leaves out. Average speed is total distance over total time, not the mean of speeds.
- Recognise the kinds of force a question may name and what each one is: weight, the normal contact force, drag such as air resistance, friction, magnetic and electrostatic forces, thrust, upthrust, lift and tension. The normal contact force acts at right angles to the surface, not vertically.
- For each kind of force, weight, drag, friction and the rest, say what decides how big it is and which way it acts.

- Read information off a force-extension graph, such as the extension at a given force and where the line stops being straight. Check which axis carries force; the gradient's meaning flips with the axes.
- Tell elastic extension, which vanishes when the load is removed, from inelastic extension, which stays, and say what the elastic limit marks. The elastic limit and the limit of proportionality are not the same point.
- Apply Hooke's law, $F = kx$, linking force, spring constant and extension, and say what the limit of proportionality means for where the law stops holding. x is the extension beyond the natural length, not the total length.
- State Newton's first law and explain it: an object keeps still, or keeps moving steadily along a straight line, until a resultant force from outside acts on it. Constant velocity means zero resultant force, even when several forces act.
- Understand mass as the measure of how much a body resists having its motion changed. Inertia is a property of the mass, not a force acting on it.
- Explain how mass differs from weight, weight being the force gravity exerts on a mass, so it changes where gravity changes while mass does not. Weight is a force in newtons; kilograms measure mass, not weight.
- Use the gravitational field strength g, taken as 10 N/kg at the surface of the Earth, in calculations. Take g as 10 N/kg, not 9.8, unless the question says otherwise.
- Apply the link between weight, mass and gravitational field strength, $W = mg$, to find any one of the three from the other two. A mass given in grams must become kilograms before multiplying by g.
- Find an object's momentum by multiplying its mass by its velocity, $p = mv$, and rearrange the formula when a question gives you the other two quantities. Momentum has direction: give each velocity a sign and keep it.
- For motion along one straight line, set the total momentum before two bodies interact equal to the total afterwards, and solve for the unknown mass or velocity. Opposite directions take opposite signs; a rebound is a negative velocity.
- Calculate a force as the change in momentum it produces divided by the time over which it acts, and rearrange to find the change or the time. The change in momentum goes on top, not the final momentum.
- Work out the work done as the force multiplied by how far the object moves along the line of that force. Only the distance along the force's line counts, not the whole path.
- Recognise that doing work on something moves energy into it, so the work done equals the energy transferred.
- Find gravitational potential energy as mgh, taking h as the difference in height between where the object starts and where it ends. h is the height change, not the height above the ground.

P4 · Thermal physics

- Explain what makes a material a good thermal conductor or a good insulator, and name examples of each. A material that feels cold is a conductor, not a colder object.
- Say which factors set how fast heat conducts through something, the material, its thickness, its area and the temperature difference across it, and predict the effect of changing one of them. Doubling thickness halves the rate; doubling area doubles it.
- Explain why heating a liquid or gas makes it expand and so lowers its density, and use this in questions. The particles spread out; they do not get bigger.
- Explain how a warmer, less dense region of a fluid rises while cooler, denser fluid sinks to take its place, setting up a convection current. Heat does not rise; the less dense fluid does.
- Recognise that thermal radiation is a form of electromagnetic wave, sitting in the infrared part of the spectrum. Every object above absolute zero radiates, not just hot ones.
- Describe how objects absorb and emit thermal radiation, and use the balance between the two to say whether something heats up or cools down. An object absorbs and emits at the same time; the net decides.
- State what sets how fast a surface absorbs and emits thermal radiation, its colour, its finish, its area and its temperature, and predict which of two objects warms or cools faster. A dull dark surface absorbs fastest; a shiny one reflects most.
- Explain how an object's temperature rises when energy is transferred into it and falls when energy leaves it. A hotter object does not always hold more thermal energy.
- Use specific heat capacity as the energy supplied divided by the mass and by the temperature change, and rearrange it to find whichever quantity is missing. A temperature change is the same in kelvin and Celsius; never add 273.

P5 · Matter

- Describe how solids, liquids and gases differ in shape, volume, whether they flow and how easily they compress. Liquids and solids barely compress; only gases do.
- Picture each state as particles with a particular arrangement, spacing and motion, and use that picture to answer questions. Liquid particles touch each other; they are not widely spaced like a gas.

- Account for the behaviour of each state, why a solid keeps its shape, a liquid flows and a gas spreads out and can be squashed, using how its particles move, how far apart they sit and how strongly they attract one another. A gas compresses because of the gaps, not because its particles shrink.
- Explain gas pressure as the force per unit area from particles colliding with the container walls, and temperature as a measure of the particles' average kinetic energy. Pressure comes from collisions with the container, not collisions between particles.
- With the temperature held fixed, use the rule that pressure multiplied by volume stays the same: halve the volume and the pressure doubles. A graph of P against V curves; P against 1/V is straight.
- Explain what a substance's melting point and its boiling point are: the fixed temperatures at which it changes state, with the reading holding steady while the change happens. Temperature does not rise while a substance melts or boils
- Define the latent heat of fusion and the latent heat of vaporisation: the energy that melts or boils a substance with no change in its temperature. Fusion here means melting, not burning or nuclear fusion
- Calculate with energy = mass times specific latent heat, rearranging it to find whichever of the three quantities is missing. Do not multiply by a temperature change; that is specific heat capacity
- Use density = mass divided by volume, and rearrange it to find the mass or the volume when the density is given. A cubic metre is a million cubic centimetres, not a hundred
- Describe how a density is measured in practice: weigh the sample, then find its volume by measuring a regular solid's dimensions, by displacement for an irregular solid, or with a measuring cylinder for a liquid. Take the rise in water level, not the final reading
- Rank solids, liquids and gases by density and explain the order through how closely their particles are packed. Ice floats: a solid can be less dense than its own liquid
- Use pressure = force divided by area, in pascals when the force is in newtons and the area in square metres, and rearrange it for force or area. Convert cm^2 to m^2 by dividing by 10 000, not 100
- Use pressure = depth times density times g for the pressure a liquid exerts at a given depth below its surface, and rearrange it for depth or density. This is the liquid's contribution only; add atmospheric pressure for the total

P6 · Waves

- Explain that a wave carries energy from place to place while the particles of the medium only oscillate about fixed positions and are not carried along with it. The medium is not transported; a floating cork only bobs
- Tell a transverse wave from a longitudinal one by whether the oscillation is across the direction of travel or along it. Sound is longitudinal; light and every electromagnetic wave are transverse
- Name the peaks and troughs of a transverse wave and the compressions and rarefactions of a longitudinal one, and point to each on a diagram. One wavelength is compression to compression, not compression to rarefaction
- Describe what happens when a wave meets a surface and is turned back, and show the reflected rays or wavefronts on a diagram. Wavefronts keep the same spacing after reflection
- Explain why a wave changes direction when it passes into a medium where it travels at a different speed, and sketch the bent ray or the turned wavefronts. Along the normal there is no bending, though the speed still changes
- State which of a wave's speed, wavelength, frequency and direction change on reflection and which on refraction, and which stay fixed in each case. Frequency never changes at a boundary; wavelength moves with the speed
- Construct and read a ray diagram for a plane mirror, using it to locate the virtual image and to read off its position, size and orientation. The image is laterally inverted, not upside down, and virtual
- Apply the rule that a ray leaves a reflecting surface at the same angle to the normal as it arrived, and use it to find missing angles in a diagram. Measure both angles from the normal, not from the mirror surface
- Draw and read a ray diagram for light crossing a flat boundary between two media, bending towards the normal on entering the optically denser one and away on leaving it. Towards the normal when slowing down, away when speeding up
- Explain how a vibrating object makes a sound: it pushes and pulls on the medium around it, sending out a train of compressions and rarefactions.
- Explain why sound cannot cross a vacuum: it travels only by passing vibrations from particle to particle, so there has to be matter for it to move through. Light needs no medium; sound does
- Connect a bigger amplitude to a louder sound and a higher frequency to a higher pitch, in words rather than through numbers. Amplitude sets loudness only; a taller trace is not a higher note
- Describe sound as a longitudinal wave, in which the particles of the medium move back and forth along the line the sound travels rather than across it.
- Know what an electromagnetic wave is: a transverse wave, and one that crosses a vacuum at the speed of light. Unlike sound, they need no medium and are not longitudinal

- Name the regions that make up the electromagnetic spectrum, beginning with radio waves. Order by wavelength runs opposite to order by frequency

P7 · Radioactivity

- Describe any atom as a set of protons, neutrons and electrons, and count how many of each a particular atom contains. Neutrons are the mass number less the atomic number, not the mass number
- Know the nuclear model, a tiny dense positive nucleus with the electrons well outside it and empty space between, and apply it to explain how atoms behave. Nearly all the mass sits in the nucleus, nearly all the volume outside it
- Give the charge and the mass of a proton, a neutron and an electron relative to one another. A neutron carries no charge yet has about the same mass as a proton
- Explain that radioactive emission comes from the nucleus, not the electrons, and that it happens because that nucleus is unstable. Radioactivity is a nuclear change, not a chemical one
- Explain that decay is random: nobody can say which nucleus will go next or when, only how likely a decay is over a given time. Random per nucleus, yet a large sample decays at a predictable rate
- Describe what alpha, beta and gamma radiation each consist of, and how the three differ from each other in nature, charge and mass. Beta particles are electrons, but they come from the nucleus, not the shells
- Rank alpha, beta and gamma by how far each gets through matter, from alpha, stopped by paper or skin, to gamma, which needs thick lead or concrete. Penetration runs opposite to ionisation: gamma travels furthest and ionises least
- Rank alpha, beta and gamma by how strongly each ionises the matter it passes through, alpha most and gamma least. Alpha's heavy double charge makes it the strongest ioniser despite its short range
- Describe in words, not numbers, what an electric or magnetic field does to each radiation: alpha and beta bend opposite ways, beta far more than alpha, gamma not at all, and say why their charge and mass make it so. Alpha bends less than beta despite double charge: its mass is far greater
- Read a decay curve, activity or undecayed nuclei against time, and read off where the graph shows the daughter product building up as the parent falls away. The product curve rises as the parent falls, and their sum stays constant
- Define half-life as the time for half the nuclei in a sample to decay, or equally for the activity to fall by half. Two half-lives leave a quarter, not none
- Carry out half-life calculations: from three of the starting amount, the amount left, the time elapsed and the half-life, find the fourth by counting half-lives. Three half-lives cut the amount to an eighth, not a third

Chemistry

C1 · Atomic structure

- Picture an atom as a tiny nucleus of protons and neutrons at its centre, with the electrons around it arranged in shells, also called energy levels. The nucleus is tiny; most of the atom is empty space.
- Give the relative mass and charge of a proton (1, +1), a neutron (1, 0) and an electron (negligible, -1), and use them to explain why almost all of an atom's mass is in its nucleus. A neutron has the same mass as a proton; only the charge differs.
- Define atomic number (the number of protons) and mass number (protons plus neutrons), and read both from an element's standard symbol. In the symbol the mass number is on top, the atomic number below.
- Work out how many protons, neutrons and electrons a given atom or ion contains from its atomic number, mass number and charge. Neutrons are the mass number minus the atomic number, never the mass number alone.
- From its atomic number, write the electron arrangement of any element from hydrogen to calcium, filling shells 2, 8, 8 in turn and giving the answer as a comma-separated list such as 2,8,8,1 for potassium. Electrons equal the atomic number for a neutral atom, not the mass number.
- Define isotopes: atoms of one element that share a proton count but differ in their number of neutrons, and so in their mass number. Isotopes share an atomic number; it is the mass number that differs.
- Use isotope data in whatever form a question gives it, including a mass spectrum, whose peaks show the mass of each isotope and how much of it there is. Peak height gives abundance; peak position gives the isotope's mass.
- Understand relative atomic mass (Ar) as the mean mass of an element's atoms, weighted by isotope abundance, on a scale where one carbon-12 atom counts as exactly 12. Ar is rarely a whole number, because it averages across isotopes.
- Calculate an Ar from isotope masses and abundances: multiply each mass by its abundance, add the results, then divide by the total abundance. Weight by abundance; do not take a plain average of the isotope masses.

C2 · The Periodic Table

- Know that a Period runs across the table as a row and a Group runs down it as a column. Row is Period, column is Group: the two get swapped.
- Know that the elements sit in order of rising atomic number. Ordering is by atomic number; a few elements are out of mass order.
- Say where the metals and non-metals sit: Group 1 holds the alkali metals, Group 2 the alkaline earth metals, Group 16 the common non-metals, Group 17 the halogens, Group 18 the noble gases, and the transition metals fill the central block. Group 16 uses the 1 to 18 numbering: oxygen and sulfur, not Group 6.
- Work out an atom's electron arrangement from its Group and Period, and place an atom in the table from its arrangement. Period number counts the shells; Group number counts outer electrons, not shells.
- Explain that elements sharing a Group react alike, and that reactivity rises down a Group of metals but falls down a Group of non-metals. The trend flips between metals and non-metals; halogens get less reactive going down.

C3 · Chemical reactions and equations

- Explain that a reaction makes new substances by reshuffling atoms and their electrons, while every nucleus survives untouched. Atoms are conserved in a reaction; do not confuse this with nuclear change.
- Give from memory the formula of a common ionic or covalent compound. Balance the charges in an ionic formula: magnesium chloride is MgCl2, not MgCl.
- Put the right state symbol after each formula in an equation: (s) solid, (l) liquid, (g) gas, (aq) dissolved in water. (aq) means dissolved in water; liquid water itself is (l).
- Build the equation for a reaction and balance it so each element has equal atoms on both sides. Balance by changing the numbers in front, never the subscripts inside a formula.
- Construct and balance ionic equations and half-equations as well, so that charge balances on both sides along with the atoms. Spectator ions drop out of an ionic equation; electrons appear in a half-equation.
- Recognise that many reactions can run backwards as well as forwards, so they stop short of using up the reactants. Reversible means both directions run at once, not that it later undoes itself.
- Know that in such a reaction only part of the starting material becomes product: in a closed system the mixture settles at equilibrium with reactants and products both present. At equilibrium both directions still run; the amounts stay steady, nothing has stopped.

C4 · Quantitative chemistry

- Add up the relative atomic masses, Ar, of every atom in a formula to find Mr, the relative formula mass. A subscript after a bracket multiplies every atom inside the bracket.
- Know that one mole of any substance holds a fixed count of particles, and that this count is Avogadro's number. Multiply moles by Avogadro's number for particles; divide particles by it for moles.
- Know that the mass of one mole is Ar or Mr written in grams, and convert a mass in grams into moles and moles back into grams. Moles equal mass divided by Mr, not Mr divided by mass.
- Carry out the same conversions when the mass is quoted in kilograms or tonnes, and know that the amount of any substance means its number of moles. A tonne is a million grams, a kilogram a thousand: convert before dividing.
- Work out the percentage of a compound's mass that each element contributes, from its formula and the Ar values you are given. Count every atom of the element in the formula before dividing by Mr.
- Know that an empirical formula shows the elements of a compound in their smallest whole-number ratio. Reduce the ratio fully: C2H4 is not empirical, CH2 is.
- Derive the empirical formula from whatever data you are given, for instance the percentage of the mass that each element contributes. Divide each mass or percentage by that element's Ar before comparing the ratios.
- From a balanced equation, use the mole ratio to find the mass of one substance from the known mass of another, whether reactant or product. Convert to moles, apply the ratio, convert back; never scale masses directly.
- Where the reactants are not supplied in the ratio the equation needs, identify which one runs out first and base the product mass on that one. Compare moles divided by coefficient, not raw mass, to find which runs out.
- Work backwards from measured reacting masses, or from gas volumes, to a mole ratio, and from that write the balanced equation. Masses must become moles first; only gas volumes give the ratio directly.
- Know that at a fixed temperature and pressure an ideal gas takes up the same volume for every mole, and that at room temperature and pressure this is 24 dm3. Use that figure to convert between the volume of a gas and its number of moles. 24 dm3 applies to gases only, and only at rtp.
- Know that concentration is quoted in moles per dm3 or in grams per dm3, and calculate it from the amount of solute, as moles or as mass, and the volume of the solution. Volumes given in cm3 must be divided by 1000 before use.
- Do titration calculations: combine the concentrations of the solutions, either given or worked out from data, with the mole ratio from the balanced equation. Apply the equation's mole ratio; the acid and base moles are often unequal.
- Find the percentage yield: use the balanced equation to predict the mass of product, then divide the mass actually obtained by that prediction and multiply by 100. Actual divided by predicted, never the reverse: yields cannot exceed 100 per cent.

C5 · Redox

- Know the simplest definition: a substance is oxidised when it gains oxygen and reduced when it loses oxygen. The substance losing its oxygen is the one being reduced.
- Treat oxidation and reduction as electron transfer: a species is oxidised when it loses electrons and reduced when it gains them, and apply this to reactions. Gains oxygen but loses electrons: both mean oxidised, do not mix the two.
- Assign an oxidation state to each atom in a simple inorganic compound, and use those states in your reasoning. States sum to zero in a compound, to the charge in an ion.
- Look at an equation and decide whether it shows oxidation alone, reduction alone, both together as redox, or neither. Neutralisation and precipitation move no electrons: they are neither oxidation nor reduction.
- Know that disproportionation is one species being oxidised and reduced in the same reaction, and pick out the reactions, or the species, where it happens. The same element must go up and down in oxidation state.
- Say what an oxidising agent and a reducing agent each do to the other reactant, and name which is which in a given equation. The oxidising agent is itself reduced: it takes the electrons.

C6 · Bonding and structure

- Define what an element, a compound and a mixture each are, and explain how the three differ from one another. A compound has fixed proportions; a mixture can have any composition.
- Know that atoms tend to react so that each ends up with the full outer shell of a Group 18 element, and that which kind of bond forms depends on which atoms are involved. Hydrogen is full at two electrons, not eight.
- Know that a metal atom hands electrons to a non-metal atom, leaving a positive ion and a negative ion, and that the pull between the opposite charges is what holds an ionic compound together. Metals lose electrons and become positive, not negative.

- Use an atom's electron configuration to predict the charge on its most stable ion, for an element in Group 1, 2, 16 or 17 and for aluminium.
- Know that a covalent bond is two atoms, nearly always both non-metals, holding a shared pair of electrons between them, and that more than one pair can be shared. Sharing is not transfer: no ions form in a covalent bond.
- Know that a covalent substance is made either of small separate molecules, such as water, ammonia or methane, or of one giant structure, such as diamond, graphite or silicon dioxide. The bonds inside a molecule stay strong even when the substance melts easily.
- Picture a solid metal as positive ions packed into a lattice, with the outer electrons free to move through the whole structure rather than tied to any one atom. The ions are positive because each atom has released its outer electrons.
- Know the physical properties metals typically share, above all melting point and conductivity, and explain each from the picture of ions in a sea of free electrons. Conduction is the free electrons moving; the ions stay put.
- Know that molecules attract one another through forces acting between them, and that it is these attractions that must be broken for a substance to melt or boil. Boiling water breaks the forces between molecules, not the O-H bonds.
- From a substance's structure and the type of bonding in it, predict and explain its physical properties, above all its melting point and whether it conducts electricity. Ionic solids conduct only when molten or dissolved, never as solids.

C7 · Groups 1, 17 and 18

- Know the physical properties and the chemical behaviour of three families: the alkali metals in Group 1, the halogens in Group 17 and the noble gases in Group 18. Group 1 metals are soft and low in density, unlike typical metals.
- Describe how the alkali metals change in reactivity and in their physical properties going down the group, and use that pattern to predict how a member you have not been told about behaves. Reactivity rises down Group 1, the opposite way to the halogens.
- Know the order in which lithium, sodium and potassium sit going down Group 1, so that the trend is applied the right way round. Lithium is at the top of the group, not partway down.
- Know which elements make up the halogens, Group 17, and treat them as one family whose members follow shared trends. Group 17 is the family older numbering called Group 7.
- State how reactivity and the physical properties shift from one halogen to the next down Group 17, and extend the pattern to predict the behaviour of any member. Down Group 17 reactivity falls while melting and boiling points rise.
- Know the order in which the halogens sit within Group 17, so that a trend can be read off in the right direction. The order downward is fluorine, chlorine, bromine, iodine.

C8 · Separation techniques

- Know that getting an element back out of a compound takes a chemical reaction, and that no physical method will do it. Filtering or distilling cannot split a compound into its elements.
- Know that a mixture is taken apart by physical methods, and that no chemical reaction is needed to do it. Separating a mixture changes no substance; each part keeps its identity.
- Cover the four cases: two liquids that mix, two that will not, a solid dissolved in a liquid, and a solid that stays undissolved, and know a physical method suited to each. Immiscible liquids settle into layers; miscible ones blend into a single liquid.
- Say which mixtures call for simple distillation, which for fractional distillation and which for paper chromatography. Close boiling points need fractional distillation, not simple.
- Calculate and interpret Rf values from a chromatogram, and know when a separating funnel, a centrifuge, dissolving or filtration is the right way to separate a mixture. Rf is spot distance over solvent front distance, and never exceeds one.
- Judge from a chromatogram whether a sample is pure: a single spot points to one substance, more than one spot to a mixture. Spot count tells you purity, matching Rf values tell you identity.

C9 · Acids and bases

- Define an acid in either of two ways: a substance that produces H+(aq) ions in water, or one that hands H+ to something else. Containing hydrogen is not enough: methane donates no H+.
- Describe how an acid reacts with a metal, a carbonate, a metal hydroxide and a metal oxide: each gives a salt, together with hydrogen, with water and carbon dioxide, or with water alone. Hydrogen comes only from the metal reaction, carbon dioxide only from carbonates.
- Define a base in either of two ways: a substance that produces OH-(aq) ions in water, or one that takes H+ from something else. A base need not contain OH: ammonia qualifies by accepting H+.

- Explain what strong, weak, dilute and concentrated mean and keep the pairs apart: strong or weak describes how fully a substance splits into ions in water, dilute or concentrated describes how much of it is dissolved in a given volume. Strong is not concentrated: a strong acid can be dilute.
- Know that some metal oxides and metal hydroxides react with water to give an alkaline solution. Non-metal oxides such as CO2 are acidic, not basic.
- Know that an acid and a base can react to neutralise each other, and that the reaction usually releases heat. Exothermic means the mixture gets warmer, so the temperature rises.

C10 · Rates of reaction

- Say whether a reaction gets faster or slower when you change the concentration, the temperature or the particle size, add a catalyst, or, for a gas, change the pressure. Smaller pieces mean more surface area, so the rate goes up, not down.
- Know that a rate can be followed by tracking how quickly a reactant is used up, how quickly a product forms, or how a physical property changes over time, and say which of these measurements suits a given reaction. Measuring mass loss only works when a gas escapes from the flask.
- Read a graph of amount against time for a reaction: the gradient at any point is the rate, and a flat line means the reaction has finished. Compare rates by gradient, not by how much product forms in the end.
- Explain any change in rate through collisions: particles react only when they collide with enough energy, so the rate rises when collisions happen more often or when a greater share of them have the energy needed to react. Heating makes collisions more energetic as well as more frequent; say both.
- Know that colliding particles react only if they carry at least a minimum energy, the activation energy (Ea), and pick out Ea on an energy level diagram as the rise from the reactants to the peak. Ea runs from the reactants up to the peak, not down to the products.
- Know that a catalyst is not consumed: the amount present at the start is still there when the reaction finishes. Never count the catalyst as a reactant when balancing the equation.
- Know that a catalyst is chemically the same substance after the reaction as before it. Chemically the same does not mean physically the same: appearance may change.
- Know that a catalyst works by opening a different pathway, a new mechanism, with a smaller activation energy than the uncatalysed route, and show this on an energy level diagram as a lower peak between the same reactant and product levels. It lowers the barrier, not the energy difference between reactants and products.

C11 · Energetics

- Tell exothermic from endothermic by the sign of the enthalpy change. Exothermic reactions give energy out and have a negative enthalpy change; endothermic reactions take energy in and have a positive one. Negative enthalpy change means exothermic: energy has left the reacting substances.
- Know that a reversible reaction which gives energy out in the forward direction takes energy in when it runs backwards, and the other way round. Reverse the equation and the sign of the enthalpy change flips too.
- Read an energy level diagram: pick out the energy levels of reactants and products, the enthalpy change between them, the activation energy, and whether energy is given out or taken in. The enthalpy change is products minus reactants, not the height of the peak.
- Work out the energy transferred in a calorimetry experiment from the mass being heated, its specific heat capacity and the temperature rise or fall. Use the mass of the water being heated, not the mass of the reactant.
- Know that breaking a bond takes energy in while making one gives energy out, and use a table of bond energies to work out the overall energy change of a reaction. Total bonds broken minus total bonds made: count every bond in every molecule.

C12 · Electrolysis

- Know the vocabulary: an electrode is the conductor dipped into the liquid where current enters or leaves it, the cathode is the negative electrode, the anode is the positive electrode, and the electrolyte is the molten substance or solution that conducts between them. Cations go to the cathode and anions to the anode: match the initials.
- Explain why electrolysis runs on direct current: the electrodes must keep a fixed sign so that each ion is always pulled the same way, and alternating current would swap them every cycle. The reason is fixed polarity, not voltage, power or safety.
- Know that at the cathode the positive ions, the cations, gain electrons, which is reduction, and become atoms or molecules; at the anode the negative ions, the anions, lose electrons, which is oxidation, and become atoms or molecules too. Gaining electrons is reduction, not oxidation: remember OIL RIG.
- Predict what is produced at each electrode when an aqueous solution, a salt solution included, is electrolysed. A solution is not the molten salt: water's own ions take part too.

- Work out which species is discharged when more than one kind of ion or molecule is pulled towards the same electrode. Hydrogen forms at the cathode unless the metal is less reactive than hydrogen.
- Write a half-equation for what happens at the cathode and another for what happens at the anode, with the electrons on the correct side of each. Electrons go on the left for reduction and on the right for oxidation.
- Explain how electrolysis puts a thin metal coat on an object: the object is made the cathode, the coating metal is the anode, and the electrolyte contains ions of that metal. The object being plated is the cathode, so it must be negative.

C13 · Organic chemistry

- Know that most hydrocarbons come from crude oil, and that fractional distillation splits the oil into fractions; you will not be asked the names or uses of individual fractions. Distillation separates by boiling point; no bonds break and no new substance forms.
- Know that the alkanes form a homologous series in which a molecule with n carbon atoms carries 2n + 2 hydrogen atoms, the general formula CnH2n+2. Saturated means every carbon to carbon bond is single.
- Name any unbranched alkane with one to six carbon atoms, and go the other way, from the name to the molecule. Butane has four carbons, not two: the first four names are irregular.
- Know that the alkenes form a homologous series whose members each hold one carbon to carbon double bond, so a molecule with n carbon atoms carries 2n hydrogen atoms, the general formula CnH2n. One letter separates alkene from alkane: read the name carefully.
- Name any unbranched alkene with two to six carbon atoms, saying where the double bond sits, as in but-1-ene and but-2-ene, and go from the name back to the molecule. There is no methene: the smallest alkene has two carbons.
- Know what addition polymerisation is: many small alkene molecules join end to end to give a polyalkene. Alkanes have no double bond, so they cannot be addition monomers
- Know that molecules carrying a C=C double bond, alkenes above all, can add to one another to build one long saturated chain, and that this product is called a polymer. The chain is saturated: every C=C opens when the monomers join
- Treat the alcohols as one homologous series: every member fits CnH2n+1OH and each differs from the next by a single CH2 group. The H in OH counts: ethanol has six hydrogens, not five
- Name any straight-chain alcohol with one to six carbons and write its structure from its name, including the number that says where the OH group sits on the chain. Number the chain from the end that gives the OH the smaller number
- Treat the carboxylic acids as one homologous series: every member fits CnH2n+1COOH and successive members differ by one CH2 group. Ethanoic acid has two carbons in total, so n is 1
- Give the name of any straight-chain carboxylic acid with one to six carbons, and write down the structure that a given name describes. The COOH carbon is part of the chain: HCOOH is methanoic acid

C14 · Metals and reactivity

- Connect how reactive a metal is to how readily its atoms lose electrons to become positive ions, and to how difficult the metal is to extract from its ore. More reactive means harder to extract, not easier
- Work out a reactivity order from the results of displacement reactions, and predict from a known order whether a given displacement will happen. No reaction is evidence too: the metal added sits below the other
- Explain why a metal is chosen for a particular job in terms of its physical properties and its chemical properties. Name the property behind the use: conducts, resists corrosion, is light
- Know the use-and-property pairings for aluminium, iron, copper, silver, gold and titanium, and know that metals are mixed into alloys to give a material the particular properties a job needs. An alloy is a mixture, not a compound
- Know that most ores hold the metal as its oxide, and that extracting a metal from any ore always involves reducing it, whatever the method. Reduction here means the oxide loses oxygen, or the metal ion gains electrons
- Know that a transition metal can form more than one stable ion, each in a different oxidation state, iron as Fe2+ or Fe3+ for example. Do not assume a metal makes just one ion: read the roman numeral
- Know that the compounds a transition metal forms are usually coloured, so colour in a solid or a solution points to a transition metal being present. It is the compounds that are coloured, not the metal itself
- Know that a transition metal is often used as a catalyst to speed up a reaction. A catalyst is not used up, so it is not a reactant

C15 · Particle model

- Describe how closely packed and how ordered the particles are in a solid, a liquid and a gas, and how they move in each: vibrating about fixed positions, sliding past one another, or travelling freely. Liquid particles still touch: their spacing is close to a solid’s

- Explain what happens to the spacing, arrangement and motion of the particles when a substance freezes, melts, boils or evaporates, and condenses. The particles themselves never expand or shrink; only spacing and motion change
- Understand that the energy needed to melt or boil a substance depends on its bonding and structure: the stronger the forces holding its particles together, intermolecular forces included, the more energy the change takes. Temperature stays constant during a change of state while energy is transferred

C16 · Chemical tests

- Test for hydrogen by holding a lit splint at the mouth of the tube: the gas burns with a squeaky pop. Lit splint for hydrogen, glowing splint for oxygen; do not swap them
- Test for oxygen with a splint that is glowing but not alight: in oxygen it bursts back into flame. The splint goes in glowing, not burning; relighting is the positive result
- Test for a carbonate by adding dilute acid: it fizzes, giving off carbon dioxide, which limewater will confirm. Carbonate plus acid gives carbon dioxide, not hydrogen; metal plus acid gives hydrogen
- Test for a halide by adding silver nitrate solution after acidifying the sample with dilute nitric acid, and read the halide from the colour of the precipitate: white for chloride, cream for bromide, yellow for iodide. Acidify with nitric acid, never hydrochloric: chloride ions would give a false positive
- Recognise that adding sodium hydroxide solution to a solution of Al3+, Ca2+ or Mg2+ ions gives a white precipitate. A white precipitate does not by itself say which of the three.
- Recognise that copper(II) ions give a blue precipitate when sodium hydroxide solution is added. The blue solid is the sign; copper solutions are blue before testing.
- Recognise that iron(II) ions give a green precipitate with sodium hydroxide solution. Green here means iron(II), not copper, whose green shows in a flame.
- Match each flame colour to its metal ion: lithium burns crimson red, sodium yellow-orange, potassium lilac, calcium red-orange and copper green. Lithium's crimson and calcium's red-orange are the pair most often swapped.
- Test for water by dropping it onto anhydrous copper(II) sulfate and watching the powder go from white to blue. The change shows water is present, not that the water is pure.

C17 · The atmosphere

- Recall the proportions of the gases in dry air, use them in a calculation, and explain that fractional distillation separates those gases. The remaining one percent is mostly argon, not carbon dioxide.
- State where greenhouse gases such as carbon dioxide and methane come from, and describe the effects they have on the climate. Do not mix greenhouse warming up with damage to the ozone layer.
- Explain where carbon monoxide, carbon dioxide, sulfur dioxide and the nitrogen oxides come from and what harm each one does. Nitrogen oxides form from air nitrogen at engine temperatures, not from the fuel.
- Explain what chlorine is for, and what fluoride ions are for, when drinking water is treated. Chlorine kills microbes and fluoride protects teeth: two purposes, keep them apart.
- Explain that rubbing an insulator leaves it carrying a charge. Rubbing moves electrons only; protons stay put, so charge is transferred, not made.

Biology

B1 · Cells and organisation

- Describe the cell membrane of an animal or plant cell, a thin partially permeable layer round the cytoplasm, and explain its job of controlling what passes into and out of the cell.
- Describe the cell wall, which plant cells have and animal cells lack, and explain what it does for the cell. The wall gives support and shape; it does not control what enters
- Describe the chloroplast, present in plant cells but not in animal cells, and explain its role in the cell. Chloroplasts photosynthesise; respiration is the mitochondrion's job
- Describe the membrane of a bacterial cell and state its role, which is to control what enters and leaves the cell.
- Describe the wall that surrounds a bacterial cell and explain what it does for the cell. Bacteria have a wall too, and it is not made of cellulose
- Know that a bacterium keeps its main DNA as a chromosome lying free in the cytoplasm, since it has no true nucleus to hold it, and say what that DNA does for the cell. Prokaryotes do have DNA; what they lack is a nucleus
- Describe plasmid DNA, a small separate loop of DNA that a bacterium carries as well as its chromosome, and say what it does for the cell. A plasmid is extra DNA; it is not the bacterial chromosome
- Place any part of an organism in the hierarchy that runs from cells, through tissues and organs, up to organ systems, and say what each level means. A tissue is many similar cells; an organ combines several tissues

B2 · Transport across membranes

- Define diffusion, osmosis and active transport, giving osmosis as the movement of water from higher to lower water potential across a partially permeable membrane, and say which of the three needs energy from the cell. State osmosis in water potential terms, high to low; 'concentration' alone is ambiguous
- Recognise these three processes in examples, some drawn from living organisms and some from settings where nothing is alive. Active transport happens only in living cells; diffusion needs no life at all

B3 · Cell division and reproduction

- Know what the mitotic cell cycle is: the repeating sequence in which a cell grows, copies its DNA and then divides once to make two identical cells.
- Split that cycle into its two parts: interphase, when the cell grows and its DNA is replicated, and mitosis, a single division that gives two daughter cells, each a genetic match for the other and for the parent cell. Interphase is not a rest: growth and DNA copying both happen there.
- Know what the meiotic cell cycle is: a cell grows, copies its DNA and then goes through two divisions to make four cells.
- Split the meiotic cycle into interphase, when the cell grows and replicates its DNA, and meiosis, two divisions one after the other that leave four daughter cells, each holding just one copy of every chromosome. DNA is copied once, before meiosis, even though two divisions follow.
- State that asexual reproduction needs only one parent and gives offspring that are genetic copies of it, unless a mutation intervenes. The offspring are identical only when no mutation has occurred.
- State that sexual reproduction needs two parents and gives offspring that differ genetically from each other and from both parents, which is where extra variation comes from.
- Give the sex chromosomes for most mammals, humans among them: XX for a female, XY for a male. The rule is stated for most mammals, not every animal.
- Use genetic data or a genetic diagram to work out the sex of an offspring and the ratio of males to females a cross is expected to give. Only the father's gamete decides sex: the mother always gives an X.

B4 · Genetics

- Know that in a eukaryotic cell the nucleus is one place where the genetic material is kept.
- Define and use each of these terms correctly: gene, allele, dominant, recessive, heterozygous, homozygous, phenotype, genotype, chromosome and autosome. Genotype is the alleles carried; phenotype is what actually shows.
- Use and read genetic diagrams for a monohybrid cross, one that follows a single gene, and interpret the genetic data such a cross gives. Each gamete carries one allele of the gene, never both.
- Read a pedigree chart to work out genotypes and the chance of a trait appearing, and give the answer as a ratio, a count, a probability or a percentage. Two unaffected parents with an affected child means the trait is recessive.

B5 · DNA and protein synthesis

- Define the genome as all the DNA an organism carries, taken together. The genome is the complete set, not a single gene or chromosome.
- State that chromosomes are the structures that hold this DNA.
- Describe a single strand of DNA (ssDNA) as a polymer: a chain built from nucleotide units linked end to end. The repeating unit is the nucleotide, not the base on its own.
- Describe double-stranded DNA (dsDNA) as a polymer too, this time two strands wound round each other in a double helix.
- Know that making a protein means joining amino acids into a chain, and that such a chain is a polypeptide.
- Know that a working protein may be a single polypeptide or several polypeptides combined. One polypeptide is not always a whole protein; some need several.
- Know that a protein's three-dimensional shape follows from the order of its amino acids.
- Know that a mutation is a change in the order of nucleotides along the DNA. A mutation is a change in the DNA itself, not in the protein.
- Know that mutations vary in their effect: the majority leave the phenotype unchanged, a few alter it slightly, and now and then one decides it outright. Most mutations are neutral; do not assume every mutation is harmful or visible.

B6 · Genetic engineering and selection

- Describe how genetic engineering works: a gene is copied from one organism's DNA and put into the DNA of a different organism, and say what restriction enzymes and ligases each do when the DNA is recombined. Restriction enzymes cut DNA at specific sites; they do not join the pieces.
- Know that some cells of the early embryo are totipotent: any one of them could grow into an entire organism made up of many cells. Totipotent is the stronger term: a whole organism, not merely many cell types.
- Know that most of the stem cells in an embryo are pluripotent: they can differentiate into any type of cell. Most, not all, embryonic stem cells are pluripotent: a few are totipotent.
- Compare natural selection with selective breeding: say what the two have in common and where they differ. Both change a population over generations; only who chooses the parents differs.
- Explain the effect that selective breeding has on a population over time. Think about the population's gene pool, not just the individuals picked to breed.

B7 · Evolution and variation

- Know that the individuals making up a population of one species usually differ from each other genetically, and to a large degree. Variation here is between members of one species, not between different species.
- Define evolution as the alteration of a population's heritable traits across generations, brought about by natural selection, which can in time produce a new species. Populations evolve; an individual cannot change its own inherited characteristics during its life.
- Recognise that differences between individuals can be genetic, passed on from parents, and that this produces a spread of phenotypes. Phenotype is the observable trait; genotype is the alleles behind it.
- Recognise that an organism's surroundings can also cause differences between individuals, so the environment too shapes the phenotypes seen. A trait picked up from the environment is not passed to offspring.

B8 · Enzymes and digestion

- Know that enzymes are mostly proteins and that they act as catalysts in living organisms. Enzymes speed reactions up without being used up or altered by them.
- Explain in outline how an enzyme works: the substrate binds to it, is converted into product, and the enzyme is released unchanged to act again. Substrate is what the enzyme acts on; product is what it makes.
- Describe the part the active site plays and explain why an enzyme is specific: only one substrate, or one kind of substrate, fits into its active site. Say the substrate fits the active site, not the enzyme fits the substrate.
- Explain how a change in temperature, or in pH, alters how quickly an enzyme works, and why. A denatured enzyme has lost its active site shape; it has not died.
- Match each digestive enzyme to what it breaks down: amylases act on carbohydrates, proteases on proteins and lipases on fats. Each enzyme digests one food type only: amylase does not touch protein.

B9 · Human physiology

- Explain what respiration is: the process going on inside every living cell, and what it achieves for the cell. Respiration is not breathing: it happens inside cells, breathing just moves air.
- Describe aerobic respiration in cells and be able to write out its word equation. Oxygen is a reactant here; in photosynthesis it is a product.
- Describe anaerobic respiration in animal cells, the process a cell uses when it has too little oxygen, and be able to write out its word equation. Anaerobic respiration still releases energy, only less of it than aerobic.
- Know which two parts make up the central nervous system: the brain together with the spinal cord. Nerves running out to the body are peripheral, not part of the central system.
- Describe how each of the three neurone types, sensory, relay and motor, is built and what it does, and do the same for a synapse and for the reflex arc. Sensory neurones carry impulses towards the central nervous system, motor neurones away from it.
- Define homeostasis as keeping conditions inside the body steady, and explain why an animal built of many cells depends on it. Steady means held within a narrow range, not fixed at one exact value.
- Explain how negative feedback keeps homeostasis going: a shift away from the normal level triggers a response that pushes it back. Negative here means opposing the change, not harmful and not switched off.
- Explain how the level of glucose in the blood is regulated, including what insulin and glucagon each do.
- Understand that each hormone is made by a particular endocrine gland, released into the blood, and carried to the organ or tissue it acts on. Hormones travel in the blood, not along nerves, so their effects are slower.
- Describe adrenaline's main job: preparing the body for sudden action, the fight or flight response, when it meets danger or stress.
- Describe what FSH, LH, oestrogen and progesterone each do across the menstrual cycle, name the hormones used in contraception, and explain how hormonal and non-hormonal methods of contraception differ.
- Know that a communicable disease is one that can be passed from person to person, and revise what causes these diseases, how they spread, how they are treated and prevented, and how new medicines and vaccines are tested. A communicable disease can be passed on; many diseases cannot be caught.
- Know that a communicable disease is caused by a pathogen, which may be a bacterium, a virus, a fungus or a protist. A pathogen is any microbe that causes disease, not only bacteria.
- Explain how a sexually transmitted infection passes from person to person, and how HIV damages the immune system so that, untreated, it leads to AIDS. Some also spread through blood or from mother to baby, not only sexually.

B10 · Ecology

- Define individual, population, community and ecosystem, and put the four in order from the single organism up to the whole system. A community is all the living things; an ecosystem adds the non-living surroundings
- Explain how a community can be changed by abiotic factors, the non-living conditions of its habitat, and by biotic factors, the effects of the other living things in it.
- Trace carbon round the carbon cycle and explain what each of photosynthesis, respiration, combustion and decomposition contributes to keeping it moving. Only photosynthesis takes carbon dioxide out of the air; the other three return it
- Explain why the water cycle matters to living things, and what they would lose without it.
- Describe how you would use quadrats and a belt transect to find out how many of an organism live in a habitat and where they are, and interpret the data such a survey gives. Random quadrats measure abundance; a transect tracks change along a line
- Work out how many organisms of one species there are in a given area from the counts in a sample of it, scaling up from the area you sampled to the whole.

B11 · Photosynthesis and plant transport

- Write photosynthesis as carbon dioxide plus water giving glucose and oxygen, and know that it is endothermic because it takes in energy from light. Photosynthesis is endothermic: energy in from light, unlike respiration
- Explain how each of temperature, light intensity and the concentration of carbon dioxide can limit the rate of photosynthesis, and what happens to the rate as each one is raised. Warmer is faster only up to a point; enzymes then denature
- Know that a plant has transport tissues, xylem and phloem, that carry substances from one part of it to another.
- Describe how xylem and phloem are built, and link each feature of their structure to the job that tissue does in the plant. Xylem is dead and hollow; phloem is living: keep their features apart
- Explain why xylem is built from dead cells with lignin in their walls, and how that suits it to carrying water and dissolved mineral ions up from the roots to the stem and leaves. Lignin is there for strength and waterproofing, not to move the water

EXAMPLE GRAMMAR SCHOOL

ESAT Mock Examination

Mathematics 1

Paper SMP-2026-09 · Version 1.5

NAME

TEACHING GROUP

DATE

Instructions

- Time allowed: **15 minutes**.
- There are **10 multiple-choice questions**. Each is worth 1 mark.
- There is no negative marking. Answer every question, even if you have to guess.
- Calculators are not permitted.
- Record your answers on the separate answer sheet. Only the answer sheet is marked.
- Formulae are not provided. The ESAT does not supply a formula sheet.

Note: the real ESAT is sat on screen at a Pearson test centre. This paper trains the mathematics and the pacing, not the on-screen interface.

Written for Example Grammar School by Lucas Rey, Engineering Science, University of Oxford; MRes, University of Cambridge.

Question 1

A winch unwinds 0.75 km of uniform steel cable of linear density 12 g cm^{-1}. Find its mass.

A. 900 kg
B. 90 kg
C. 625 kg
D. 9 kg
E. 90000 kg
F. 9000 kg
G. 0.9 kg
H. 0.09 kg

Question 2

A ski tow climbs a uniform slope in a straight line. The cable rises 1.5 m for every 2 m of horizontal distance. Find the height gained along 20 m of cable.

A. 15 m
B. 12 m
C. 6 m
D. 16 m
E. 8 m

Question 3

240 g of alloy occupies 30 cm^3, each to the nearest ten. Find the greatest density.

A. $9.4\,\text{g cm}^{-3}$
B. $9.8\,\text{g cm}^{-3}$
C. $8\,\text{g cm}^{-3}$
D. $10\,\text{g cm}^{-3}$
E. $7\,\text{g cm}^{-3}$
F. $9.6\,\text{g cm}^{-3}$
G. $12\,\text{g cm}^{-3}$
H. $12.5\,\text{g cm}^{-3}$

Question 4

A start-up's cash balance C, in thousands of pounds, at the end of month x is modelled by $C = (x-1)(x-5)$, valid for months 0 to 6. Treating x as continuous, what is the lowest balance predicted?

A. $-\$3000$
B. $\$0$
C. $-\$9000$
D. $\$5000$
E. $-\$4000$
F. $\$1000$
G. $-\$14000$
H. $\$3000$

Question 5

Decrease 80 by 15%.

A. 92
B. 68
C. 65
D. 94.12
E. 12

Question 6

A circular ornamental garden has a diameter of 36 m, with a sundial at its centre. A wedge-shaped vegetable patch has its point at the sundial and reaches out to the garden's edge, and it covers $54\pi\ \text{m}^2$ of the ground. Rabbit fencing is fitted along the curved edge of the patch and nowhere else. What length of the garden's boundary is left unfenced?

A. 33π m
B. 36π m
C. 60π m
D. 42π m
E. 69π m
F. 30π m
G. 15π m
H. 6π m

Question 7

A camera's light meter plots the correct exposure time t, in milliseconds, against the aperture area A, in mm^2. The plotted curve is a hyperbola in the first quadrant, falling steeply and approaching both axes without ever meeting them. One recorded setting has $A = 8$ and $t = 45$. What exposure time does the curve give when $A = 12$?

A. 41 ms
B. 22.5 ms
C. 20 ms
D. 101.25 ms
E. 90 ms
F. 3.75 ms
G. 67.5 ms
H. 30 ms

Question 8

A battery is $\frac{3}{4}$ charged. Using 0.35 of its capacity leaves what percentage?

A. 40%
B. 110%
C. 48.75%
D. 60%
E. 74.65%
F. 71.5%
G. 65%
H. 26.25%

Question 9

A vineyard's yield falls by 20% each year while a disease goes untreated. Three harvests after it appeared the yield is 2560 kg. How large was the last healthy harvest?

A. 6400 kg
B. 4480 kg
C. 3200 kg
D. 4096 kg
E. 5120 kg
F. 4000 kg
G. 6250 kg
H. 5000 kg

Question 10

A courier walks 4 blocks east and 2 north. How many routes are possible?

A. 8
B. 360
C. 30
D. 6
E. 64
F. 15
G. 720
H. 10

END OF PAPER

Questions 11 to 27 are in the full paper

This is an extract. The 10 questions before this page are real questions from a commissioned question paper, shown in full and with nothing withheld, so that the standard can be judged rather than described.

The rest of the paper is not hidden behind this page. It was never in this file. Each commissioned paper is written for one school, recorded against it, and issued to no other, so the questions in it are not published anywhere, including here.

A commissioned paper is NOT this paper with the missing questions restored. Every question in it is different from every question here, the 10 shown above included. Your school's paper is drawn fresh, and no question that has ever been published, on this page or anywhere else, can appear in it.

A full paper runs to 27 questions in this module alone, with a question paper, an answer sheet, a mark scheme and a worked solution for every question, across all five ESAT modules.

This page is in the full paper.

lucasrey.com/teaching/for-schools/

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EXAMPLE GRAMMAR SCHOOL

ESAT Mock: Answer Sheet

Mathematics 1

Paper SMP-2026-09 · Version 1.5

NAME

TEACHING GROUP

DATE

Fill in one bubble per question. If you change your mind, cross out the old mark clearly and fill the new bubble. Unfilled or doubly filled rows score zero.

Q	A	B	C	D	E	F	G	H
1	◯	◯	◯	◯	◯	◯	◯	◯
2	◯	◯	◯	◯	◯			
3	◯	◯	◯	◯	◯	◯	◯	◯
4	◯	◯	◯	◯	◯	◯	◯	◯
5	◯	◯	◯	◯	◯			
6	◯	◯	◯	◯	◯	◯	◯	◯
7	◯	◯	◯	◯	◯	◯	◯	◯
8	◯	◯	◯	◯	◯	◯	◯	◯
9	◯	◯	◯	◯	◯	◯	◯	◯
10	◯	◯	◯	◯	◯	◯	◯	◯

MARKER INITIALS

TOTAL OUT OF 10

EXAMPLE GRAMMAR SCHOOL

ESAT Mock: Mark Scheme

Mathematics 1

Paper SMP-2026-09 · Version 1.5

Answer grid

Q	Ans	Q	Ans	Q	Ans
1	**A**	10	**F**		
2	**B**				
3	**B**				
4	**E**				
5	**B**				
6	**F**				
7	**H**				
8	**A**				
9	**H**				

Totals

10 marks available, 1 mark per question. There is no negative marking, so a wrong answer scores zero and never a negative. Only the answer sheet is marked.

Diagnostic table

Use this to see which topics the cohort is losing marks on.

Q	Ans	Node	Group	Topic	Difficulty
1	**A**	M1.1	M1	Units and compound measures	★★☆☆☆
2	**B**	M5.18	M5	Geometry	★★★☆☆
3	**B**	M2.12	M2	Number	★★★☆☆
4	**E**	M4.11	M4	Algebra	★★★☆☆
5	**B**	M3.8	M3	Ratio and proportion	★★☆☆☆
6	**F**	M5.16	M5	Geometry	★★★☆☆
7	**H**	M4.12	M4	Algebra	★★★☆☆
8	**A**	M2.10	M2	Number	★★☆☆☆
9	**H**	M3.11	M3	Ratio and proportion	★★★☆☆
10	**F**	M2.5	M2	Number	★★★☆☆

Marks available by topic group

Group	Topic	Marks
M1	Units and compound measures	1
M2	Number	3
M3	Ratio and proportion	2
M4	Algebra	2
M5	Geometry	2
Total		**10**

Worked solutions for every question are in the separate worked solutions document.

EXAMPLE GRAMMAR SCHOOL

ESAT Mock: Worked Solutions

Mathematics 1

Paper SMP-2026-09 · Version 1.5

Staff copy

This document contains the answers. Keep it away from candidates until the paper has been sat. Marking guidance is in the separate mark scheme.

Question 1

M1.1 · Units and compound measures · ★★☆☆☆ · 85 s

A winch unwinds 0.75 km of uniform steel cable of linear density 12 g cm^{-1}. Find its mass.

- **A.** 900 kg
- **B.** 90 kg
- **C.** 625 kg
- **D.** 9 kg
- **E.** 90000 kg
- **F.** 9000 kg
- **G.** 0.9 kg
- **H.** 0.09 kg

Answer A. Key idea

A compound unit is a recipe for a division, and every unit inside it has to be rescaled separately before the number in front of it means anything. A linear density written in grams per centimetre carries a mass unit on the top and a length unit on the bottom, so moving it to kilograms per metre multiplies by one factor and divides by another. Once the rate is expressed in units that match the length being measured, the mass is a single multiplication, because mass equals rate multiplied by length.

Worked solution

1. Rewrite the linear density in kg m^{-1}

A linear density of 12 g cm^{-1} says that every centimetre of cable weighs 12 g. A metre contains 100 of those centimetres, so one metre of cable weighs

$12 \times 100 = 1200$ g

and 1200 g = 1.2 kg, so the cable is 1.2 kg m^{-1}.

Both unit changes are doing work here and they pull in opposite directions: the length unit contributes a factor of 100 on the bottom, the mass unit a factor of $\frac{1}{1000}$ on the top, and the survivor is a net division by 10. That is why 12 becomes 1.2 and not 1200 or 0.012.

2. Convert the length and multiply

0.75 km = $0.75 \times 1000 = 750$ m, and the rate is now per metre, so the two agree:

$m = 1.2 \times 750 = 750 + 150 = 900$ kg

Check: run it backwards. 900 kg spread over 750 m is 1.2 kg per metre, which is 1200 g per metre, which is 12 g per centimetre, exactly the figure given. A three quarter kilometre of heavy steel cable weighing close to a tonne is also the right order of magnitude for a winch drum.

Matches Option **A**.

Why the other options are wrong

- **B** (90 kg): Took 0.75 km as 75 m, using a factor of 100 for kilometres to metres instead of 1000: $1.2 \times 75 = 90$. A kilometre is a thousand metres, and the factor of 100 belongs to centimetres. *(Wrong power of ten for km)*
- **C** (625 kg): Reached 1.2 kg m^{-1} and converted the length correctly, then divided by the rate instead of multiplying by it: $750 \div 1.2 = 7500 \div 12 = 625$. A rate in kilograms per metre multiplies a number of metres; dividing a number of metres by it leaves square metres per kilogram, which is not a mass at all. *(Rate inverted, divided instead of multiplied)*
- **D** (9 kg): Converted the mass but not the length: 12 g = 0.012 kg, then $0.012 \times 750 = 9$. That multiplies a per centimetre rate by a number of metres, so the answer is short by the factor of 100 that the length unit carries. *(Half a compound unit converted)*
- **E** (90000 kg): Knew that a single factor of ten survives between g cm^{-1} and kg m^{-1} but multiplied by it instead of dividing, reading 12 g cm^{-1} as 120 kg m^{-1}, then $120 \times 750 = 90000$. Dividing by 1000 for the mass outweighs multiplying by 100 for the length, so the net move is a division and the rate is 1.2, not 120. *(Net factor applied the wrong way)*
- **F** (9000 kg): Read the numeral 12 as though it were already 12 kg m^{-1} and multiplied: $12 \times 750 = 9000$. Neither the gram nor the centimetre was rescaled, so the figure is ten times the true rate. *(No conversion at all)*
- **G** (0.9 kg): Reached 1.2 kg m^{-1} correctly but multiplied by 0.75 instead of 750, leaving the length in kilometres: $1.2 \times 0.75 = 0.9$. The rate is per metre, so the length must be in metres. *(Length unit not converted)*
- **H** (0.09 kg): Divided for both unit changes: $12 \div 1000 = 0.012$ kg cm^{-1}, then divided again by 100 for centimetres to metres to give 0.00012 kg m^{-1}, and $0.00012 \times 750 = 0.09$. The length unit sits on the bottom of the rate, so going from a per centimetre rate to a per metre rate multiplies by 100. *(Both conversions taken as divisions)*

Common mistake. Rescaling the mass unit and leaving the length unit alone, or the other way round. Turning 12 g cm^{-1} into 0.012 kg cm^{-1} and then multiplying straight by 750 treats a per centimetre rate as a per metre rate, and the answer comes out a hundred times too small. Both halves of a compound unit have to be converted before the number is used.

Takeaway. The Cheat Code: 1 g cm^{-1} = 0.1 kg m^{-1}, because dividing by 1000 for the mass and multiplying by 100 for the length leaves a single division by 10. Fix the rate first, fix the length second, then multiply once.

Time-saving tip. Speed Heuristic: do not chase the powers of ten one at a time. Grams to kilograms is $\div 1000$ and per centimetre to per metre is $\times 100$, a net $\div 10$, so 12 g cm^{-1} is 1.2 kg m^{-1} on sight. Then 1.2×750 is 750 plus a fifth of 750, that is $750 + 150 = 900$.

Question 2

M5.18 · Geometry · ★★★☆☆ · 90 s

A ski tow climbs a uniform slope in a straight line. The cable rises 1.5 m for every 2 m of horizontal distance. Find the height gained along 20 m of cable.

- **A.** 15 m
- **B.** 12 m
- **C.** 6 m
- **D.** 16 m
- **E.** 8 m

Answer B. Key idea

For a fixed angle the three ratios sin, cos and tan are fixed numbers, so a slope quoted as a vertical rise per horizontal run hands you tan directly. A distance measured along the slope itself is the hypotenuse, not the adjacent side, so the ratio that connects it to the height is sin. Once the two given lengths are reduced to the 3, 4, 5 side lengths, both ratios are available and the whole conversion is exact arithmetic.

Worked solution

Step 1: Build the right-angled triangle

The cable, the horizontal ground and the vertical rise form a right-angled triangle. Taking θ as the angle of climb at the bottom of the slope, the rise is the side opposite θ and the horizontal distance is the side adjacent to θ, so opposite = 1.5 m and adjacent = 2 m.

Step 2: Read off the ratio the slope statement gives

TOA: $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{1.5}{2} = \dfrac{3}{4}$.

Step 3: Switch to the ratio that matches the given length

The 20 m is measured along the cable, which is the hypotenuse, so the height needs sin, not tan. From opposite : adjacent = 3 : 4 the hypotenuse is $\sqrt{3^2+4^2} = 5$, so the sides are in the ratio 3 : 4 : 5 and $\sin\theta = \dfrac{3}{5}$.

Step 4: Apply sin to the cable length

SOH: height $= 20 \times \dfrac{3}{5} = 12$ m.

Sanity check: the same 20 m of cable covers $20 \times \dfrac{4}{5} = 16$ m horizontally, and $\dfrac{12}{16} = \dfrac{3}{4}$, which is the $\tan\theta$ we started from. The height is also comfortably less than the cable length, as it must be.

Matches Option **B**.

Why the other options are wrong

- **A** (15 m): Uses tan on the cable length: $20 \times \dfrac{3}{4} = 15$ m. That is the height only if the 20 m were the horizontal distance, but it is measured along the slope, so it is the hypotenuse. *(Conceptual Error)*
- **C** (6 m): Mixes the ratio numbers with the measured lengths: takes the hypotenuse as 5 from the 3 : 4 : 5 ratio while keeping the rise as the raw 1.5 m, giving $\sin\theta = \dfrac{1.5}{5} = 0.3$ and $20 \times 0.3 = 6$ m. The ratio must be built from two lengths of the same triangle. *(Scaling Error)*
- **D** (16 m): Picks CAH instead of SOH: $20 \times \dfrac{4}{5} = 16$ m. That is the horizontal distance the tow covers, not the height it gains. *(Conceptual Error)*

- **E** (8 m): Stops at the scale factor. The small triangle has hypotenuse 2.5 m, so $20 \div 2.5 = 8$, and the 8 is reported as the answer instead of being used to scale the rise: $8 \times 1.5 = 12$ m. *(Incomplete Solution)*

Common mistake. Treating the 20 m of cable as a horizontal distance and multiplying by $\tan\theta$, giving $20 \times \frac{3}{4} = 15$ m. The 20 m runs along the slope, so it is the hypotenuse and sin is the ratio that applies.

Takeaway. The Cheat Code: Before choosing a ratio, decide which side of the triangle the given number actually is. A rise per horizontal run fixes tan, but any distance measured along the slope is a hypotenuse, so the ratio that turns it into a height is sin.

Time-saving tip. Speed Heuristic: 1.5 : 2 scales to 3 : 4, so this is a 3, 4, 5 triangle. For any length measured along the slope, divide by 5 and multiply by 3: $20 \div 5 = 4$, then $4 \times 3 = 12$, with no fractions written down.

Question 3

M2.12 · Number · ★★★☆☆ · 90 s

240 g of alloy occupies 30 cm^3, each to the nearest ten. Find the greatest density.

- **A.** 9.4 g cm^{-3}
- **B.** 9.8 g cm^{-3}
- **C.** 8 g cm^{-3}
- **D.** 10 g cm^{-3}
- **E.** 7 g cm^{-3}
- **F.** 9.6 g cm^{-3}
- **G.** 12 g cm^{-3}
- **H.** 12.5 g cm^{-3}

Answer B. Key idea

A figure rounded to the nearest ten is only guaranteed to lie within half of that ten, so each measurement becomes an interval five units wide on either side of the stated value. Density is a quotient, and a quotient is at its largest when the numerator is at its largest and the denominator at its smallest, so the two measurements must be pushed to opposite ends of their intervals rather than to the same end.

Worked solution

1. Turn each rounding into an interval

A value given to the nearest ten carries half of ten, that is 5, of slack on either side. Writing m for the mass and V for the volume,

$235 \le m < 245$ and $25 \le V < 35$.

The two intervals are the same width in absolute terms, but the volume interval is far wider as a proportion of its measurement, which is why the volume controls the answer.

2. Choose the ends that maximise a quotient

Density is $\rho = \dfrac{m}{V}$. A quotient grows when the numerator grows and when the denominator shrinks, so the greatest possible density pairs the heaviest allowed mass with the smallest allowed volume:

$$\rho_{\max} = \frac{245}{25}.$$

3. Evaluate and check

$$\frac{245}{25} = \frac{490}{50} = \frac{49}{5} = 9.8 \text{ g cm}^{-3}.$$

Sanity check: the stated figures give $240 \div 30 = 8\ \text{g cm}^{-3}$, and the smallest allowed density is $235 \div 35 = 6.71\ldots\ \text{g cm}^{-3}$. An upper bound has to sit above the nominal value and it does, since $9.8 > 8$.

Matches Option **B**.

Why the other options are wrong

- **A** ($9.4\ \text{g cm}^{-3}$): Paired the two lower bounds: $235 \div 25 = 9.4$. The numerator has to be at its upper bound for the quotient to be greatest. *(Both lower bounds used)*
- **C** ($8\ \text{g cm}^{-3}$): Divided the stated figures and ignored the rounding altogether: $240 \div 30 = 8$. That is the nominal density, which sits inside the interval rather than at its top. *(Bounds ignored)*
- **D** ($10\ \text{g cm}^{-3}$): Gave the mass the full ten grams of slack instead of half of it: $250 \div 25 = 10$. The nearest ten guarantees only 5 either way, so 250 is outside the possible range. *(Whole quantum instead of half)*
- **E** ($7\ \text{g cm}^{-3}$): Pushed both measurements to the top of their intervals, as though the greatest density meant the largest number everywhere: $245 \div 35 = 7$. The denominator has to be at its smallest for a quotient to be greatest, so the volume belongs at 25, not 35. *(Both upper bounds used)*
- **F** ($9.6\ \text{g cm}^{-3}$): Took the volume down to its lower bound but left the mass at the figure printed in the stem: $240 \div 25 = 9.6$. The mass is rounded to the nearest ten as well, so it carries the same five grams of slack and must go up to 245. *(Only the denominator moved)*
- **G** ($12\ \text{g cm}^{-3}$): Gave the volume the full ten of slack while leaving the mass alone: $240 \div 20 = 12$. Rounding to the nearest ten guarantees only five either way, so the volume cannot fall below 25, and the mass is free to rise to 245. *(Whole quantum on the volume, mass unmoved)*
- **H** ($12.5\ \text{g cm}^{-3}$): Used the full ten of slack on both measurements: $250 \div 20 = 12.5$. Both intervals are twice as wide as the rounding allows. *(Whole quantum instead of half)*

Common mistake. Pushing both measurements to the same end of their intervals, as though "largest" meant taking the largest number everywhere. For a quotient the two ends must be opposite: 245 over 25, not 245 over 35 and not 235 over 25.

Takeaway. The Cheat Code: For the maximum of a quotient take the top of the numerator's interval over the bottom of the denominator's, and get each interval from half of the stated rounding unit, so nearest ten means five either way.

Time-saving tip. Speed Heuristic: Only one pairing needs evaluating, so write $245 \div 25$ straight down and double both parts to $490 \div 50 = 9.8$. There is no need to work out the nominal or minimum density first.

Question 4

M4.11 · Algebra · ★★★☆☆ · 90 s

A start-up's cash balance C, in thousands of pounds, at the end of month x is modelled by $C = (x-1)(x-5)$, valid for months 0 to 6. Treating x as continuous, what is the lowest balance predicted?

A. $-\$3000$
B. $\$0$
C. $-\$9000$
D. $\$5000$
E. $-\$4000$
F. $\$1000$
G. $-\$14000$
H. $\$3000$

Answer E. Key idea

A quadratic with a positive squared term has a single minimum, and completing the square puts it in the form $(x-p)^2+q$, where the bracket cannot be negative, so the least value is q and it occurs at $x=p$. The brackets of a factorised quadratic give the values where the curve is zero, not where it is least; the minimum sits exactly midway between them because the curve is symmetric about its turning point.

Worked solution

1. Put the model in standard form

$$C=(x-1)(x-5)=x^2-6x+5$$

2. Complete the square

$$x^2-6x+5=(x-3)^2-9+5=(x-3)^2-4$$

3. Read off the turning point

$(x-3)^2$ is never negative and is zero only at $x=3$, so the least value of C is -4, reached in month 3. The same x comes from $-\dfrac{b}{2a}=\dfrac{6}{2}=3$, and substituting back gives $9-18+5=-4$.

4. Check the range, then convert the units

Month 3 lies inside the valid span of months 0 to 6, so the model is being used where it applies. Either side, $C(2)=(1)(-3)=-3$ and $C(4)=(3)(-1)=-3$, both above -4, which confirms $x=3$ as the trough. Since C is in thousands of pounds, the lowest balance is $-\$4000$.

Matches Option **E**.

Why the other options are wrong

- **A** ($-\$3000$): The midpoint of the roots was found as half their difference, $(5-1)\div 2=2$, rather than half their sum. Then $C(2)=(2-1)(2-5)=-3$, giving $-\$3000$, which is a real balance on the curve but not the least one. *(Midpoint from the difference)*
- **B** ($\$0$): Read the factorised form as handing the answer over: $C=0$ at $x=1$ and at $x=5$, and the balance there was quoted as the lowest. Those are the two months the balance passes through zero, and the curve dips below zero between them, reaching $(3-1)(3-5)=-4$ thousand in month 3. *(Balance at a root taken as the minimum)*
- **C** ($-\$9000$): The square was completed to $(x-3)^2-9+5$ and the working stopped at the -9, so the $+5$ from the original constant was never carried through. That reports $-\$9000$ instead of -4 thousand. *(Constant not carried through)*
- **D** ($\$5000$): The opening balance was taken as the lowest point: $C(0)=(0-1)(0-5)=5$, that is $\$5000$. The curve falls after month 0, so its starting value is the highest point in the first half, not the lowest. *(Endpoint taken as minimum)*
- **F** ($\$1000$): The brackets were expanded as x^2-4x+5 by subtracting the roots instead of adding them. That puts the turning point at $x=2$ with value $4-8+5=1$, giving $\$1000$. Expanding correctly gives $-6x$, since $-1-5=-6$. *(Expansion sign slip)*
- **G** ($-\$14000$): Expanded $(x-1)(x-5)$ as x^2-6x-5, taking $(-1)\times(-5)$ as -5. Completing the square on that gives $(x-3)^2-9-5=(x-3)^2-14$, so the minimum reads as -14 thousand. Two negatives multiply to $+5$, and $-9+5=-4$. *(Sign slip on the product of the constants)*

- **H** ($3000): Located the turning point correctly, since $(x-3)^2 - 4$ is least at $x = 3$, then reported the 3 instead of the value there. The 3 is the month the trough falls in; substituting it gives $C = (3-1)(3-5) = -4$, that is −$4000. *(Month of the minimum reported as the balance)*

Common mistake. Quoting a root, or the balance at a root, as the lowest point. The factors give $x = 1$ and $x = 5$, where the balance is zero, and zero is higher than the trough between them. Roots answer where the curve crosses, never how low it goes.

Takeaway. The Cheat Code: For a factorised quadratic the turning point sits midway between the roots, so average them and substitute once. Completing the square says the same thing: the bracket vanishes at the midpoint and the number left outside is the minimum.

Time-saving tip. Speed Heuristic: average the roots, $(1+5) \div 2 = 3$, then feed that straight back into the brackets already given: $(3-1)(3-5) = 2 \times (-2) = -4$. No expansion, no formula and no completing the square are needed.

Question 5

M3.8 · Ratio and proportion · ★★☆☆☆ · 45 s

Decrease 80 by 15%.

A. 92
B. 68
C. 65
D. 94.12
E. 12

Answer B. Key idea

Percentage increases and decreases are handled fastest with a single decimal multiplier. A 15% decrease leaves 85% of the original, so the multiplier is 0.85 and $80 \times 0.85 = 68$.

Worked solution

1. Choose the multiplier

A decrease of 15% leaves 100% − 15% = 85% of the original amount, so the multiplier is 0.85.

2. Apply it in one step

$$80 \times 0.85 = 68$$

Doing it in two steps (15% of 80 = 12, then $80 - 12 = 68$) gives the same value, but the single multiplier is quicker and leaves nothing to forget.

3. Sense-check the direction

The question asks for a decrease, so the answer must be smaller than 80. That rules out 92 and 94.12 before any arithmetic is checked.

4. Why each of the other options fails

- **A** 92: applied the change the wrong way, 80×1.15, giving an increase.
- **C** 65: subtracted 15 as a raw number rather than 15% of 80, which is 12.
- **D** 94.12: worked backwards, $80 \div 0.85$, as if 80 were the value after the decrease.
- **E** 12: found 15% of 80 but never took it away from the original.

Matches Option **B**.

Why the other options are wrong

- **A** (92): Applied the opposite change. *(Conceptual Error)*
- **C** (65): Added/subtracted the percentage as a raw number. *(Conceptual Error)*
- **D** (94.12): Calculated original value before change (reverse percentage). *(Conceptual Error)*
- **E** (12): Calculated the percentage, but didn't apply the change. *(Conceptual Error)*

Common mistake. Forgetting to add/subtract from the original.

Takeaway. **The Cheat Code:** Do not calculate the percentage and then add/subtract it in two separate steps. Use a single multiplier (e.g., Original $\times$ 0.85 = New for a 15% decrease). For reverse percentages, NEVER add the percentage back on to the new amount; instead, divide by the multiplier (New / 0.85 = Original).

Time-saving tip. Decide first whether the quantity scales up or down, then check your answer against that expectation before you look at the options.

Question 6

M5.16 · Geometry · ★★★☆☆ · 90 s

A circular ornamental garden has a diameter of 36 m, with a sundial at its centre. A wedge-shaped vegetable patch has its point at the sundial and reaches out to the garden's edge, and it covers 54π m^2 of the ground. Rabbit fencing is fitted along the curved edge of the patch and nowhere else. What length of the garden's boundary is left unfenced?

- **A.** 33π m
- **B.** 36π m
- **C.** 60π m
- **D.** 42π m
- **E.** 69π m
- **F.** 30π m
- **G.** 15π m
- **H.** 6π m

Answer F. Key idea

A sector is a fixed fraction of its circle, and the single fraction $\frac{\theta}{360}$ governs the sector's area, its arc and its angle alike. A patch that takes one sixth of the disc therefore has one sixth of the circumference as its curved edge, whether or not the angle is ever written down. When an area is given and a length is wanted, that fraction is the bridge: read it off by comparing the sector area with the area of the whole circle, then apply it to the circumference. The part of the boundary outside the sector takes the complementary share of the same circumference.

Worked solution

Step 1: Halve the diameter and size the whole garden

The garden is 36 m across, so the radius is $r = 18$ m and the whole circle covers

$\pi r^2 = \pi \times 18^2 = 324\pi$ m^2.

Step 2: Read off the share taken by the patch

The patch is a sector, so its area is $\frac{\theta}{360}$ of the disc:

$\dfrac{\theta}{360} \times 324\pi = 54\pi$, so $\dfrac{\theta}{360} = \dfrac{54}{324} = \dfrac{1}{6}$.

Step 3: Apply the same share to the boundary

The boundary of the garden has length $2\pi r = 2\pi \times 18 = 36\pi$ m, and the patch's curved edge is the same sixth of it:

$\dfrac{1}{6} \times 36\pi = 6\pi$ m carries fencing.

Step 4: Remove the fenced arc

$36\pi - 6\pi = 30\pi$ m of boundary is left unfenced.

Check: 30π is five sixths of 36π, which is the share of the boundary lying outside a patch that takes one sixth of the garden. Numerically the answer is about 94 m out of a full boundary of about 113 m, so the fenced piece is the small part, as a one sixth share must be.

Matches Option **F**.

Why the other options are wrong

- **A** (33π m): Slipped the arc formula's factor of 2π into the area, using $\dfrac{\theta}{360} \times 2\pi r^2 = 54\pi$. That gives $\dfrac{\theta}{360} = \dfrac{54}{648} = \dfrac{1}{12}$, an arc of $\dfrac{1}{12} \times 36\pi = 3\pi$, and $36\pi - 3\pi = 33\pi$ m. A sector's area is $\dfrac{\theta}{360}\pi r^2$. *(Area formula confusion)*
- **B** (36π m): Gave the whole boundary, $2\pi \times 18 = 36\pi$ m, without removing the 6π m that the rabbit fencing occupies. *(Subtraction omitted)*
- **C** (60π m): Halved the 36 m for the area, so $\pi \times 18^2 = 324\pi$ and the share is $\dfrac{54}{324} = \dfrac{1}{6}$, then reached for the printed 36 again in the length step: $2\pi \times 36 = 72\pi$. That makes the fenced arc $\dfrac{1}{6} \times 72\pi = 12\pi$ and leaves $72\pi - 12\pi = 60\pi$ m. The radius is 18 m in every step, not just the first. *(Diameter reused as the radius in the boundary step)*
- **D** (42π m): Both lengths were right, the boundary $2\pi \times 18 = 36\pi$ m and the fenced arc $\dfrac{1}{6} \times 36\pi = 6\pi$ m, but they were combined the wrong way: $36\pi + 6\pi = 42\pi$ m. The patch reaches the garden's edge, so its curved edge is part of the boundary and has to come off it, giving $36\pi - 6\pi = 30\pi$ m. *(Arc added instead of removed)*
- **E** (69π m): Took the 36 m as the radius. The circle then appears to be $\pi \times 36^2 = 1296\pi$ m^2, so the patch looks like $\dfrac{54}{1296} = \dfrac{1}{24}$ of it, and $\dfrac{1}{24}$ of the inflated boundary 72π is 3π, leaving $72\pi - 3\pi = 69\pi$ m. *(Diameter used as radius)*
- **G** (15π m): Found the share correctly, $\dfrac{54\pi}{324\pi} = \dfrac{1}{6}$, then measured the boundary with $\pi r = 18\pi$ m instead of $2\pi r = 36\pi$ m. The fenced arc then looks like $\dfrac{1}{6} \times 18\pi = 3\pi$, and $18\pi - 3\pi = 15\pi$ m. The circumference is πd, that is $2\pi r$, so a 36 m garden has a 36π m boundary. *(Circumference taken as πr)*
- **H** (6π m): This is the fenced arc itself, $\dfrac{1}{6} \times 36\pi = 6\pi$ m. The question asks for the length the fencing does not cover, which is the other five sixths of the boundary. *(Answered the complement)*

Common mistake. Using the 36 m as the radius because it is the length printed in the stem. It is the diameter, so the disc is 324π m^2 rather than 1296π m^2 and the boundary is 36π m rather than 72π m. The slip corrupts both the fraction and the circumference, and it lands on 69π m.

Takeaway. The Cheat Code: In a sector, the share of the area, the share of the arc and the share of the full turn are all the same fraction. Find that fraction once, apply it to whichever quantity the question wants, and if the question asks for the rest of the circle, use one minus the fraction instead of recomputing.

Time-saving tip. Speed Heuristic: cancel π on sight, so the areas give $\frac{54}{324} = \frac{1}{6}$ immediately. The unfenced boundary is then five sixths of 36π, that is 30π, in one multiplication, and the angle of $60°$ never has to be written down at all.

Question 7

M4.12 · Algebra · ★★★☆☆ · 90 s

A camera's light meter plots the correct exposure time t, in milliseconds, against the aperture area A, in mm^2. The plotted curve is a hyperbola in the first quadrant, falling steeply and approaching both axes without ever meeting them. One recorded setting has $A = 8$ and $t = 45$. What exposure time does the curve give when $A = 12$?

- **A.** 41 ms
- **B.** 22.5 ms
- **C.** 20 ms
- **D.** 101.25 ms
- **E.** 90 ms
- **F.** 3.75 ms
- **G.** 67.5 ms
- **H.** 30 ms

Answer H. Key idea

A hyperbola that lies in the first quadrant and has both axes as asymptotes is the graph of the reciprocal function, $t = \frac{k}{A}$. Its defining property is that the product of the two coordinates is the same at every point, so one recorded pair fixes k and every other point then follows by a single division. Because k is not zero, neither coordinate can ever reach zero, which is exactly why the curve closes on the axes without touching them.

Worked solution

1. Name the curve from its description

A hyperbola in the first quadrant with both axes as asymptotes is the reciprocal graph, so the meter is using

$$t = \frac{k}{A}, \qquad \text{that is} \qquad tA = k$$

2. Fix the constant from the recorded setting

$$k = 8 \times 45 = 360$$

3. Substitute the new area

$$t = \frac{360}{12} = 30\,\text{ms}$$

4. Check it against the shape of the curve

The area rose, and the curve falls, so the time must come out below 45 ms, and 30 does. The product test also passes: $12 \times 30 = 360$, the same as 8×45. Finally the value is positive and finite, as it must be for a curve that never reaches either axis.

Matches Option **H**.

Why the other options are wrong

- **A** (41 ms): The sum was held constant instead of the product: $8 + 45 = 53$, then $53 - 12 = 41$. A constant sum is a straight line of gradient -1, which would cut the A axis at 53 rather than run alongside it. *(Constant sum instead of product)*
- **B** (22.5 ms): The area rose by half, from 8 to 12, so the time was cut by half: $45 \times 0.5 = 22.5$. Under $tA = k$ the time is scaled by the ratio of the areas, $\frac{8}{12} = \frac{2}{3}$, which is a cut of a third and not a half. The product test also fails, since $12 \times 22.5 = 270$ rather than 360. *(Percentage change mirrored)*
- **C** (20 ms): An inverse square rule was used instead of the reciprocal: $45 \times \left(\frac{8}{12}\right)^2 = 45 \times \frac{4}{9} = 20$. That curve does approach both axes, but it is not a hyperbola and it fails the constant product test, since $12 \times 20 = 240$. *(Inverse square used)*
- **D** (101.25 ms): The area ratio was squared and applied the way the area grows: $45 \times \left(\frac{12}{8}\right)^2 = 45 \times 2.25 = 101.25$. That is the curve $t \propto A^2$, which climbs away from both axes rather than closing on them, and it fails the constant product test since $12 \times 101.25 = 1215$. *(Direct square proportion)*
- **E** (90 ms): The constant was found correctly, $k = 8 \times 45 = 360$, but then divided by the increase in area instead of the area itself: $\frac{360}{12-8} = 90$. The curve gives the time at $A = 12$, so the division is $\frac{360}{12} = 30$, and a wider aperture cannot need a longer exposure than the 45 ms already recorded. *(Divided by the change in area)*
- **F** (3.75 ms): The recorded time was divided by the new area, $45 \div 12 = 3.75$. The division must be applied to the constant 360, not to a time; dividing a time by an area does not even leave a time. *(Divided the wrong quantity)*
- **G** (67.5 ms): The time was scaled up with the area, $45 \times \frac{12}{8} = 67.5$, which is direct proportion. That is a straight line through the origin, and it contradicts a curve that falls as the area grows. *(Direct proportion assumed)*

Common mistake. Treating a falling curve as a falling straight line. A straight line through the recorded point would cross the A axis at some finite area, which would mean a real setting needing no exposure time at all. The stated asymptotes rule that out, and the stated hyperbola then forces the reciprocal model.

Takeaway. The Cheat Code: A hyperbola with asymptotes on both axes means the product of the coordinates is constant. Multiply the pair you are given, divide by the new value, and you are done in two operations.

Time-saving tip. Speed Heuristic: skip the constant. The area is multiplied by $\frac{12}{8} = \frac{3}{2}$, so on a reciprocal curve the time is multiplied by $\frac{2}{3}$, and two thirds of 45 is 30.

Question 8

M2.10 · Number · ★★☆☆☆ · 90 s

A battery is $\frac{3}{4}$ charged. Using 0.35 of its capacity leaves what percentage?

A. 40%
B. 110%
C. 48.75%
D. 60%
E. 74.65%
F. 71.5%
G. 65%
H. 26.25%

Answer A. Key idea

A fraction, a decimal and a percentage are three notations for the same kind of number, and they can only be combined once they are all written against the same reference quantity. Here both the charge held and the amount used are measured against the battery's full capacity, so converting each to a percentage of that capacity makes the calculation a single subtraction rather than a mixture of unlike quantities.

Worked solution

1. Write the starting charge as a percentage

A percentage is a number of parts per hundred, so multiply the fraction by 100:

$$\frac{3}{4} = \frac{3}{4} \times 100\% = 75\%.$$

The quarter is 25%, and three of them give 75%, so the battery starts at 75% of capacity.

2. Write the amount used as a percentage of the same thing

$$0.35 = \frac{35}{100} = 35\%.$$

This is stated as a share of the capacity, the same reference as the 75%, so the two figures can now be compared directly. Had it been a share of the charge present it would have had to be multiplied by the 75% instead.

3. Subtract and check

$75\% - 35\% = 40\%.$

Sanity check in decimals: $0.75 - 0.35 = 0.40$, which is 40% of capacity. The answer is below the starting 75% and above zero, as it must be, and the battery is left a little under half charged.

Matches Option **A**.

Why the other options are wrong

- **B** (110%): Added the amount used instead of subtracting it: $75\% + 35\% = 110\%$, a charge above full capacity, which is impossible. *(Wrong operation)*
- **C** (48.75%): Read the 0.35 as a share of the charge present rather than of the capacity, so 35% of 75% was removed: $75 \times 0.65 = 48.75$, giving 48.75%. *(Wrong reference quantity)*
- **D** (60%): Answered how much of the capacity is empty rather than how much charge remains: the battery starts $100\% - 75\% = 25\%$ empty and a further 35% is used, so $25 + 35 = 60$. What is left is the other part, $100\% - 60\% = 40\%$. *(Gave the empty share)*
- **E** (74.65%): Subtracted the raw decimal from the percentage: $75 - 0.35 = 74.65$. The two figures have to be written in the same notation before they can be combined, and 0.35 of the capacity is 35% of it, so the subtraction is $75\% - 35\% = 40\%$. *(Decimal subtracted without converting)*
- **F** (71.5%): Converted 0.35 to 3.5% by multiplying by 10 rather than 100: $75\% - 3.5\% = 71.5\%$. *(Decimal to percentage slip)*
- **G** (65%): Subtracted from a full battery instead of a three quarters full one: $100\% - 35\% = 65\%$, ignoring the starting charge entirely. *(Starting value ignored)*
- **H** (26.25%): Combined the two figures with a product rather than a difference: $0.75 \times 0.35 = 0.2625$, that is 26.25%. That number is the charge used if the 0.35 were a share of the charge present, so it is an amount removed and not an amount left. *(Multiplied instead of subtracting)*

Common mistake. Subtracting 0.35 from 75 without converting, giving 74.65%, or converting 0.35 to 3.5% and giving 71.5%. A decimal becomes a percentage by multiplying by 100, so 0.35 is 35%, not 3.5% and not 0.35%.

Takeaway. The Cheat Code: Put every quantity into one notation before combining them: fraction to percentage multiply by 100, decimal to percentage multiply by 100, and check that both figures are measured against the same whole before you add or subtract.

Time-saving tip. Speed Heuristic: Stay in decimals for the whole item. $\frac{3}{4}$ is 0.75, so the answer is $0.75 - 0.35 = 0.4$, and only the final number needs the single conversion to 40%.

Question 9

M3.11 · Ratio and proportion · ★★★☆☆ · 90 s

A vineyard's yield falls by 20% each year while a disease goes untreated. Three harvests after it appeared the yield is 2560 kg. How large was the last healthy harvest?

- **A.** 6400 kg
- **B.** 4480 kg
- **C.** 3200 kg
- **D.** 4096 kg
- **E.** 5120 kg
- **F.** 4000 kg
- **G.** 6250 kg
- **H.** 5000 kg

Answer H. Key idea

A quantity that falls by a fixed percentage each year is multiplied by the same factor every year, so three years of decay amount to one multiplication by that factor cubed. Undoing the decay therefore means dividing by the cube, not adding the percentage back on. The multiplier for a 20% fall is $1 - 0.20 = 0.8$, and because each year's loss is taken from a smaller amount than the year before, repeated falls never simply add: three falls of 20% remove 48.8%, not 60%.

Worked solution

1. Write down the yearly multiplier

A fall of 20% leaves 80% of the yield standing, so each year multiplies the yield by

$$1 - 0.20 = 0.8$$

2. Compound it over the three harvests

Three successive falls multiply the healthy yield Y by

$$0.8^3 = 0.8 \times 0.8 \times 0.8 = 0.512$$

so $0.512Y = 2560$.

3. Reverse the multiplication

$$Y = \frac{2560}{0.512} = 5000 \text{ kg}$$

Dividing by 0.512 is awkward on paper, so divide by 0.8 three times instead, which is the same as multiplying by 1.25 three times: $2560 \to 3200 \to 4000 \to 5000$.

Sanity check by running the model forwards from 5000 kg: the three harvests come out as 4000 kg, then 3200 kg, then 2560 kg, each one a fifth down on the one before, and the last matches the figure given. The total loss is $1 - 0.512 = 0.488$, that is 48.8%, so the healthy harvest should be a little under twice the present one, and 5000 sits just below $2 \times 2560 = 5120$.

Matches Option **H**.

Why the other options are wrong

- **A** (6400 kg): Three falls of 20% were added into one fall of 60%, giving $2560 \div 0.4 = 6400$. Compounded over three years a 20% annual fall removes 48.8% in total, not 60%. *(Percentages added)*
- **B** (4480 kg): The correct reverse multiplier $\frac{1}{0.8} = 1.25$ was used, but its effect was added as a fixed amount three times: 25% of 2560 is 640, so $2560 + 3 \times 640 = 4480$. Compounding takes 25% of a larger figure each time, so the true steps up are 640, then 800, then 1000, reaching 5000 kg. *(Reverse rise added as a flat amount)*
- **C** (3200 kg): Only one year of decay was reversed: $2560 \div 0.8 = 3200$. That is the yield at the second harvest of the outbreak, not the healthy one, because two further falls still stand between it and the start. *(Only one year reversed)*
- **D** (4096 kg): The fall was treated as a fixed amount rather than a fixed percentage, using 20% of the final yield each time: 20% of 2560 is 512, and $2560 + 3 \times 512 = 4096$. Percentage decay strips a shrinking amount each year, so the earlier losses were larger than 512 kg. *(Fixed amount instead of fixed percentage)*
- **E** (5120 kg): The three year factor $0.8^3 = 0.512$ was rounded to 0.5, so the final yield was simply doubled: $2560 \div 0.5 = 5120$. The 0.012 is worth 120 kg here, and dividing by 0.512 exactly gives 5000 kg. *(Compound factor rounded to a half)*
- **F** (4000 kg): Only two of the three falls were undone: $2560 \div 0.8^2 = 2560 \div 0.64 = 4000$. That is the yield at the first harvest after the disease appeared, and one further division by 0.8 is needed to get back to the healthy one, giving 5000 kg. *(Two years reversed)*
- **G** (6250 kg): Four years of decay were reversed instead of three: $2560 \div 0.8^4 = 2560 \div 0.4096 = 6250$. The disease appeared three harvests before the one quoted, so the factor to undo is $0.8^3 = 0.512$. *(One year too many)*

Common mistake. Treating three years of 20% as a single fall of 60% and dividing by 0.4 to reach 6400 kg. Each year's 20% is taken from a smaller yield than the year before, so the losses cannot be added: their combined effect is $1 - 0.512 = 0.488$, a fall of 48.8%.

Takeaway. The Cheat Code: Repeated percentage change is repeated multiplication, so undoing it is repeated division by the same factor. Dividing by 0.8 is the same as multiplying by 1.25, and running the chain forwards at the end costs seconds and catches every slip.

Time-saving tip. Speed Heuristic: never divide by 0.512. Multiply by 1.25 three times instead, which is just adding a quarter each time: $2560 \to 3200 \to 4000 \to 5000$. Every step is a quarter of a round number, so the whole reversal is mental arithmetic.

Question 10

M2.5 · Number · ★★★☆☆ · 90 s

A courier walks 4 blocks east and 2 north. How many routes are possible?

A. 8
B. 360
C. 30
D. 6
E. 64
F. 15
G. 720
H. 10

Answer F. Key idea

A route is nothing more than the order in which the individual block moves are made, so the walk is a list of six moves of which four are east and two are north. Because the four east moves are indistinguishable from one another and so are the two north moves, a route is fixed as soon as you say which positions in the list the north moves occupy. Counting positions rather than listing whole routes turns an awkward enumeration into a short, systematic count.

Worked solution

1. Describe a route as a list of moves

Every route uses exactly 4 east moves and 2 north moves, so it is a list of 6 single block moves. No route can be longer or shorter, since the courier must gain four blocks east and two blocks north in total.

2. Fix a route by the positions of the north moves

The east moves are identical to each other and so are the north moves, so the whole route is determined once you say which two of the six places in the list hold the north moves. Listing those pairs of positions systematically, starting with the first north move in place 1:

$(1,2),(1,3),(1,4),(1,5),(1,6)$, then $(2,3),(2,4),(2,5),(2,6)$, then $(3,4),(3,5),(3,6)$, then $(4,5),(4,6)$, then $(5,6)$.

3. Count the pairs and check

That is $5+4+3+2+1=15$ pairs. The same total comes from $\frac{6\times 5}{2}=15$, where the division by 2 removes the double counting caused by the two north moves being interchangeable.

Sanity check by building the count junction by junction. Along the bottom row of junctions the counts are $1,1,1,1,1$; the middle row is $1,2,3,4,5$; the top row is $1,3,6,10,15$, since each junction is reached from the one to its left plus the one below it. The far corner reads 15, which agrees.

Matches Option **F**.

Why the other options are wrong

- **A** (8): Multiplied the leg lengths, $4\times 2=8$, as though a route were one choice of an east block paired with one choice of a north block. The two legs are not independent single choices. *(Product rule misapplied)*
- **B** (360): All six moves were ordered, $6!=720$, and the count was then divided by the $2!$ for the two identical north moves but not by the $4!$ for the four identical east moves: $\frac{720}{2}=360$. Both sets of repeats have to come out, and $\frac{720}{4!\times 2!}=\frac{720}{48}=15$. *(Only one repeat divided out)*

- **C** (30): Chose the two north positions in order: $6 \times 5 = 30$. Every route is then counted twice, once for each order of the two identical north moves, so this is exactly double the answer. *(Double counting identical moves)*
- **D** (6): Added the two leg lengths, $4 + 2 = 6$. That counts the number of blocks walked, not the number of orders in which they can be walked. *(Counted steps, not routes)*
- **E** (64): Treated each of the 6 moves as a free choice between east and north: $2^6 = 64$. Most of those sequences do not contain exactly four easts and two norths, so they do not end at the right corner. *(Constraint ignored)*
- **G** (720): The six moves were counted as six distinct objects and their orderings totalled: $6! = 720$. Swapping two east moves does not change the route walked, so each route has been counted $4! \times 2! = 48$ times over, and $\frac{720}{48} = 15$. *(Moves treated as distinguishable)*
- **H** (10): The two north moves were placed in different gaps around the four east moves. There are 5 such gaps, one before each east move and one after the last, so this gives $\frac{5 \times 4}{2} = 10$ ways. It discards the 5 routes in which both north moves are taken one straight after the other, both sitting in the same gap, and $10 + 5 = 15$. *(Adjacent north moves excluded)*

Common mistake. Treating the six moves as six distinguishable objects and counting orderings of them, or at the other extreme multiplying the two leg lengths together. The four east moves cannot be told apart, so swapping two of them does not create a new route.

Takeaway. The Cheat Code: A journey made of two kinds of identical steps is counted by choosing which places in the sequence the rarer step occupies: with 6 steps of which 2 are north, that is $\frac{6 \times 5}{2}$, and the junction by junction addition grid gives the same number as a free check.

Time-saving tip. Speed Heuristic: Count the rarer move, not the common one. Two north moves among six places gives $\frac{6 \times 5}{2} = 15$ in one line, whereas working with the four east moves needs $\frac{6 \times 5 \times 4 \times 3}{4 \times 3 \times 2}$ for the same answer.

END OF SOLUTIONS

Questions 11 to 27 are in the full paper

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EXAMPLE GRAMMAR SCHOOL

ESAT Mock Examination

Mathematics 2

Paper SMP-2026-09 · Version 1.5

NAME TEACHING GROUP DATE

Instructions

- Time allowed: **15 minutes**.
- There are **10 multiple-choice questions**. Each is worth 1 mark.
- There is no negative marking. Answer every question, even if you have to guess.
- Calculators are not permitted.
- Record your answers on the separate answer sheet. Only the answer sheet is marked.
- Formulae are not provided. The ESAT does not supply a formula sheet.

Note: the real ESAT is sat on screen at a Pearson test centre. This paper trains the mathematics and the pacing, not the on-screen interface.

Written for Example Grammar School by Lucas Rey, Engineering Science, University of Oxford; MRes, University of Cambridge.

Question 1

Simplify $\dfrac{x^{5/2} \cdot x^{-1/2}}{x^{3/2}}$.

A. 1
B. x^2
C. $x^{3/2}$
D. x
E. $x^{1/2}$
F. $x^{7/2}$
G. $x^{9/2}$
H. $x^{-1/2}$

Question 2

A pressure sensor's calibration line is $y = 2x + 5$. After the sensor is re-tuned its calibration line has a gradient 50% larger and an intercept on the y-axis 3 units lower. Give the coordinates of the point where the two calibration lines meet.

A. $(2,\ 9)$
B. $(-3,\ -1)$
C. $(-7,\ -9)$
D. $(5,\ 15)$
E. $(-3,\ -4)$
F. $(3,\ 11)$
G. $(0,\ 5)$
H. $(6,\ 17)$

Question 3

A lens surface is machined so that its cross-section satisfies $y = \dfrac{3x^6 + 5x^2\sqrt{x} - 2}{x^2}$ for $x > 0$, with x and y in millimetres. Find the gradient of the cross-section at $x = 1$.

A. $\dfrac{49}{2}$
B. 21
C. 11
D. $\dfrac{61}{4}$
E. $\dfrac{21}{2}$
F. $\dfrac{37}{2}$
G. $\dfrac{29}{2}$
H. $\dfrac{77}{4}$

Question 4

Rationalise the denominator of $\dfrac{\sqrt{7} - \sqrt{3}}{\sqrt{7} + \sqrt{3}}$ and simplify fully.

A. $\dfrac{5 + \sqrt{21}}{2}$
B. $\dfrac{5 - \sqrt{21}}{2}$
C. $\dfrac{10 - \sqrt{21}}{2}$
D. $\dfrac{5 - \sqrt{21}}{5}$
E. $\dfrac{2 - \sqrt{21}}{2}$

Question 5

Functions f and g are integrable on $1 \le x \le 9$. The integral of f is 14 over $1 \le x \le 5$ and 22 over $5 \le x \le 9$. Given $\int_1^9 (f(x) + g(x))\,dx = 92$, find $\int_1^9 g(x)\,dx$.

- **A.** 74
- **B.** 84
- **C.** 56
- **D.** 70
- **E.** 128
- **F.** 100
- **G.** 78
- **H.** 36

Question 6

A curve $y = a^x$ has $a > 1$. The chord joining its points at $x = 1$ and $x = 4$ has gradient 26. Find y when $x = -2$.

- **A.** 9
- **B.** $\frac{1}{6}$
- **C.** $\frac{1}{3}$
- **D.** $\frac{1}{81}$
- **E.** $-\frac{1}{9}$
- **F.** $\frac{1}{27}$
- **G.** -9
- **H.** $\frac{1}{9}$

Question 7

A horizontal line $y = k$ never meets the curve $y = x^2 - 2x - 7$. Find all possible values of k.

- **A.** $k < -8$
- **B.** $k < -6$
- **C.** $k > -8$
- **D.** $k > 8$
- **E.** $k < -7$
- **F.** $k > -6$
- **G.** $k < -11$
- **H.** $k \le -8$

Question 8

Each of the eight functions below is defined for all real x. Which is one-to-one?

A. $f(x) = \sin x$
B. $f(x) = x^4 + x$
C. $f(x) = |x| - 4$
D. $f(x) = x^3 - 3x$
E. $f(x) = x^3 + 2x$
F. $f(x) = 2^x - x$
G. $f(x) = 2^{x^2}$
H. $f(x) = x^2 - 6x$

Question 9

Let $\log_a 2 = p$ and $\log_a 3 = q$, where $a > 1$ is a fixed base. Express $\log_a\left(\frac{9\sqrt{a}}{8}\right)$ in terms of p and q.

A. $3q - 2p + \frac{1}{2}$
B. $2q - 3p + 2$
C. $q^2 - p^3 + \frac{1}{2}$
D. $\frac{2q}{3p} + \frac{1}{2}$
E. $2q + 3p + \frac{1}{2}$
F. $2q - 3p + \frac{1}{2}$
G. $2q - 3p + 1$
H. $3p - 2q - \frac{1}{2}$

Question 10

For an acute angle θ, $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = \frac{25}{12}$. Find the exact value of $\tan\theta$.

A. $\frac{3}{4}$
B. $\frac{24}{25}$
C. $\frac{4}{3}$
D. $\frac{24}{7}$
E. $\frac{7}{24}$

END OF PAPER

Questions 11 to 27 are in the full paper

This is an extract. The 10 questions before this page are real questions from a commissioned question paper, shown in full and with nothing withheld, so that the standard can be judged rather than described.

The rest of the paper is not hidden behind this page. It was never in this file. Each commissioned paper is written for one school, recorded against it, and issued to no other, so the questions in it are not published anywhere, including here.

A commissioned paper is NOT this paper with the missing questions restored. Every question in it is different from every question here, the 10 shown above included. Your school's paper is drawn fresh, and no question that has ever been published, on this page or anywhere else, can appear in it.

A full paper runs to 27 questions in this module alone, with a question paper, an answer sheet, a mark scheme and a worked solution for every question, across all five ESAT modules.

Written by researchers in engineering, physics and mathematics at Oxford and Cambridge, and checked question by question before it is issued.

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EXAMPLE GRAMMAR SCHOOL

ESAT Mock: Answer Sheet

Mathematics 2

Paper SMP-2026-09 · Version 1.5

NAME

TEACHING GROUP

DATE

Fill in one bubble per question. If you change your mind, cross out the old mark clearly and fill the new bubble. Unfilled or doubly filled rows score zero.

Q	A	B	C	D	E	F	G	H
1	○	○	○	○	○	○	○	○
2	○	○	○	○	○	○	○	○
3	○	○	○	○	○	○	○	○
4	○	○	○	○	○			
5	○	○	○	○	○	○	○	○
6	○	○	○	○	○	○	○	○
7	○	○	○	○	○	○	○	○
8	○	○	○	○	○	○	○	○
9	○	○	○	○	○	○	○	○
10	○	○	○	○	○			

MARKER INITIALS

TOTAL OUT OF 10

EXAMPLE GRAMMAR SCHOOL

ESAT Mock: Mark Scheme

Mathematics 2

Paper SMP-2026-09 · Version 1.5

Answer grid

Q	Ans	Q	Ans	Q	Ans
1	**E**	10	**D**		
2	**F**				
3	**F**				
4	**B**				
5	**C**				
6	**H**				
7	**A**				
8	**E**				
9	**F**				

Totals

10 marks available, 1 mark per question. There is no negative marking, so a wrong answer scores zero and never a negative. Only the answer sheet is marked.

Diagnostic table

Use this to see which topics the cohort is losing marks on.

Q	Ans	Node	Group	Topic	Difficulty
1	**E**	MM1.1	MM1	Algebra	★★☆☆☆
2	**F**	MM8.3	MM8	Graphs and transformations	★★★☆☆
3	**F**	MM6.2	MM6	Differentiation	★★★☆☆
4	**B**	MM1.2	MM1	Algebra and functions	★★★☆☆
5	**C**	MM7.4	MM7	Integration	★★☆☆☆
6	**H**	MM5.1	MM5	Exponentials and logarithms	★★★☆☆
7	**A**	MM1.3	MM1	Algebra and functions	★★★☆☆
8	**E**	MM1.7	MM1	Algebra and functions	★★☆☆☆
9	**F**	MM5.2	MM5	Exponentials and logarithms	★★★☆☆
10	**D**	MM4.5	MM4	Trigonometry	★★★★☆

Marks available by topic group

Group	Topic	Marks
MM1	Algebra and functions	4
MM4	Trigonometry	1
MM5	Exponentials and logarithms	2
MM6	Differentiation	1
MM7	Integration	1
MM8	Graphs and transformations	1
Total		**10**

Worked solutions for every question are in the separate worked solutions document.

EXAMPLE GRAMMAR SCHOOL

ESAT Mock: Worked Solutions

Mathematics 2

Paper SMP-2026-09 · Version 1.5

Staff copy

This document contains the answers. Keep it away from candidates until the paper has been sat. Marking guidance is in the separate mark scheme.

Question 1

MM1.1 · Algebra · ★★☆☆☆ · 30 s

Simplify $\dfrac{x^{5/2} \cdot x^{-1/2}}{x^{3/2}}$.

- **A.** 1
- **B.** x^2
- **C.** $x^{3/2}$
- **D.** x
- **E.** $x^{1/2}$
- **F.** $x^{7/2}$
- **G.** $x^{9/2}$
- **H.** $x^{-1/2}$

Answer E. Key idea

Numerator: $x^{5/2+(-1/2)} = x^2$. Then $x^2/x^{3/2} = x^{1/2}$.

Worked solution

1. Simplify the numerator

$x^{5/2} \cdot x^{-1/2} = x^{5/2-1/2} = x^{4/2} = x^2$

2. Divide by the denominator

$\dfrac{x^2}{x^{3/2}} = x^{2-3/2} = x^{1/2}$

Matches Option **E**.

Why the other options are wrong

- **A** (1): The negative index is used twice: $x^{-1/2}$ lowers the numerator to $x^{5/2-1/2} = x^2$ and is also moved into the denominator as $x^{1/2}$, giving $x^2/x^{3/2+1/2} = x^2/x^2 = 1$. The factor belongs either above with index $-1/2$ or below with index $+1/2$, never both. *(Calculation / Conceptual Error)*
- **B** (x^2): Simplifies the numerator correctly to $x^{5/2-1/2} = x^2$ and stops there, never dividing by the denominator. The index of $x^{3/2}$ still has to be subtracted: $2 - 3/2 = 1/2$. *(Calculation / Conceptual Error)*
- **C** ($x^{3/2}$): Reads $x^{-1/2}$ as $x^{1/2}$, dropping its minus sign, so the numerator becomes $x^{5/2+1/2} = x^3$ and dividing gives $x^{3-3/2} = x^{3/2}$. With the sign kept the numerator is x^2 and the result is $x^{1/2}$. *(Calculation / Conceptual Error)*

- **D** (x): The $x^{-1/2}$ factor is lost when the fraction is copied, leaving $x^{5/2}/x^{3/2} = x^{5/2-3/2} = x^1$, printing x. The missing factor is worth $-1/2$ on the index, which is exactly the difference between x and the key $x^{1/2}$. *(Factor dropped from the numerator)*
- **F** ($x^{7/2}$): Added the denominator exponent instead of subtracting it. *(Sign Error)*
- **G** ($x^{9/2}$): Every index is added and both negatives are dropped, the $-1/2$ in the numerator and the subtraction owed to the denominator: $5/2 + 1/2 + 3/2 = 9/2$, printing $x^{9/2}$. This is one step past the $x^{7/2}$ slip, which at least keeps the sign on the $-1/2$. *(Both minus signs ignored)*
- **H** ($x^{-1/2}$): The numerator is simplified correctly to $x^{5/2-1/2} = x^2$, then the last subtraction is taken the wrong way round: the numerator index is subtracted from the denominator index, $3/2 - 2 = -1/2$, printing $x^{-1/2}$. Dividing takes numerator index minus denominator index, $2 - 3/2 = 1/2$. *(Subtraction reversed at the division step)*

Common mistake. Adding all the exponents (including the denominator): $5/2 - 1/2 + 3/2 = 7/2$.

Takeaway. The Cheat Code: Combine every index in one line, adding the indices of factors multiplied together and subtracting the index of anything divided, with each sign kept: here $5/2 - 1/2 - 3/2 = 1/2$.

Time-saving tip. Speed Heuristic: All three indices share the denominator 2, so work with the numerators alone: $5 - 1 - 3 = 1$ gives the index $1/2$ and the answer $x^{1/2}$, with no intermediate power written.

Question 2

MM8.3 · Graphs and transformations · ★★★☆☆ · 90 s

A pressure sensor's calibration line is $y = 2x + 5$. After the sensor is re-tuned its calibration line has a gradient 50% larger and an intercept on the y-axis 3 units lower. Give the coordinates of the point where the two calibration lines meet.

A. $(2,\ 9)$
B. $(-3,\ -1)$
C. $(-7,\ -9)$
D. $(5,\ 15)$
E. $(-3,\ -4)$
F. $(3,\ 11)$
G. $(0,\ 5)$
H. $(6,\ 17)$

Answer F. Key idea

Altering m and altering c do two different things to a straight line. The gradient m controls steepness, so changing it pivots the line about the point where it crosses the y-axis, while c is that crossing itself, so changing it slides every point of the line vertically by the same amount. Once both parameters of the second line are known, the point the two lines have in common is found by setting their expressions for y equal.

Worked solution

Step 1: Read the two parameters of the original line.

Written in the form $y = mx + c$, the line $y = 2x + 5$ has $m = 2$ and $c = 5$, so its gradient is 2 and it crosses the y-axis at $(0, 5)$.

Step 2: Alter m.

A gradient 50% larger means multiplying by $\frac{3}{2}$, so $m' = \frac{3}{2} \times 2 = 3$. This change pivots the line about its y-axis crossing, so the crossing is still at 5 after this step alone.

Step 3: Alter c.

Lowering the intercept by 3 gives $c' = 5 - 3 = 2$, a pure vertical translation. The re-tuned calibration line is therefore $y = 3x + 2$.

Step 4: Set the two expressions for y equal.

$2x + 5 = 3x + 2$, so $5 - 2 = 3x - 2x$, giving $x = 3$.

Step 5: Find y and check on both lines.

The original line gives $y = 2(3) + 5 = 11$ and the re-tuned line gives $y = 3(3) + 2 = 11$, so the meeting point is $(3, 11)$. Sanity check: at $x = 0$ the new line sits 3 below the old one, and it gains $3 - 2 = 1$ unit on it for every unit of x, so it needs exactly 3 units of x to catch up, which agrees with $x = 3$.

Matches Option **F**.

Why the other options are wrong

- **A** ($(2,\ 9)$): Reading '3 units lower' as an intercept of 3 rather than $5 - 3 = 2$ gives $y = 3x + 3$. Then $2x + 5 = 3x + 3$ gives $x = 2$, and $y = 2(2) + 5 = 9$, printing $(2,\ 9)$. *(Misread of the intercept change)*
- **B** ($(-3,\ -1)$): Lifting the intercept instead of lowering it gives $c' = 5 + 3 = 8$ and the line $y = 3x + 8$. Then $2x + 5 = 3x + 8$ gives $-3 = x$, and $y = 2(-3) + 5 = -1$, printing $(-3,\ -1)$. *(Wrong direction for the change in c)*
- **C** ($(-7,\ -9)$): A sign slip when collecting terms: writing $2x - 3x = 2 + 5$ instead of $2x - 3x = 2 - 5$ gives $-x = 7$, so $x = -7$ and $y = 2(-7) + 5 = -9$, printing $(-7,\ -9)$. *(Sign error moving a constant across the equals sign)*
- **D** ($(5,\ 15)$): Steepening is taken to swing the line about $(0, 0)$ rather than about its y-axis crossing, so the re-tuned line is written as $y = 3x$ with no intercept at all. Then $2x + 5 = 3x$ gives $x = 5$, and $y = 2(5) + 5 = 15$, printing $(5,\ 15)$. Changing m pivots about $(0, 5)$, and the intercept only moves when c is altered. *(Line pivoted about the origin)*
- **E** ($(-3,\ -4)$): The intercept drop is applied to the original line and only the steepening to the re-tuned one, giving $y = 2x + 2$ and $y = 3x + 5$. Then $2x + 2 = 3x + 5$ gives $x = -3$, and $y = 2(-3) + 2 = -4$, printing $(-3,\ -4)$. Both changes describe the line after re-tuning, so both belong to the same equation. *(The two changes applied to opposite lines)*
- **G** ($(0,\ 5)$): Applying only the gradient change leaves c at 5, giving $y = 3x + 5$. Two lines sharing a y-intercept meet on the y-axis, so $2x + 5 = 3x + 5$ gives $x = 0$ and $y = 5$, printing $(0,\ 5)$. *(Second alteration omitted)*
- **H** ($(6,\ 17)$): The gradient 50% larger is read as adding 0.5 to it rather than multiplying by $\frac{3}{2}$, giving $m' = 2 + 0.5 = 2.5$ and the line $y = 2.5x + 2$. Then $2x + 5 = 2.5x + 2$ gives $0.5x = 3$, so $x = 6$ and $y = 2(6) + 5 = 17$, printing $(6,\ 17)$. Fifty per cent of 2 is 1, so the new gradient is 3. *(Percentage added instead of multiplied)*

Common mistake. Assuming that steepening a line also moves where it crosses the y-axis. Changing m pivots the line about $(0, c)$, so the crossing stays at 5 until c itself is altered. Candidates who instead pivot about the origin write the new line as $y = 3x$, solve $2x + 5 = 3x$ and report $(5, 15)$.

Takeaway. The Cheat Code: In $y = mx + c$, m is a pivot and c is a lift: change m and the line turns about $(0, c)$, change c and every point slides vertically. Two lines then close their intercept gap at a rate equal to the difference of their gradients.

Time-saving tip. Speed Heuristic: Skip the simultaneous equations. The gradients differ by 1 and the intercepts differ by 3, so the lower line closes the gap at 1 per unit of x and they meet at $x = 3$; one substitution gives $y = 11$.

Question 3

MM6.2 · Differentiation · ★★★☆☆ · 90 s

A lens surface is machined so that its cross-section satisfies $y = \dfrac{3x^6 + 5x^2\sqrt{x} - 2}{x^2}$ for $x > 0$, with x and y in millimetres. Find the gradient of the cross-section at $x = 1$.

A. $\dfrac{49}{2}$
B. 21
C. 11
D. $\dfrac{61}{4}$
E. $\dfrac{21}{2}$
F. $\dfrac{37}{2}$
G. $\dfrac{29}{2}$
H. $\dfrac{77}{4}$

Answer F. Key idea

The power rule differentiates x^n to nx^{n-1} for any rational n, positive, negative or fractional, but it applies only to a sum of separate powers. An expression written as one fraction is not in that form, so the first move is to divide the denominator into each term and rewrite roots and reciprocals as indices. Only then does term-by-term differentiation apply, and a gradient at a particular point is the value of that derivative there.

Worked solution

1. Simplify into a sum of powers

Dividing each term of the numerator by x^2, and using $x^2\sqrt{x} = x^{5/2}$:

$$y = \frac{3x^6}{x^2} + \frac{5x^{5/2}}{x^2} - \frac{2}{x^2} = 3x^4 + 5x^{1/2} - 2x^{-2}$$

In plainer form this is $y = 3x^4 - \dfrac{2}{x^2} + 5\sqrt{x}$.

2. Differentiate term by term

$$\frac{dy}{dx} = 3(4)x^3 + 5\left(\tfrac{1}{2}\right)x^{-1/2} - 2(-2)x^{-3}$$

$$\frac{dy}{dx} = 12x^3 + 4x^{-3} + \frac{5}{2}x^{-1/2} = 12x^3 + \frac{4}{x^3} + \frac{5}{2\sqrt{x}}$$

Note the middle sign: a negative coefficient multiplied by a negative index gives a positive term.

3. Substitute the required value

At $x = 1$ every power of x is 1, so

$$\frac{dy}{dx} = 12(1) + \frac{4}{1} + \frac{5}{2 \times 1} = 12 + 4 + \frac{5}{2}$$

4. Combine

$$12 + 4 + \frac{5}{2} = 16 + \frac{5}{2} = \frac{32 + 5}{2} = \frac{37}{2}$$

Sanity check on the simplification: multiplying back, $x^2\left(3x^4+5x^{1/2}-2x^{-2}\right)=3x^6+5x^{5/2}-2$, which is the numerator given. All three derivative terms are positive at $x=1$, so the surface is rising steeply there, and a gradient of $\frac{37}{2}$, that is 18.5, is consistent with the dominant $12x^3$ term.

Matches Option **F**.

Why the other options are wrong

- **A** $(\frac{49}{2})$: The x^2 is cancelled against $3x^6$ and the rest of the numerator is left as it stands, giving $y=3x^4+5x^{5/2}-2$. Differentiating that gives $12x^3+\frac{25}{2}x^{3/2}$ with the constant vanishing, and at $x=1$ the value is $12+\frac{25}{2}=\frac{49}{2}$. Every term of the numerator must be divided by x^2. *(Denominator divided into the first term only)*
- **B** (21): Writes the derivative of $5x^{1/2}$ as $5x^{-1/2}$, lowering the index without multiplying by it, so the last term contributes 5 instead of $\frac{5}{2}$ and the total is $12+4+5=21$. The rule multiplies by the old index as well as reducing it. *(Index reduced but coefficient not multiplied)*
- **C** (11): $x^2\sqrt{x}$ is read as $x^{2\times1/2}=x$ instead of $x^{2+1/2}=x^{5/2}$, so the middle term becomes $\frac{5x}{x^2}=5x^{-1}$, whose derivative $-5x^{-2}$ is worth -5 at $x=1$. The total is $12-5+4=11$. Multiplying powers of the same base adds the indices. *(Indices multiplied when powers are multiplied)*
- **D** $(\frac{61}{4})$: Differentiates top and bottom separately: the numerator gives $18x^5+\frac{25}{2}x^{3/2}$ and the denominator gives $2x$, and at $x=1$ that is $\frac{18+12.5}{2}=\frac{61}{4}$. Differentiation does not distribute over a division, so the fraction must be simplified into separate powers first. *(Quotient differentiated piecewise)*
- **E** $(\frac{21}{2})$: Differentiates $-2x^{-2}$ to $-4x^{-3}$, keeping the minus sign, so the value becomes $12-4+\frac{5}{2}=\frac{21}{2}$. Bringing the index -2 down onto the coefficient -2 multiplies two negatives, so the term is $+4x^{-3}$ and contributes $+4$. *(Sign error on the reciprocal term)*
- **G** $(\frac{29}{2})$: Reads $-\frac{2}{x^2}$ as a constant because its numerator carries no x, so the middle term is dropped and the total is $12+0+\frac{5}{2}=\frac{29}{2}$. The variable sits in the denominator, so the term is $-2x^{-2}$ and its derivative is $+4x^{-3}$, worth 4 at $x=1$. *(Constant-looking term differentiated to zero)*
- **H** $(\frac{77}{4})$: Dividing by x^2 is carried out by halving each index rather than subtracting 2: x^6 becomes x^3 and $x^{5/2}$ becomes $x^{5/4}$, giving $y=3x^3+5x^{5/4}-2x^{-2}$. Its derivative $9x^2+\frac{25}{4}x^{1/4}+4x^{-3}$ is $9+\frac{25}{4}+4=\frac{77}{4}$ at $x=1$. Division subtracts indices, so the terms are x^4 and $x^{1/2}$. *(Indices divided instead of subtracted)*

Common mistake. Differentiating the numerator and the denominator separately, as though a quotient could be handled a piece at a time. Dividing through first is what makes the power rule legal here, and it costs one line because every term of the numerator divides cleanly by x^2.

Takeaway. **The Cheat Code:** Get the expression into the form sum of kx^n before differentiating anything. Divide out the denominator, write roots as fractional indices and reciprocals as negative indices, and the whole problem reduces to applying nx^{n-1} three times.

Time-saving tip. Speed Heuristic: The gradient is wanted at $x=1$, where every power equals 1, so only the coefficients matter. Simplify to $3x^4+5x^{1/2}-2x^{-2}$ and read off 3×4, $5\times\frac{1}{2}$ and $(-2)\times(-2)$, then add: no powers need evaluating at all.

Question 4

MM1.2 · Algebra and functions · ★★★☆☆ · 85 s

Rationalise the denominator of $\dfrac{\sqrt{7}-\sqrt{3}}{\sqrt{7}+\sqrt{3}}$ and simplify fully.

A. $\dfrac{5+\sqrt{21}}{2}$

B. $\dfrac{5-\sqrt{21}}{2}$

C. $\dfrac{10-\sqrt{21}}{2}$

D. $\dfrac{5-\sqrt{21}}{5}$

E. $\dfrac{2-\sqrt{21}}{2}$

Answer B. Key idea

A denominator containing a sum of two surds is cleared by multiplying above and below by its conjugate, because a conjugate pair multiplies out as a difference of two squares and leaves a rational number behind. When the numerator happens to be that same conjugate, the multiplication squares it, so the cross term of the expansion survives and the simplified answer still carries a surd on top.

Worked solution

1. Multiply by the conjugate of the denominator

The denominator is $\sqrt{7}+\sqrt{3}$, whose conjugate is $\sqrt{7}-\sqrt{3}$. Multiplying numerator and denominator by the same quantity leaves the value unchanged:

$$\frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}+\sqrt{3}} = \frac{(\sqrt{7}-\sqrt{3})(\sqrt{7}-\sqrt{3})}{(\sqrt{7}+\sqrt{3})(\sqrt{7}-\sqrt{3})} = \frac{(\sqrt{7}-\sqrt{3})^2}{(\sqrt{7})^2-(\sqrt{3})^2} = \frac{(\sqrt{7}-\sqrt{3})^2}{7-3}$$

The denominator is now the rational number 4.

2. Expand the numerator and divide through

$$(\sqrt{7}-\sqrt{3})^2 = 7-2\sqrt{7}\sqrt{3}+3 = 10-2\sqrt{21}$$

so the fraction becomes

$$\frac{10-2\sqrt{21}}{4} = \frac{2\left(5-\sqrt{21}\right)}{4} = \frac{5-\sqrt{21}}{2}$$

Both terms of the numerator carry the factor 2, so the factor cancels out of the whole bracket, not out of one term only.

Sanity check

With $\sqrt{7}\approx 2.646$ and $\sqrt{3}\approx 1.732$, the original quotient is about $0.914 \div 4.378 \approx 0.209$. Using $\sqrt{21}\approx 4.583$, the answer gives $(5-4.583)\div 2\approx 0.209$, which agrees. The value must also be positive and smaller than 1, since the numerator is the smaller of the two surd expressions, and it is.

Matches Option **B**.

Why the other options are wrong

- **A** ($\frac{5+\sqrt{21}}{2}$): The cross term was expanded with the wrong sign: $(\sqrt{7}-\sqrt{3})^2$ was written as $7+2\sqrt{21}+3=10+2\sqrt{21}$. Dividing by the correct denominator 4 then gives $\frac{10+2\sqrt{21}}{4}=\frac{5+\sqrt{21}}{2}\approx 4.79$, which is far larger than 1 even though the original fraction has the smaller expression on top. *(Cross-term sign slip)*
- **C** ($\frac{10-\sqrt{21}}{2}$): From the correct $\frac{10-2\sqrt{21}}{4}$, the factor 2 was cancelled against the 4 but applied only to the surd term, leaving $\frac{10-\sqrt{21}}{2}$. The 10 must be halved as well, since the whole numerator is being divided. *(Partial cancellation)*
- **D** ($\frac{5-\sqrt{21}}{5}$): The denominator was multiplied out as $\sqrt{7}\times\sqrt{7}+\sqrt{3}\times\sqrt{3}=7+3=10$ rather than as the difference of two squares $7-3=4$. That gives $\frac{10-2\sqrt{21}}{10}=\frac{5-\sqrt{21}}{5}\approx 0.083$, which is well below the true value of about 0.209. *(Denominator added instead of differenced)*
- **E** ($\frac{2-\sqrt{21}}{2}$): The second square was subtracted rather than added when expanding the numerator: $7-2\sqrt{21}-3=4-2\sqrt{21}$. Dividing by 4 gives $\frac{4-2\sqrt{21}}{4}=\frac{2-\sqrt{21}}{2}\approx -1.29$, a negative value, although both $\sqrt{7}-\sqrt{3}$ and $\sqrt{7}+\sqrt{3}$ are positive. *(Second square subtracted in the expansion)*

Common mistake. Multiplying numerator and denominator by the denominator itself instead of by its conjugate. That produces $\frac{(\sqrt{7}-\sqrt{3})(\sqrt{7}+\sqrt{3})}{(\sqrt{7}+\sqrt{3})^2}=\frac{4}{10+2\sqrt{21}}$, which still has a surd underneath, so nothing has been rationalised and the work has to be started again.

Takeaway. The Cheat Code: When the top and bottom of a surd fraction are conjugates of one another, one multiplication does everything: the denominator collapses to a plain integer and the numerator becomes a perfect square. Expect the answer in the form $\frac{p-q\sqrt{r}}{s}$, and divide every term of the numerator by that integer.

Time-saving tip. Speed Heuristic: compute the denominator first. Here $(\sqrt{7})^2-(\sqrt{3})^2=4$, and the expanded numerator $10-2\sqrt{21}$ is even, so the printed answer must sit over 2, which discards the option written over 5 at once. A single decimal estimate near 0.21 then separates the four remaining forms without any further algebra.

Question 5

MM7.4 · Integration · ★★☆☆☆ · 90 s

Functions f and g are integrable on $1\le x\le 9$. The integral of f is 14 over $1\le x\le 5$ and 22 over $5\le x\le 9$. Given $\int_1^9 (f(x)+g(x))\,dx=92$, find $\int_1^9 g(x)\,dx$.

A. 74
B. 84
C. 56
D. 70
E. 128
F. 100
G. 78
H. 36

Answer C. Key idea

> Definite integrals over ranges that meet end to end add to give the integral over the joined range, and over one shared range the integral of a sum of functions equals the sum of the two integrals. Both facts come from areas stacking, so a question built on them needs no formula for either function.

Worked solution

1. Join the two ranges for f

$[1, 5]$ and $[5, 9]$ are contiguous: they meet at $x = 5$ and between them cover $[1, 9]$, so their integrals add.

2. Total the integral of f

$$\int_1^9 f(x)\,dx = \int_1^5 f(x)\,dx + \int_5^9 f(x)\,dx = 14 + 22 = 36$$

3. Split the combined integrand over the equal range

Both integrals now run over the same range $[1, 9]$, so

$$\int_1^9 (f(x) + g(x))\,dx = \int_1^9 f(x)\,dx + \int_1^9 g(x)\,dx$$

giving $92 = 36 + \int_1^9 g(x)\,dx$.

4. Solve for the unknown integral

$$\int_1^9 g(x)\,dx = 92 - 36 = 56$$

Sanity check

Rebuilding the combined value from the two parts: $36 + 56 = 92$, the figure given in the question.

Matches Option **C**.

Why the other options are wrong

- **A** (74): Because $[1, 5]$ and $[5, 9]$ each cover half of $[1, 9]$, the two values are averaged rather than added: $\frac{14 + 22}{2} = 18$ is taken as $\int_1^9 f$, and $92 - 18 = 74$. Each figure is already the whole integral over its own sub-range, so ranges that meet end to end add: $14 + 22 = 36$. *(Contiguous pieces averaged instead of added)*
- **B** (84): The contiguous pieces were subtracted rather than added, $22 - 14 = 8$, and then $92 - 8 = 84$. Ranges that join end to end add. *(Contiguous ranges subtracted)*
- **D** (70): Only the second piece was removed, $92 - 22 = 70$, leaving out the 14 that f contributes over $[1, 5]$. *(First sub-range dropped)*
- **E** (128): The two pieces of f are joined correctly to $14 + 22 = 36$, then $92 = 36 + \int_1^9 g$ is rearranged by adding rather than subtracting: $92 + 36 = 128$. The integral of g is what is left of the combined 92 after f's share is removed, so it must be smaller than 92, not larger. *(Sign slip rearranging for the unknown)*
- **F** (100): Written out in full the equation is $92 = 14 + 22 + \int_1^9 g$. Moving the 14 across correctly but carrying the 22 over with its sign unchanged gives $\int_1^9 g = 92 - 14 + 22 = 100$. Both pieces belong to f, so both are subtracted. *(Only one piece of f changed sign)*
- **G** (78): Only the first piece was removed, $92 - 14 = 78$, leaving out the 22 that f contributes over $[5, 9]$. *(Second sub-range dropped)*
- **H** (36): The work stopped after joining the contiguous pieces: $14 + 22 = 36$ is $\int_1^9 f(x)\,dx$, and the question asks for the integral of g. *(Answered the wrong quantity)*

Common mistake. Subtracting the two pieces of f instead of adding them. The ranges $[1,5]$ and $[5,9]$ follow one after the other rather than one sitting inside the other, so their values combine as $14+22$.

Takeaway. The Cheat Code: Two rules cover this whole family: ranges that meet end to end add their integrals, and over one shared range the integral of a sum is the sum of the integrals. Neither rule asks what f or g actually is.

Time-saving tip. Speed Heuristic: Collapse the two pieces of f to 36 before reading anything else, and the question reduces to the single subtraction $92-36$.

Question 6

MM5.1 · Exponentials and logarithms · ★★★☆☆ · 90 s

A curve $y = a^x$ has $a > 1$. The chord joining its points at $x = 1$ and $x = 4$ has gradient 26. Find y when $x = -2$.

A. 9
B. $\frac{1}{6}$
C. $\frac{1}{3}$
D. $\frac{1}{81}$
E. $-\frac{1}{9}$
F. $\frac{1}{27}$
G. -9
H. $\frac{1}{9}$

Answer H. Key idea

A curve $y = a^x$ with $a > 1$ is completely determined by its base: it passes through $(0,1)$, multiplies its height by a at every unit step to the right, and stays strictly positive because a positive base raised to any power is positive. That means the coordinates of any point on it are (x, a^x), so a geometric condition such as the gradient of a chord becomes an ordinary algebraic equation in a. Negative values of x are handled by the reciprocal rule $a^{-n} = \frac{1}{a^n}$, which is why the curve falls towards the axis on the left without ever reaching it.

Worked solution

1. Write down the two points

Every point of the curve has the form $(x,\ a^x)$, so the chord joins

$(1,\ a)$ and $(4,\ a^4)$

2. Turn the gradient into an equation in the base

$$\text{gradient} = \frac{a^4 - a}{4-1} = 26$$

$$a^4 - a = 78$$

3. Solve for the base

Factorise the left side as $a(a^3-1) = 78$. Testing simple bases greater than one: $a = 2$ gives $2(8-1) = 14$, which is too small, and $a = 3$ gives $3(27-1) = 3 \times 26 = 78$, which is exact. So $a = 3$.

This is the only solution with $a > 1$, because both a and $a^3 - 1$ increase as a increases, so their product is strictly increasing and can hit 78 only once.

4. Evaluate the curve at the required point

$y = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

Sanity check: the curve is $y = 3^x$, which passes through $(1, 3)$ and $(4, 81)$, and the chord between them has gradient $\frac{81-3}{3} = \frac{78}{3} = 26$ as required. At $x = -2$ the curve lies to the left of $(0, 1)$, so the height must be positive and smaller than 1, and $\frac{1}{9}$ is.

Matches Option **H**.

Why the other options are wrong

- **A** (9): Evaluates $3^2 = 9$, ignoring the minus sign in $x = -2$. That is the height of the curve at $x = +2$, and the two are reciprocals of one another, so the required value is $\frac{1}{9}$. *(Sign of the exponent dropped)*
- **B** ($\frac{1}{6}$): Finds $a = 3$ correctly, then reads 3^{-2} as $\frac{1}{3 \times 2} = \frac{1}{6}$, multiplying the base by the index instead of raising it. The index counts repeated multiplication, so $3^2 = 9$ and the height is $\frac{1}{9}$. *(Index treated as a multiplier)*
- **C** ($\frac{1}{3}$): Works back from the known point $(1, 3)$, where each unit step to the left divides the height by 3. The gap from $x = 1$ to $x = -2$ is three steps, giving $3 \div 3^3 = \frac{1}{9}$, but counting it as $|-2| = 2$ steps gives $3 \div 3^2 = \frac{1}{3}$. The number of steps is $1 - (-2) = 3$, not the size of the target x. *(Steps back along the curve miscounted)*
- **D** ($\frac{1}{81}$): Carries the value $3^4 = 81$ across from the trial in the previous step and writes $\frac{1}{81}$. That is the height at $x = -4$, not $x = -2$; the required power is $3^2 = 9$. *(Wrong power reciprocated)*
- **E** ($-\frac{1}{9}$): Finds $a = 3$ correctly, then writes $3^{-2} = -\frac{1}{3^2} = -\frac{1}{9}$, applying the reciprocal rule and carrying the minus sign into the value on top of it. The negative index does one job only, inverting the power, and $y = 3^x$ is positive at every x, so the height is $+\frac{1}{9}$. *(Reciprocal taken and the minus kept as well)*
- **F** ($\frac{1}{27}$): Finds $a = 3$, then evaluates with the run of the chord, $4 - 1 = 3$, in place of the 2 in $x = -2$, giving $3^{-3} = \frac{1}{27}$. The 3 was used up in forming the gradient equation $a^4 - a = 78$; the point asked for is $x = -2$, so the power needed is $3^2 = 9$. *(Chord run substituted for the required x)*
- **G** (-9): Interprets 3^{-2} as $-3^2 = -9$. A negative index inverts, it does not negate: $y = 3^x$ is positive for every x, so no point of the curve can lie below the horizontal axis. *(Negative index read as a negative value)*

Common mistake. Treating the chord gradient as though the curve were a straight line and writing $a = 26$, or dividing by the wrong run. The rise is the difference of two powers, $a^4 - a$, and the run is $4 - 1 = 3$, so the base satisfies a quartic condition rather than reading off directly.

Takeaway. The Cheat Code: Any point on $y = a^x$ is (x, a^x), so geometric statements about the graph become index equations in a. Once the base is known, a negative exponent is only a reciprocal: $a^{-n} = \frac{1}{a^n}$, never a negative height.

Time-saving tip. Speed Heuristic: Write $a^4 - a = 78$ as $a(a^3 - 1) = 78$ and split $78 = 3 \times 26$. Since $26 = 27 - 1 = 3^3 - 1$, the base $a = 3$ falls out with no trial at all, and the answer is then just $\frac{1}{3^2}$.

Question 7

MM1.3 · Algebra and functions · ★★★☆☆ · 90 s

A horizontal line $y = k$ never meets the curve $y = x^2 - 2x - 7$. Find all possible values of k.

- **A.** $k < -8$
- **B.** $k < -6$
- **C.** $k > -8$
- **D.** $k > 8$
- **E.** $k < -7$
- **F.** $k > -6$
- **G.** $k < -11$
- **H.** $k \le -8$

Answer A. Key idea

A horizontal line $y = k$ crosses a curve exactly where the curve's expression equals k, so "never meets" is the statement that $x^2 - 2x - 7 = k$ has no real solution. There are two equivalent ways to pin that down. Completing the square rewrites an upward parabola as a square plus a constant, and since a square is never negative, that constant is the lowest height the curve ever reaches; heights below it are the ones the curve misses. The discriminant says the same thing formally: rearrange to the form $ax^2 + bx + c = 0$ and demand $b^2 - 4ac < 0$. Both routes must give the same inequality, and using one to check the other is the safest way to get the direction right.

Worked solution

1. Complete the square

$y = x^2 - 2x - 7$ $y = (x-1)^2 - (-1)^2 - 7$ $y = (x-1)^2 - 1 - 7$ $y = (x-1)^2 - 8$

The bracket $(x-1)^2$ is never negative and equals 0 at $x = 1$, so the lowest point of the curve is $(1, -8)$ and its least y-value is -8.

2. Turn "never meets" into a condition on the roots

The line $y = k$ meets the curve wherever $x^2 - 2x - 7 = k$, so "never meets" means that equation has no real solution. Written in standard form,

$$x^2 - 2x - (7 + k) = 0$$

so the discriminant is

$$D = (-2)^2 - 4(1)(-(7+k)) = 4 + 4(7+k) = 32 + 4k$$

No real roots requires $D < 0$:

$$32 + 4k < 0 \implies 4k < -32 \implies k < -8$$

3. Read it off the graph and check

The completed square gives the same test in one line: $(x-1)^2 = k + 8$ can be solved only when $k + 8 \ge 0$, so the line misses the curve exactly when $k < -8$.

Sanity check: $k = -9$ lies below the lowest point of the curve, and $D = 32 - 36 = -4 < 0$, so that line misses. $k = -8$ is the height of the lowest point itself, where the line touches at $(1, -8)$, so -8 is correctly excluded by the strict inequality. $k = 0$ gives $x^2 - 2x - 7 = 0$ with $D = 32 > 0$, two crossings, as expected for a height above the minimum.

Matches Option **A**.

Why the other options are wrong

- **B** ($k < -6$): A sign slip on the correction term: adding instead of subtracting gives $y = (x-1)^2 + 1 - 7 = (x-1)^2 - 6$, a least value of -6 and the range $k < -6$. Substituting $x = 1$ into the original expression settles it, since $1 - 2 - 7 = -8$ and not -6. *(Sign error completing the square)*
- **C** ($k > -8$): The minimum -8 is found correctly and then the inequality is pointed the wrong way, treating heights above the lowest point as the ones the curve avoids. In fact every $k > -8$ gives $D = 32 + 4k > 0$ and two crossings, so this range describes the lines that cut the curve, not the lines that miss it. *(Inequality reversed)*
- **D** ($k > 8$): Moving k to the wrong side before forming the discriminant: from $x^2 - 2x - 7 + k = 0$ the discriminant becomes $D = 4 - 4(k-7) = 32 - 4k$, and $32 - 4k < 0$ delivers $k > 8$. Testing $k = 9$ against the curve exposes it, since a line nine units above the axis clearly cuts an upward parabola whose lowest point is at -8. *(Sign error rearranging before the discriminant)*
- **E** ($k < -7$): Completing the square but never subtracting the square of the halved coefficient: writing $y = (x-1)^2 - 7$ leaves the constant untouched at -7, so the least value looks like -7 and the range comes out as $k < -7$. The check kills it: at $k = -7.5$ the discriminant is $32 + 4(-7.5) = 2 > 0$, so that line does cross the curve twice. *(Incomplete completing the square)*
- **F** ($k > -6$): Quotes the discriminant as $b^2 + 4ac$ instead of $b^2 - 4ac$. From $x^2 - 2x - (7+k) = 0$, with $a = 1$, $b = -2$ and $c = -7 - k$, that gives $4 + 4(-7-k) = -24 - 4k$, and $-24 - 4k < 0$ rearranges to $k > -6$. The correct form gives $4 + 28 + 4k = 32 + 4k$, and $k = 0$ exposes the slip, since $x^2 - 2x - 7 = 0$ plainly has two roots and so needs a positive discriminant. *(Sign error in the discriminant formula)*
- **G** ($k < -11$): Completes the square with the whole coefficient instead of half of it, writing $y = (x-2)^2 - 2^2 - 7 = (x-2)^2 - 11$, so the least value looks like -11 and the range prints as $k < -11$. Halving is what makes the bracket reproduce the $-2x$ term, since $(x-2)^2$ expands to $x^2 - 4x + 4$ and not $x^2 - 2x + 4$. The discriminant kills it: at $k = -10$, $D = 32 + 4(-10) = -8 < 0$, so that line does miss the curve yet this range excludes it. *(Coefficient of x not halved)*
- **H** ($k \le -8$): Writes the no-real-roots condition as $D \le 0$ rather than $D < 0$, so $32 + 4k \le 0$ gives $k \le -8$. The equality case is exactly the one that has to go: $k = -8$ makes $D = 0$, a repeated root at $x = 1$, and the line $y = -8$ touches the curve at its vertex $(1, -8)$. Touching is meeting, so -8 itself is not a value the line may take. *(Tangency wrongly included)*

Common mistake. Locating the lowest point correctly at height -8 and then choosing the wrong side of it. Values of k above -8 are heights the curve does reach, and each such line cuts the curve twice; it is the heights below the minimum that are never attained. Reading the answer straight off the vertex without asking which side of it the line has to sit is what turns a correct -8 into a wrong inequality.

Takeaway. **The Cheat Code:** Once a curve is written as $y = (x-p)^2 + q$, its y-values are exactly $y \ge q$. A horizontal line $y = k$ then cuts it twice when $k > q$, touches it once when $k = q$, and misses it when $k < q$. Sorting the three cases by comparing k with q is faster and safer than forming the discriminant from scratch, and it tells you at once whether the inequality should be strict.

Time-saving tip. Speed Heuristic: You never need the roots, and you never need the full discriminant here. Halve the coefficient of x to get the bracket $(x-1)^2$, then subtract the square of that half from the constant: $-7 - 1 = -8$ is the vertex height. The answer is everything strictly below it, so scan the options for the one reading $k < -8$.

Question 8

MM1.7 · Algebra and functions · ★★☆☆☆ · 85 s

Each of the eight functions below is defined for all real x. Which is one-to-one?

A. $f(x) = \sin x$
B. $f(x) = x^4 + x$
C. $f(x) = |x| - 4$
D. $f(x) = x^3 - 3x$
E. $f(x) = x^3 + 2x$
F. $f(x) = 2^x - x$
G. $f(x) = 2^{x^2}$
H. $f(x) = x^2 - 6x$

Answer E. Key idea

A mapping is one-to-one only when no two different inputs share an output, so a single pair of distinct inputs giving equal outputs settles the matter against a function immediately. Proving the positive case takes more care: a function that increases strictly across its whole domain can never repeat an output, because a larger input always produces a larger output.

Worked solution

1. Hunt for a repeated output

A single pair of distinct inputs with the same output is enough to rule a function out, so test the candidates rather than trying to prove anything first.

Option A: $\sin 0^\circ = 0$ and $\sin 180^\circ = 0$.

Option B: $f(0) = 0 + 0 = 0$ and $f(-1) = (-1)^4 + (-1) = 0$.

Option C: $f(3) = |3| - 4 = -1$ and $f(-3) = |-3| - 4 = -1$.

Option D: $f(1) = 1 - 3 = -2$ and $f(-2) = -8 + 6 = -2$.

Option F: $f(0) = 2^0 - 0 = 1$ and $f(1) = 2^1 - 1 = 1$.

Option G: $f(2) = 2^4 = 16$ and $f(-2) = 2^4 = 16$.

Option H: $f(0) = 0 - 0 = 0$ and $f(6) = 36 - 36 = 0$.

Each of those seven sends two different inputs to one output, so seven of the eight are many-to-one and one function is left.

2. Confirm the survivor is one-to-one

$f(x) = x^3 + 2x$ is the sum of x^3 and $2x$, and each of those grows strictly as x grows. A sum of strictly increasing functions is strictly increasing, so $x_1 < x_2$ forces $f(x_1) < f(x_2)$ and two inputs can never share an output.

Sanity check. Suppose $x_1^3 + 2x_1 = x_2^3 + 2x_2$. Subtracting and factorising gives $(x_1 - x_2)(x_1^2 + x_1x_2 + x_2^2 + 2) = 0$. The second bracket equals $\left(x_1 + \frac{1}{2}x_2\right)^2 + \frac{3}{4}x_2^2 + 2$, which is never smaller than 2, so the first bracket must vanish and $x_1 = x_2$. The mapping is one-to-one.

Matches Option **E**.

Why the other options are wrong

- **A** ($f(x) = \sin x$): The sine function repeats every 360°, and even inside one period $\sin 0^\circ = 0$ and $\sin 180^\circ = 0$. Infinitely many inputs therefore share each output value. *(Overlooked periodicity)*

- **B** ($f(x) = x^4 + x$): Reasons that x^4 on its own repeats outputs but the added x destroys the symmetry, so the sum must be one-to-one. The fold survives, merely shifted off the axis: $f(0) = 0 + 0 = 0$ and $f(-1) = (-1)^4 + (-1) = 1 - 1 = 0$, two different inputs with the same output. An even leading power forces a minimum, and any graph with a minimum turns back on itself. *(Assumed a linear term removes the fold)*
- **C** ($f(x) = |x| - 4$): The modulus folds the negative half of the number line onto the positive half, so inputs of equal size give identical outputs: $f(3) = 3 - 4 = -1$ and $f(-3) = 3 - 4 = -1$. Two inputs, one output. *(Ignored the reflection built into the modulus)*
- **D** ($f(x) = x^3 - 3x$): Treating every cubic as one-to-one because x^3 is. The $-3x$ term creates two turning points: $f(1) = 1 - 3 = -2$ and $f(-2) = -8 + 6 = -2$, so this graph repeats outputs. *(Over-generalised from $y = x^3$)*
- **F** ($f(x) = 2^x - x$): Takes it that anything built from 2^x inherits its strict increase and therefore cannot repeat an output. Close to the origin the $-x$ term wins: $f(0) = 2^0 - 0 = 1$ and $f(1) = 2^1 - 1 = 1$. One output, two inputs, so the mapping is many-to-one; the graph dips between $x = 0$ and $x = 1$ before the exponential takes over. *(Assumed the exponential forces strict increase)*
- **G** ($f(x) = 2^{x^2}$): Applies the rule that $y = a^x$ is one-to-one without checking what sits in the index. The index here is x^2, which already sends inputs of equal size to the same number: $f(2) = 2^4 = 16$ and $f(-2) = 2^4 = 16$. The exponential step is one-to-one, but the squaring that feeds it is not, and a many-to-one first stage is enough to make the whole mapping many-to-one. *(Even index inside the exponent overlooked)*
- **H** ($f(x) = x^2 - 6x$): Every quadratic is symmetric about its vertex. Factorising, $x^2 - 6x = x(x-6)$, so $f(0) = 0$ and $f(6) = 0$: the outputs pair up about $x = 3$. *(Overlooked quadratic symmetry)*

Common mistake. Assuming that every cubic is one-to-one because $y = x^3$ is. A cubic $x^3 - kx$ with $k > 0$ doubles back on itself: for $x^3 - 3x$, $f(1) = -2$ and $f(-2) = -2$, so it repeats outputs exactly as the quadratic and the modulus do.

Takeaway. **The Cheat Code:** One counterexample kills injectivity, so go looking for a repeated output before attempting any proof. Symmetry (a modulus, an even power) and repetition (the trigonometric functions) hand you the pair on sight, and for $x^3 - kx$ with $k > 0$ the pair $f(0) = f(\sqrt{k}) = 0$ ends it in one line.

Time-saving tip. Speed Heuristic: sort the options by shape before substituting anything. A built-in fold repeats an output on sight, and five of these have one: the sine repeats, the modulus and the quadratic are symmetric, the quartic has a minimum, and the squared index inside 2^{x^2} sends x and $-x$ to the same place. That leaves the two cubics and $2^x - x$. One substitution pair settles $x^3 - 3x$, another settles $2^x - x$, and $x^3 + 2x$ is a sum of strictly increasing terms.

Question 9

MM5.2 · Exponentials and logarithms · ★★★☆☆ · 90 s

Let $\log_a 2 = p$ and $\log_a 3 = q$, where $a > 1$ is a fixed base. Express $\log_a\left(\frac{9\sqrt{a}}{8}\right)$ in terms of p and q.

A. $3q - 2p + \frac{1}{2}$
B. $2q - 3p + 2$
C. $q^2 - p^3 + \frac{1}{2}$
D. $\frac{2q}{3p} + \frac{1}{2}$
E. $2q + 3p + \frac{1}{2}$
F. $2q - 3p + \frac{1}{2}$
G. $2q - 3p + 1$
H. $3p - 2q - \frac{1}{2}$

Answer F. Key idea

The laws of logarithms convert the arithmetic inside a logarithm into simpler arithmetic outside it: a product becomes a sum, a quotient becomes a difference, and an index becomes a multiplying factor. So an argument that is built from powers of a few known numbers can always be dismantled into multiples of the logarithms of those numbers. A surd is only a fractional index, and a logarithm of the base itself is 1, which together fix the contribution of any root of the base without needing its value.

Worked solution

1. Split the quotient

The subtraction law turns the division inside the logarithm into a subtraction outside it:

$$\log_a\left(\frac{9\sqrt{a}}{8}\right) = \log_a(9\sqrt{a}) - \log_a 8$$

2. Split the remaining product

$$\log_a(9\sqrt{a}) = \log_a 9 + \log_a \sqrt{a}$$

3. Bring every index down as a factor

$$\log_a 9 = \log_a 3^2 = 2\log_a 3 = 2q$$

$$\log_a 8 = \log_a 2^3 = 3\log_a 2 = 3p$$

$$\log_a \sqrt{a} = \log_a a^{1/2} = \frac{1}{2}\log_a a = \frac{1}{2}$$

4. Reassemble

$$\log_a\left(\frac{9\sqrt{a}}{8}\right) = 2q + \frac{1}{2} - 3p = 2q - 3p + \frac{1}{2}$$

Sanity check with a concrete base. Take $a = 2$, so $p = \log_2 2 = 1$ and the expression predicts $2q - 3 + \frac{1}{2} = 2q - \frac{5}{2}$. Working directly, $\log_2\left(\frac{9\sqrt{2}}{8}\right) = \log_2 9 + \log_2\sqrt{2} - \log_2 8 = 2\log_2 3 + \frac{1}{2} - 3 = 2q - \frac{5}{2}$, the same expression, so the dismantling is consistent.

Matches Option **F**.

Why the other options are wrong

- **A** ($3q - 2p + \frac{1}{2}$): Recognises the indices 2 and 3 but pairs them with the wrong logarithms, writing $\log_a 9 = 3q$ and $\log_a 8 = 2p$ to give $3q - 2p + \frac{1}{2}$. The multiplier comes from the index of the number itself: $9 = 3^2$ contributes $2q$ and $8 = 2^3$ contributes $3p$. *(Coefficients attached to the wrong letters)*
- **B** ($2q - 3p + 2$): Converts the surd with the index the wrong way up, taking $\log_a\sqrt{a} = 2\log_a a = 2$ instead of $\frac{1}{2}\log_a a = \frac{1}{2}$, so the total becomes $2q - 3p + 2$. A square root is the index $\frac{1}{2}$: $\sqrt{a} \times \sqrt{a} = a$ forces the two indices to add to 1, so each is one half, and the index 2 belongs to a^2. *(Root index inverted)*
- **C** ($q^2 - p^3 + \frac{1}{2}$): Raises the logarithm to the index instead of multiplying by it, taking $\log_a 3^2 = (\log_a 3)^2 = q^2$ and $\log_a 2^3 = (\log_a 2)^3 = p^3$, for $q^2 - p^3 + \frac{1}{2}$. The law reads $\log_a x^k = k\log_a x$, so the index becomes a factor in front, not an index on the answer. *(Power law applied to the logarithm)*

- **D** $\left(\frac{2q}{3p}+\frac{1}{2}\right)$: Handles $\sqrt{a}$ correctly but writes $\log_a \frac{9}{8} = \frac{\log_a 9}{\log_a 8} = \frac{2q}{3p}$. Division inside a logarithm becomes subtraction outside it, so that part is $2q-3p$; a ratio of two logarithms is a change of base, which is a different statement entirely. *(Quotient inside turned into a quotient outside)*
- **E** $\left(2q+3p+\frac{1}{2}\right)$: Applies the product law to a quotient, writing $\log_a\left(\frac{9\sqrt{a}}{8}\right) = \log_a 9 + \log_a \sqrt{a} + \log_a 8 = 2q + \frac{1}{2} + 3p$. Dividing inside a logarithm subtracts outside it, so the 8 contributes $-3p$. Testing $a=2$, where $p=1$, this reads $2q+\frac{7}{2}$ against the true $2q-\frac{5}{2}$, an error of 6, which is exactly $2\log_a 8$. *(Division inside turned into addition)*
- **G** $(2q-3p+1)$: Treats $\sqrt{a}$ as though it were a itself, so the term contributes $\log_a a = 1$ and the total becomes $2q-3p+1$. Since $\sqrt{a}=a^{1/2}$, the index $\frac{1}{2}$ comes down as a factor and the term is worth $\frac{1}{2}$, so this exceeds the correct value by exactly $\frac{1}{2}$. *(Root of the base mishandled)*
- **H** $\left(3p-2q-\frac{1}{2}\right)$: Subtracts the numerator's logarithm from the denominator's, writing $\log_a 8 - \log_a(9\sqrt{a}) = 3p - \left(2q+\frac{1}{2}\right) = 3p-2q-\frac{1}{2}$. That is the logarithm of $\frac{8}{9\sqrt{a}}$, the reciprocal of the argument asked for, so every term has the wrong sign. The quotient law reads top minus bottom. *(Subtraction taken in the wrong order)*

Common mistake. Turning the division inside the logarithm into a division of logarithms, writing $\log_a \frac{9}{8}$ as $\frac{\log_a 9}{\log_a 8}$. The law says the logarithm of a quotient is the difference of the logarithms, so the two terms are subtracted, never divided.

Takeaway. The Cheat Code: Rewrite every number inside the logarithm as a power of something you already have a letter for, then let each index step outside as a multiplier. Roots are just indices too, and any root of the base itself contributes only its fractional index.

Time-saving tip. Speed Heuristic: Deal with $\sqrt{a}$ first, because a root of the base always contributes its index alone, here $\frac{1}{2}$, whatever the base is. That fixes the constant immediately, and the only remaining question is which of 2 and 3 multiplies which letter: $9=3^2$ gives $2q$, and $8=2^3$ gives $3p$.

Question 10

MM4.5 · Trigonometry · ★★★★☆ · 90 s

For an acute angle θ, $\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=\frac{25}{12}$. Find the exact value of $\tan\theta$.

A. $\frac{3}{4}$
B. $\frac{24}{25}$
C. $\frac{4}{3}$
D. $\frac{24}{7}$
E. $\frac{7}{24}$

Answer D. Key idea

> The Pythagorean identity is what converts information about one trigonometric ratio into information about the others, since $\sin^2\theta + \cos^2\theta = 1$ ties them together for every angle. That means an expression built from sine and cosine can often be collapsed to a single ratio before any value is extracted, which is much cheaper than solving for both at once. Once one ratio is pinned down, the quadrant fixes the sign of the other, and $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ converts the pair into the tangent that was asked for.

Worked solution

1. Put the two fractions over a common denominator

For an acute angle neither $\sin\theta$ nor $1+\cos\theta$ is zero, so the sum may be combined:

$$\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = \frac{\sin^2\theta + (1+\cos\theta)^2}{\sin\theta(1+\cos\theta)}$$

2. Collapse the numerator with the Pythagorean identity

Expanding the square and using $\sin^2\theta + \cos^2\theta = 1$:

$$\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta = 1 + 1 + 2\cos\theta = 2(1+\cos\theta)$$

3. Cancel and read off the sine

The factor $1+\cos\theta$ appears above and below, so the whole left side is

$$\frac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)} = \frac{2}{\sin\theta}$$

Hence $\dfrac{2}{\sin\theta} = \dfrac{25}{12}$, giving $\sin\theta = \dfrac{24}{25}$.

4. Recover the cosine, then the tangent

$$\cos^2\theta = 1 - \left(\frac{24}{25}\right)^2 = \frac{625-576}{625} = \frac{49}{625}$$

The angle is acute, so $\cos\theta = +\dfrac{7}{25}$ and

$$\tan\theta = \frac{24/25}{7/25} = \frac{24}{7}$$

Sanity check: with these values the first fraction is $\dfrac{24/25}{32/25} = \dfrac{3}{4}$ and the second is its reciprocal $\dfrac{4}{3}$, and $\dfrac{3}{4} + \dfrac{4}{3} = \dfrac{9+16}{12} = \dfrac{25}{12}$, the value given. The triple 7, 24, 25 also satisfies $49 + 576 = 625$, so the identity holds exactly.

Matches Option **D**.

Why the other options are wrong

- **A** ($\frac{3}{4}$): Evaluates the first fraction once $\sin\theta = \dfrac{24}{25}$ and $\cos\theta = \dfrac{7}{25}$ are known: $\dfrac{24/25}{1+7/25} = \dfrac{24/25}{32/25} = \dfrac{3}{4}$, and reports that. It is a genuine value in the problem, but it is $\dfrac{\sin\theta}{1+\cos\theta}$, not $\dfrac{\sin\theta}{\cos\theta}$. *(Intermediate fraction quoted as the answer)*
- **B** ($\frac{24}{25}$): Solves $\dfrac{2}{\sin\theta} = \dfrac{25}{12}$ to get $\sin\theta = \dfrac{24}{25}$ and stops there. The sine is only the halfway point: the cosine still has to come from $\cos^2\theta = 1 - \dfrac{576}{625} = \dfrac{49}{625}$ before the ratio $\dfrac{24}{7}$ can be formed. *(Answers the wrong quantity)*

- **C** ($\frac{4}{3}$): Uses $1 - \cos\theta$ in place of $\cos\theta$: $\frac{24/25}{1-7/25} = \frac{24/25}{18/25} = \frac{4}{3}$. The tangent divides by the cosine itself, $\frac{7}{25}$, which gives $\frac{24}{7}$. *(Wrong denominator used for the tangent)*
- **E** ($\frac{7}{24}$): Reads the collapsed left side as $\frac{2}{\cos\theta}$, so $\cos\theta = \frac{24}{25}$ and $\sin\theta = \frac{7}{25}$, giving $\tan\theta = \frac{7}{24}$. The factor that cancels is $1 + \cos\theta$, leaving $\sin\theta$ alone in the denominator, so it is the sine that equals $\frac{24}{25}$. *(Sine and cosine interchanged)*

Common mistake. Expanding $(1 + \cos\theta)^2$ as $1 + \cos^2\theta$ and losing the cross term $2\cos\theta$. Without it the numerator becomes the constant 2, nothing cancels, and the equation turns into a messy one in two unknowns instead of collapsing to $\frac{2}{\sin\theta}$.

Takeaway. The Cheat Code: When a sine and cosine expression is a fraction plus its own reciprocal, combine before you substitute anything. The cross term from the square is exactly what $\sin^2\theta + \cos^2\theta = 1$ needs to produce a repeated factor, and that factor cancels the whole problem down to one ratio.

Time-saving tip. Speed Heuristic: The second fraction is the reciprocal of the first, and combining always leaves $\frac{2}{\sin\theta}$, so $\sin\theta = \frac{24}{25}$ can be written down in one line. Then recognise 7, 24, 25 as a Pythagorean triple rather than taking a square root, and the tangent is immediate.

END OF SOLUTIONS

Questions 11 to 27 are in the full paper

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A full paper runs to 27 questions in this module alone, with a question paper, an answer sheet, a mark scheme and a worked solution for every question, across all five ESAT modules.

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EXAMPLE GRAMMAR SCHOOL

ESAT Mock Examination

Physics

Paper SMP-2026-09 · Version 1.5

NAME TEACHING GROUP DATE

Instructions

- Time allowed: **15 minutes**.
- There are **10 multiple-choice questions**. Each is worth 1 mark.
- There is no negative marking. Answer every question, even if you have to guess.
- Calculators are not permitted.
- Record your answers on the separate answer sheet. Only the answer sheet is marked.
- Formulae are not provided. The ESAT does not supply a formula sheet.

Note: the real ESAT is sat on screen at a Pearson test centre. This paper trains the mathematics and the pacing, not the on-screen interface.

Written for Example Grammar School by Lucas Rey, Engineering Science, University of Oxford; MRes, University of Cambridge.

Question 1

A textbook claims that virtually all of an atom's mass sits in its tiny central nucleus, and a student decides to check that claim numerically for the single neutral atom shown in the diagram. Treat the atom's mass as the sum of the masses of its individual particles. To a good approximation, what fraction of the atom's total mass is carried by its electrons?

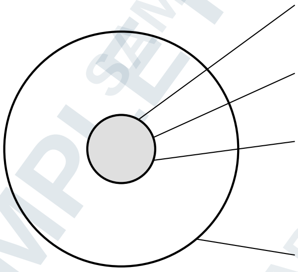
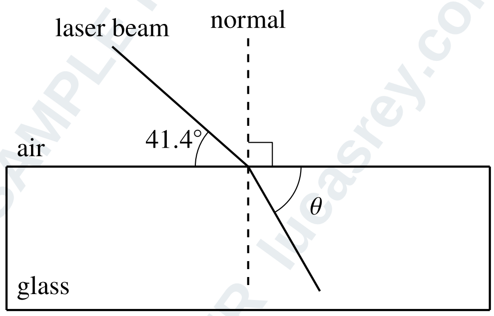


diagram not to scale

A. $\frac{1}{2000}$
B. $\frac{1}{8000}$
C. $\frac{1}{6000}$
D. $\frac{1}{400}$
E. $\frac{1}{100}$
F. $\frac{1}{4000}$
G. $\frac{1}{40000}$
H. $\frac{1}{80000}$

Question 2

An optical technician sets a rectangular crown glass block on a bench with its top face horizontal. The block is known to bend a beam arriving at 30° to the normal into a path 19.5° from the normal inside the glass. She now aims her laser at the same face as shown, and the refracted beam inside the glass makes an angle θ with the face. Take $\sin 19.5° = 0.33$ and $\sin 48.6° = 0.75$. What is θ?



- **A.** 70.5°
- **B.** 60.0°
- **C.** 57.6°
- **D.** 41.4°
- **E.** 30.0°

Question 3

A soft drink is made by stirring sugar syrup of density 1200 kg m^{-3} into water of density 1000 kg m^{-3}. A sealed carton holds 1.05 kg of the finished drink, and the drink's density is 1050 kg m^{-3}. Assume the two liquids mix without any change in total volume and that the carton itself has negligible mass. What volume of syrup does the carton contain?

- **A.** 350 cm^3
- **B.** 250 cm^3
- **C.** 750 cm^3
- **D.** 1000 cm^3
- **E.** 300 cm^3
- **F.** 500 cm^3
- **G.** 50 cm^3
- **H.** 875 cm^3

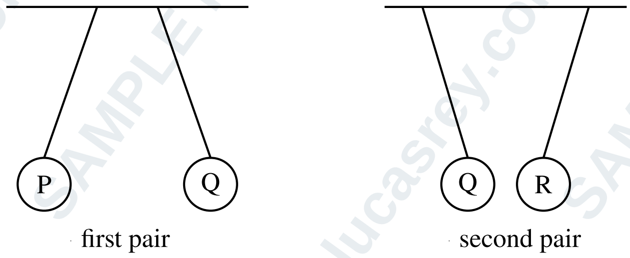
Question 4

An object of mass 5 kg is dropped from rest from a height of 45 m above the ground. Assuming negligible air resistance and $g = 10$ N kg^{-1}, what is its speed just before hitting the ground?

- **A.** 15 m s^{-1}
- **B.** 150 m s^{-1}
- **C.** 21 m s^{-1}
- **D.** 3 m s^{-1}
- **E.** 900 m s^{-1}
- **F.** 90 m s^{-1}
- **G.** 450 m s^{-1}
- **H.** 30 m s^{-1}

Question 5

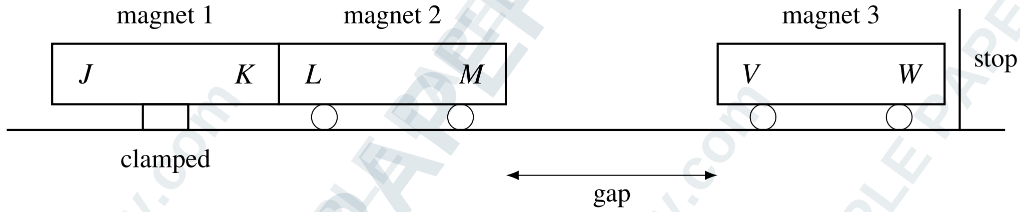
Three light insulating spheres hang on nylon threads in a dry laboratory. Sphere Q was charged by stroking it with a silk cloth, and the cloth was afterwards found to be negatively charged. Spheres P and R had each been given a charge of their own beforehand. The spheres are then brought close in pairs, and they hang as shown. Assume no charge leaks away to the air or along the threads during the tests. Which statement about sphere R is correct?



A. R is positively charged, having lost electrons when it was charged.
B. R is negatively charged, having gained electrons when it was charged.
C. R is negatively charged, having lost electrons when it was charged.
D. R is positively charged, having gained electrons when it was charged.
E. R is uncharged, and is pulled towards Q only because Q is charged.

Question 6

Three identical bar magnets sit on low-friction trolleys on a straight horizontal rail, as shown. The left-hand magnet is clamped, and the other two are free to slide. When they are released, magnets 1 and 2 close up until they touch, while magnet 3 runs away from magnet 2 and rests against an end stop. A compass test shows that end J is a north pole. Which two lettered ends are both north poles?



A. K and M
B. J and M
C. L and V
D. J and L
E. J and K

Question 7

A wire of length 0.1 m carries a current of 3 A perpendicular to a magnetic field of flux density 1.2 T. Find the force on the wire.

A. 0.25 N
B. 4.30 N
C. 0.04 N
D. 0.36 N
E. 0.12 N
F. 3.60 N
G. 0.30 N
H. 36.00 N

Question 8

An acetate rod rubbed with cloth becomes POSITIVE because. . .

A. both objects become positively charged.
B. the rod created new electrons.
C. electrons transferred FROM the rod TO the cloth.
D. positive charges moved onto the cloth.
E. protons moved between the materials.

Question 9

A camping stove's electric plate draws 1200 W from a generator, and 360 W of this is lost to the pan and the surroundings. The pan holds 2.0 litres of water at 20 °C. Water has density 1000 kg m^{-3} and specific heat capacity 4200 J kg^{-1} K^{-1}. Assume the plate reaches its steady output immediately and the loss stays constant. How long does the water take to reach 80 °C?

A. 600 s
B. 1000 s
C. 200 s
D. 800 s
E. 1050 s
F. 294 s
G. 1400 s
H. 420 s

Question 10

A paraffin lamp burns inside a tall glass chimney that is open at the top and pierced by a ring of small holes around its base; these openings are the only ways in or out. The lamp stands in a draught-free room whose air stays at a steady 20°C, while the flame keeps the air inside the chimney much hotter. Which statement correctly describes how air moves through the chimney, and why?

- **A.** Air enters at the top and leaves through the base holes, because the cooler, denser room air sinks into the chimney and pushes the warm air out below it.
- **B.** Air enters at the top and leaves through the base holes, because warming the air raises its density, so the hot air sinks and spills out through the holes.
- **C.** No air passes through either opening, because the difference in density between the hot and the cool air holds the two bodies of air apart.
- **D.** Air enters through the base holes and leaves at the top, because warming the air lowers its density, so the denser room air displaces it upwards.
- **E.** Air enters through the base holes and leaves at the top, because hot gas always rises, whatever the density of the air surrounding it.

END OF PAPER

Questions 11 to 27 are in the full paper

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EXAMPLE GRAMMAR SCHOOL

ESAT Mock: Answer Sheet

Physics

Paper SMP-2026-09 · Version 1.5

NAME

TEACHING GROUP

DATE

Fill in one bubble per question. If you change your mind, cross out the old mark clearly and fill the new bubble. Unfilled or doubly filled rows score zero.

Q	A	B	C	D	E	F	G	H
1	○	○	○	○	○	○	○	○
2	○	○	○	○	○			
3	○	○	○	○	○	○	○	○
4	○	○	○	○	○	○	○	○
5	○	○	○	○	○			
6	○	○	○	○	○			
7	○	○	○	○	○	○	○	○
8	○	○	○	○	○			
9	○	○	○	○	○	○	○	○
10	○	○	○	○	○			

MARKER INITIALS

TOTAL OUT OF 10

EXAMPLE GRAMMAR SCHOOL

ESAT Mock: Mark Scheme

Physics

Paper SMP-2026-09 · Version 1.5

Answer grid

Q	Ans	Q	Ans	Q	Ans
1	**F**	10	**D**		
2	**B**				
3	**B**				
4	**H**				
5	**B**				
6	**D**				
7	**D**				
8	**C**				
9	**A**				

Totals

10 marks available, 1 mark per question. There is no negative marking, so a wrong answer scores zero and never a negative. Only the answer sheet is marked.

Diagnostic table

Use this to see which topics the cohort is losing marks on.

Q	Ans	Node	Group	Topic	Difficulty
1	**F**	P7.1	P7	Radioactivity	★★★☆☆
2	**B**	P6.3	P6	Waves	★★★☆☆
3	**B**	P5.4	P5	Matter	★★★☆☆
4	**H**	P3.7	P3	Energy	★★☆☆☆
5	**B**	P1.1	P1	Electricity	★★☆☆☆
6	**D**	P2.1	P2	Magnetism and electromagnetism	★★☆☆☆
7	**D**	P2.3	P2	The motor effect	★★☆☆☆
8	**C**	P1.1	P1	Electrostatics	★★☆☆☆
9	**A**	P4.4	P4	Thermal physics	★★★☆☆
10	**D**	P4.2	P4	Thermal physics	★★★☆☆

Marks available by topic group

Group	Topic	Marks
P1	Electricity	2
P2	Magnetism and electromagnetism	2
P3	Mechanics	1
P4	Thermal physics	2
P5	Matter	1
P6	Waves	1
P7	Radioactivity	1
Total		**10**

Worked solutions for every question are in the separate worked solutions document.

EXAMPLE GRAMMAR SCHOOL

ESAT Mock: Worked Solutions

Physics

Paper SMP-2026-09 · Version 1.5

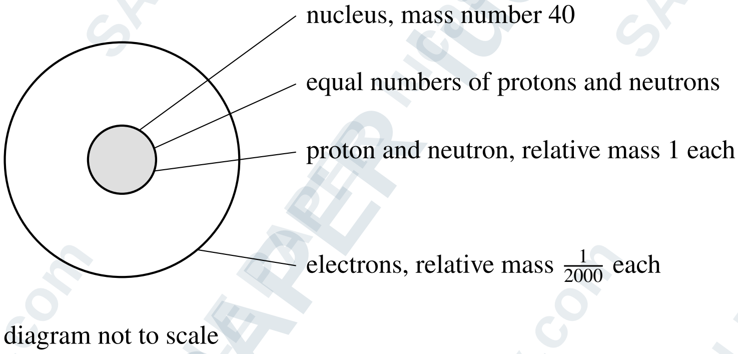
Staff copy

This document contains the answers. Keep it away from candidates until the paper has been sat. Marking guidance is in the separate mark scheme.

Question 1

P7.1 · Radioactivity · ★★★☆☆ · 90 s

A textbook claims that virtually all of an atom's mass sits in its tiny central nucleus, and a student decides to check that claim numerically for the single neutral atom shown in the diagram. Treat the atom's mass as the sum of the masses of its individual particles. To a good approximation, what fraction of the atom's total mass is carried by its electrons?



A. $\frac{1}{2000}$
B. $\frac{1}{8000}$
C. $\frac{1}{6000}$
D. $\frac{1}{400}$
E. $\frac{1}{100}$
F. $\frac{1}{4000}$
G. $\frac{1}{40000}$
H. $\frac{1}{80000}$

Answer F. Key idea

On the relative mass scale a proton and a neutron each count as 1, so the mass number is also the relative mass of the nucleus. An electron is lighter than either by a factor of about two thousand, so although a neutral atom has as many electrons as protons, those electrons carry a total mass smaller than that of the protons by that same factor. Comparing the two totals means counting the particles of each type first, weighting each count by its relative mass, and only then forming the ratio.

Worked solution

1. Count the three kinds of particle

The mass number counts nucleons only, and the protons and neutrons are equal in number, so

$$p = n = \frac{40}{2} = 20$$

The atom is neutral, so it carries one electron for every proton, giving 20 electrons.

2. Add up the mass held in the nucleus

A proton and a neutron each have relative mass 1, so the 40 nucleons contribute

$$20 \times 1 + 20 \times 1 = 40$$

3. Add up the mass held by the electrons

Each electron has relative mass $\frac{1}{2000}$, so the twenty of them contribute

$$20 \times \frac{1}{2000} = \frac{1}{100} = 0.01$$

4. Form the fraction and check it

$$\frac{0.01}{40 + 0.01} = \frac{0.01}{40.01} = \frac{1}{4001} \approx \frac{1}{4000}$$

Sanity check: the nucleons outnumber the electrons only two to one, while each electron is two thousand times lighter than a nucleon, so the answer should be about $\frac{1}{2} \times \frac{1}{2000}$. Whether the 0.01 is kept in the denominator or dropped makes no difference at this precision, which is the point of the claim being tested: the electrons are a rounding error in the mass of an atom.

Matches Option **F**.

Why the other options are wrong

- **A** ($\frac{1}{2000}$): The neutrons were left out of the atom's mass, so the nucleus was given a relative mass of 20 rather than 40, and the fraction came out as $\frac{0.01}{20} = \frac{1}{2000}$. Neutrons carry no charge but they carry the same relative mass as protons, and here they supply half the 40 mass units. *(Neutron mass omitted from the nucleus)*
- **B** ($\frac{1}{8000}$): Halved 40 to get 20 protons and 20 neutrons, then halved again on the way to the electrons and used 10 of them: $10 \times \frac{1}{2000} = \frac{1}{200}$, and $\frac{1}{200} \div 40 = \frac{1}{8000}$. A neutral atom carries one electron for every proton, and there are 20 protons here, so the electron count is 20. *(Mass number halved a second time)*
- **C** ($\frac{1}{6000}$): The electron mass $\frac{1}{100}$ was divided by the number of particles in the atom, $20 + 20 + 20 = 60$, giving $\frac{1}{6000}$. The denominator must be a relative mass, not a head count: the twenty electrons add only 0.01 to it, not twenty units. *(Particle count used in place of relative mass)*
- **D** ($\frac{1}{400}$): Reached $\frac{0.01}{40}$ correctly and then divided by 4 rather than by 40, which is the same as placing the decimal point one column late: $0.01 \div 40 = 0.00025 = \frac{1}{4000}$, whereas $0.0025 = \frac{1}{400}$. The atom's relative mass is 40, not 4. *(Decimal place lost in the final division)*
- **E** ($\frac{1}{100}$): The total relative mass of the electrons, $20 \times \frac{1}{2000} = \frac{1}{100}$, was quoted as the answer without ever being divided by the mass of the whole atom. That figure is a mass on the relative scale, not a share of the atom's mass, and dividing it by 40 is the step that turns it into one. *(Electron mass not expressed as a fraction)*
- **G** ($\frac{1}{40000}$): Cancelled $20 \times \frac{1}{2000}$ to $\frac{1}{1000}$ instead of $\frac{1}{100}$, losing a factor of ten, then divided by the atom's relative mass: $\frac{1}{1000} \div 40 = \frac{1}{40000}$. Since $2000 \div 20 = 100$, the twenty electrons together weigh $\frac{1}{100}$. *(Cancelling slip in the electron total)*

- **H** ($\frac{1}{80000}$): Only a single electron was weighed, giving $\frac{1}{2000} \div 40 = \frac{1}{80000}$. A neutral atom holds one electron for every proton, so there are twenty of them here and the numerator must be twenty times as large. *(One electron used instead of all twenty)*

Common mistake. Building the atom's mass out of the protons alone, dividing 0.01 by 20 and answering $\frac{1}{2000}$. The neutron has no charge, which makes it easy to forget, but on the relative mass scale it weighs the same as a proton, so half of the 40 mass units in this atom are neutrons. Charge and mass are separate properties and the neutron is only absent from one of them.

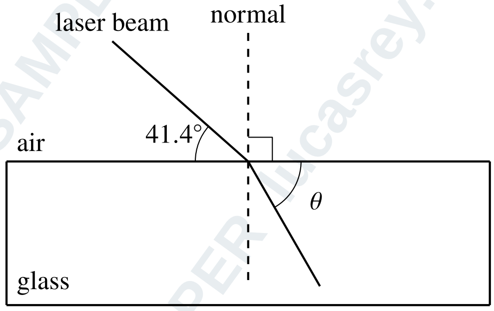
Takeaway. **The Cheat Code:** count, then weight. Write down how many protons, neutrons and electrons there are, multiply each count by its relative mass (1, 1 and $\frac{1}{2000}$), and let the mass number stand in for the whole nucleus rather than adding nucleons one at a time.

Time-saving tip. Speed Heuristic: with equal protons and neutrons the electron count is half the mass number, so the electron mass is $\frac{A}{2} \times \frac{1}{2000} = \frac{A}{4000}$ against a nuclear mass of A. The A cancels and the answer is $\frac{1}{4000}$ whatever the mass number is, so you never need the 40 at all.

Question 2

P6.3 · Waves · ★★★☆☆ · 90 s

An optical technician sets a rectangular crown glass block on a bench with its top face horizontal. The block is known to bend a beam arriving at 30° to the normal into a path 19.5° from the normal inside the glass. She now aims her laser at the same face as shown, and the refracted beam inside the glass makes an angle θ with the face. Take $\sin 19.5° = 0.33$ and $\sin 48.6° = 0.75$. What is θ?



- **A.** 70.5°
- **B.** 60.0°
- **C.** 57.6°
- **D.** 41.4°
- **E.** 30.0°

Answer B. Key idea

Refraction at a plane boundary obeys $n = \dfrac{\sin i}{\sin r}$, in which both angles are measured from the normal, the line drawn perpendicular to the surface. One measured pair of angles fixes n for that boundary, and the same n then sets the refracted direction for any other angle of incidence at that boundary. An angle given from the surface has to be turned into an angle from the normal by subtracting it from 90°, and turned back the same way when the answer is wanted from the surface.

Worked solution

Step 1: Read the angle of incidence from the diagram

The beam is marked at 41.4° to the face, but refraction is worked from the normal, drawn perpendicular to that face. The angle of incidence is therefore $i = 90° - 41.4° = 48.6°$.

Step 2: Get the refractive index from the check ray

The check ray crosses the same face at 30° to the normal and runs at 19.5° to the normal inside the glass, so $n = \dfrac{\sin 30°}{\sin 19.5°} = \dfrac{0.50}{0.33} = 1.5$ to two significant figures, a value typical of crown glass.

Step 3: Refract the laser beam

The same n governs every ray crossing this boundary, so $\sin r = \dfrac{\sin i}{n} = \dfrac{0.75}{1.5} = 0.50$, giving $r = 30°$ measured from the normal.

Step 4: Convert to the angle asked for

θ is the angle to the face, not to the normal, so $\theta = 90° - 30° = 60.0°$.

Sanity check: the beam arrives 48.6° from the normal and continues at 30° from it, so it has bent towards the normal on entering the glass, which is the direction expected for light passing from air into a denser medium. It is bent through 18.6° against the check ray's 10.5°, and the larger bend belongs to the larger angle of incidence.

Matches Option **B**.

Why the other options are wrong

- A (70.5°): This carries the check ray's path over to the laser beam, keeping $r = 19.5°$ from the normal and converting it to the face: $90° - 19.5° = 70.5°$. The refracted angle is not a fixed property of the glass; only n is, and here the beam arrives at 48.6° rather than 30°. *(Conceptual Error)*
- C (57.6°): This divides the angle instead of its sine: $48.6° \div 1.5 = 32.4°$, then $90° - 32.4° = 57.6°$. Snell's law relates the sines, so the division to make is $\sin r = 0.75 \div 1.5$. *(Misapplied Formula)*
- D (41.4°): This treats the boundary as a mirror and applies angle of incidence = angle of reflection to the beam that crosses it, leaving the angle to the face unchanged at $90° - 48.6° = 41.4°$. Equal angles hold for the part of the light reflected back into the air, not for the part refracted into the glass. *(Conceptual Error)*
- E (30.0°): This is r itself, measured from the normal: $\sin r = 0.75 \div 1.5 = 0.50$ gives $r = 30°$. The diagram marks θ between the refracted beam and the face, which is $90° - 30°$. *(Misread the Question)*

Common mistake. Stopping at the angle measured from the normal. Snell's law returns $r = 30°$, but the diagram marks θ between the refracted beam and the face, so the final $90° - 30°$ is still owed.

Takeaway. The Cheat Code: Every angle in $n = \sin i / \sin r$ is measured from the normal. When a ray diagram marks an angle to the surface instead, subtract it from 90° on the way in, and subtract from 90° again on the way out whenever the answer is wanted from the surface.

Time-saving tip. Speed Heuristic: Stay in sines and never write n as a decimal. Here $0.50 \div 0.33$ sits close to $3/2$, and 0.75 is exactly $3/2 \times 0.50$, so the refracted ray is at 30° to the normal and only $90 - 30$ is left to do.

Question 3

P5.4 · Matter · ★★★☆☆ · 90 s

A soft drink is made by stirring sugar syrup of density 1200 kg m^{-3} into water of density 1000 kg m^{-3}. A sealed carton holds 1.05 kg of the finished drink, and the drink's density is 1050 kg m^{-3}. Assume the two liquids mix without any change in total volume and that the carton itself has negligible mass. What volume of syrup does the carton contain?

A. 350 cm^3
B. 250 cm^3
C. 750 cm^3
D. 1000 cm^3
E. 300 cm^3
F. 500 cm^3
G. 50 cm^3
H. 875 cm^3

Answer B. Key idea

Density is a mass divided by a volume, so a mixture can only be handled through its total mass and its total volume: the two ingredient densities cannot be combined directly. When the volumes simply add, every cubic centimetre of the mixture belongs to one liquid or the other, and the total mass is the sum of what each contributes. Comparing the mixture's mass with the mass the same volume of the lighter liquid alone would have shows how much extra mass the denser liquid brought, and dividing that surplus by the difference in densities gives its volume.

Worked solution

1. Volume of drink in the carton

$\rho = \dfrac{m}{V}$, so $V = \dfrac{m}{\rho} = \dfrac{1.05}{1050} = 1.0 \times 10^{-3}\ \text{m}^3 = 1000\ \text{cm}^3$.

2. Move to convenient units

$1200\ \text{kg m}^{-3} = 1.2\ \text{g cm}^{-3}$, $1000\ \text{kg m}^{-3} = 1.0\ \text{g cm}^{-3}$, and the drink is $1.05\ \text{g cm}^{-3}$. The carton therefore holds $1000 \times 1.05 = 1050$ g of drink.

3. Account for the extra mass

If the carton held nothing but water it would hold 1000 g. The syrup is responsible for the surplus of 50 g, and each 1 cm^3 of syrup that takes the place of 1 cm^3 of water adds $1.2 - 1.0 = 0.2$ g. Hence

$V_{\text{syrup}} = \dfrac{50}{0.2} = 250\ \text{cm}^3$.

4. Rebuild the mixture as a check

250 cm^3 of syrup and 750 cm^3 of water have mass $250 \times 1.2 + 750 \times 1.0 = 300 + 750 = 1050$ g in 1000 cm^3, a density of $1.05\ \text{g cm}^{-3}$ as stated. Notice that 1050 sits a quarter of the way from 1000 to 1200, and the syrup is a quarter of the volume.

Matches Option **B**.

Why the other options are wrong

- **A** (350 cm^3): Read the 1.05 kg as the mass of the water, giving 1050 cm^3 of it, then solved $\dfrac{1050 + 1.2V}{1050 + V} = 1.05$ for the syrup: $1050 + 1.2V = 1102.5 + 1.05V$, so $0.15V = 52.5$ and $V = 350\ \text{cm}^3$. The stated mass belongs to the finished drink, which fixes the carton at 1000 cm^3 before any splitting. *(Stated mass assigned to the water)*
- **C** (750 cm^3): Found the syrup correctly as 250 cm^3 and then quoted what was left over, $1000 - 250 = 750\ \text{cm}^3$, which is the volume of water rather than of syrup. *(Answered the complement)*
- **D** (1000 cm^3): Divided the 50 g surplus by $1.05 - 1.00 = 0.05\ \text{g cm}^{-3}$ instead of by $1.20 - 1.00 = 0.20\ \text{g cm}^{-3}$: $\dfrac{50}{0.05} = 1000\ \text{cm}^3$. The difference that matters is the one between the two liquids, since that is what each cubic centimetre of syrup gains over the water it displaces, and 1000 cm^3 is the whole carton with no water left in it. *(Wrong density difference used)*
- **E** (300 cm^3): Found the mass of syrup, $250 \times 1.2 = 300$ g, and read that number straight off as a volume in cm^3. That step is only valid for a liquid of density $1.0\ \text{g cm}^{-3}$, which the syrup is not. *(Mass read as a volume)*
- **F** (500 cm^3): Split the carton in half on the assumption that a mixture always lies midway between its ingredients. Half of each would give $\dfrac{1200 + 1000}{2} = 1100\ \text{kg m}^{-3}$, which is not the density the carton is stated to have. *(Assumed equal volumes)*
- **G** (50 cm^3): Divided the surplus 50 g by water's $1.0\ \text{g cm}^{-3}$ to get 50 cm^3, instead of by the $0.2\ \text{g cm}^{-3}$ that each cubic centimetre of syrup actually adds when it displaces water. *(Wrong density used)*
- **H** (875 cm^3): Read the 1.05 kg as the mass of the syrup rather than of the finished drink, and converted it with the syrup's density: $\dfrac{1050}{1.2} = 875\ \text{cm}^3$. The carton holds 1050 g of drink altogether, of which only 300 g is syrup. *(Stated mass assigned to the syrup)*

Common mistake. Assuming a mixture always sits halfway between its ingredients and splitting the carton equally. Equal volumes would give $\dfrac{1200 + 1000}{2} = 1100\ \text{kg m}^{-3}$, not the $1050\ \text{kg m}^{-3}$ stated: the drink is only a quarter of the way up from water, so only a quarter of it is syrup.

Takeaway. The Cheat Code: when volumes add, the mixture's density lands between the two ingredient densities in exactly the proportion of the volumes. Measure how far along the gap from the lighter liquid to the denser one the mixture sits, and that fraction is the fraction of the volume the denser liquid occupies.

Time-saving tip. Speed Heuristic: switch to g cm^{-3} and read the position on a scale rather than solving an equation. The gap from 1.00 to 1.20 is 0.20, the drink sits 0.05 along it, and $\frac{0.05}{0.20} = \frac{1}{4}$ of 1000 cm^3 is 250 cm^3.

Question 4

P3.7 · Energy · ★★☆☆☆ · 25 s

An object of mass 5 kg is dropped from rest from a height of 45 m above the ground. Assuming negligible air resistance and $g = 10$ N kg^{-1}, what is its speed just before hitting the ground?

- **A.** 15 m s^{-1}
- **B.** 150 m s^{-1}
- **C.** 21 m s^{-1}
- **D.** 3 m s^{-1}
- **E.** 900 m s^{-1}
- **F.** 90 m s^{-1}
- **G.** 450 m s^{-1}
- **H.** 30 m s^{-1}

Answer H. Key idea

Assuming no energy is lost to heat/friction, the total mechanical energy is conserved. Initial gravitational potential energy, mgh, converts entirely into kinetic energy, $\frac{1}{2}mv^2$, during the fall.

Worked solution

1. Apply conservation of energy

Initial GPE = Final KE $mgh = \frac{1}{2}mv^2$

2. Cancel mass and solve for v

$$gh = \frac{1}{2}v^2 \implies v^2 = 2gh \implies v = \sqrt{2gh}$$

3. Substitute values

$$v = \sqrt{2 \times 10 \times 45} = \sqrt{900} = 30 \text{ m s}^{-1}$$

Matches Option **H**.

Why the other options are wrong

- **A** (15 m s^{-1}): Moved the half across the energy equation as a halving instead of a doubling: from $mgh = \frac{1}{2}mv^2$ wrote $v^2 = \frac{1}{2}gh = \frac{1}{2} \times 10 \times 45 = 225$, so $v = \sqrt{225} = 15$. The half multiplies v^2, so it crosses over as $v^2 = 2gh = 900$ and $v = 30$. The same value comes from quoting the average speed of the fall, $h \div t = 45 \div 3 = 15$, which is half the final speed for a body starting from rest. *(Factor of two on the wrong side)*
- **B** (150 m s^{-1}): Multiplied the correct speed by the mass: $5 \times 30 = 150$, which is the momentum in kg m s^{-1}, not a speed. The mass cancels from $mgh = \frac{1}{2}mv^2$, so it never multiplies the answer. *(Calculation / Conceptual Error)*

- **C** (21 m s^{-1}): Lost the factor of two when clearing the half, writing $v^2 = gh = 450$ and $v = \sqrt{450} \approx 21$. The half multiplies v^2, so $v^2 = 2gh = 900$ and $v = 30$. *(Calculation / Conceptual Error)*
- **D** (3 m s^{-1}): Stopped one line early and reported the time of flight as though it were the speed: $45 = \frac{1}{2} \times 10 \times t^2$ gives $t^2 = 9$ and $t = 3$, which is the number of seconds the fall takes. The speed still has to be found from $v = gt = 10 \times 3 = 30$. *(Fall time quoted as the speed)*
- **E** (900 m s^{-1}): Stopped at $v^2 = 2gh = 2 \times 10 \times 45 = 900$ and quoted that as the speed. The square root is still owed: $v = \sqrt{900} = 30$. *(Calculation / Conceptual Error)*
- **F** (90 m s^{-1}): Took the fall time from $h = \frac{1}{2}gt^2$ but never rooted it: wrote $t = 2h/g = 90 \div 10 = 9$ s instead of $t = \sqrt{2h/g} = \sqrt{9} = 3$ s, then $v = gt = 10 \times 9 = 90$. With the correct $t = 3$ s the speed is $10 \times 3 = 30$. *(Square root dropped when finding the fall time)*
- **G** (450 m s^{-1}): Dropped both the factor of two and the square root, quoting $gh = 10 \times 45 = 450$. The speed is $v = \sqrt{2gh} = \sqrt{900} = 30$. *(Calculation / Conceptual Error)*

Common mistake. Multiplying by the mass or forgetting to square root 2gh.

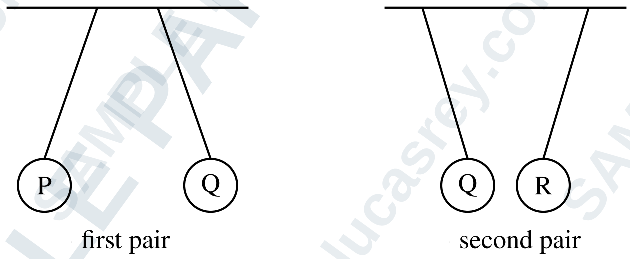
Takeaway. The Cheat Code: When all of an object's GPE becomes its own KE, the mass cancels: $mgh = \frac{1}{2}mv^2$ gives $v = \sqrt{2gh}$, so the 5 kg is not needed for the speed here. The mass is needed as soon as a force other than the weight is given in newtons, an energy loss is given in joules or an energy is asked for, so check before ignoring it.

Time-saving tip. Speed Heuristic: Look at the relative magnitudes and units in the multiple-choice options to eliminate extreme or impossible values immediately.

Question 5

P1.1 · Electricity · ★★☆☆☆ · 85 s

Three light insulating spheres hang on nylon threads in a dry laboratory. Sphere Q was charged by stroking it with a silk cloth, and the cloth was afterwards found to be negatively charged. Spheres P and R had each been given a charge of their own beforehand. The spheres are then brought close in pairs, and they hang as shown. Assume no charge leaks away to the air or along the threads during the tests. Which statement about sphere R is correct?



A. R is positively charged, having lost electrons when it was charged.
B. R is negatively charged, having gained electrons when it was charged.
C. R is negatively charged, having lost electrons when it was charged.
D. R is positively charged, having gained electrons when it was charged.
E. R is uncharged, and is pulled towards Q only because Q is charged.

Answer B. Key idea

Rubbing two insulators together can move electrons and nothing else, so the material that ends up holding the extra electrons is the negative one and its partner is left positive. Once spheres carry charge, the way they hang shows their signs, because like charges repel and unlike charges attract.

Worked solution

1. Fix the sign of sphere Q

Charging by rubbing moves electrons only. The silk cloth finished negatively charged, so the silk gained electrons, and those electrons came off sphere Q. Sphere Q is therefore short of electrons and carries a positive charge.

2. Read the second pair, then count electrons

In the second pair the threads lean inwards, so Q and R are pulling towards each other: this is attraction. Both spheres carry a charge, and unlike charges attract, so R must carry the opposite sign to Q. Q is positive, so R is negative, and a negative sphere is one holding more electrons than protons. R gained electrons when it was charged.

Sanity check: the first pair hangs apart, which is repulsion, so P carries the same sign as Q, positive. That makes the three spheres positive, positive and negative, and exactly one of the two pairs tested holds unlike charges, which is the one drawn leaning together.

Matches Option **B**.

Why the other options are wrong

- A (R is positively charged, having lost electrons when it was charged.): Treats the inward lean of Q and R as evidence that their charges match, which makes R positive like Q, and then correctly pairs a positive charge with lost electrons. The lean is inwards, so that pair attracts and its charges are unlike. *(Interaction rule inverted)*
- C (R is negatively charged, having lost electrons when it was charged.): Gets the sign right, R negative, but books it as a loss of electrons. Losing electrons leaves a shortage of negative charge and so gives a positive sphere; a negative sphere is one that has gained electrons. *(Sign attached to the wrong electron move)*
- D (R is positively charged, having gained electrons when it was charged.): Makes both slips at once: takes the inward lean as repulsion, so R is called positive like Q, and then attaches that positive charge to a gain of electrons. Gaining electrons gives a negative sphere, so the two halves of the statement contradict each other. *(Interaction rule and electron count both inverted)*
- E (R is uncharged, and is pulled towards Q only because Q is charged.): Leans on the fact that a charged object can pull an uncharged one towards it, so the attraction is read as needing no charge on R. The stem states that R had already been given a charge of its own, so this is an attraction between two charged spheres and R cannot be neutral. *(Attraction assumed to show an uncharged sphere)*

Common mistake. Reading the inward lean of the second pair as a sign that Q and R match. Threads that lean together show attraction, and attraction between two charged spheres means opposite signs; it is the pair hanging apart, P and Q, that shares a sign.

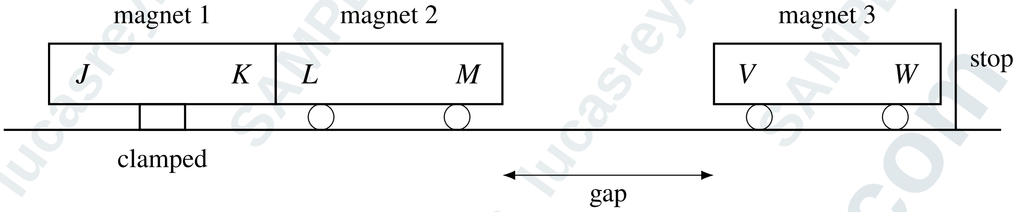
Takeaway. **The Cheat Code:** Only electrons move, so negative means extra electrons and positive means missing ones. Then let the threads speak: for two charged spheres, leaning together is unlike, and hanging apart is like.

Time-saving tip. Speed Heuristic: settle the cloth first. Whatever the silk gained, Q lost, so Q is positive before you even look at the diagram, and the second pair then fixes R in a single move.

Question 6

P2.1 · Magnetism and electromagnetism · ★★☆☆☆ · 85 s

Three identical bar magnets sit on low-friction trolleys on a straight horizontal rail, as shown. The left-hand magnet is clamped, and the other two are free to slide. When they are released, magnets 1 and 2 close up until they touch, while magnet 3 runs away from magnet 2 and rests against an end stop. A compass test shows that end J is a north pole. Which two lettered ends are both north poles?



A. K and M
B. J and M
C. L and V
D. J and L
E. J and K

Answer D. Key idea

Every bar magnet carries a north pole at one end and a south pole at the other, so fixing one end of a magnet fixes its other end automatically. Between two magnets the rule runs one way only: unlike poles attract and like poles repel, so watching what a free magnet does identifies the pair of faces nearest to each other. Chaining those two facts along a row lets one known pole propagate through every magnet in the line, because each observed movement settles a junction and each magnet then settles its own far end.

Worked solution

1. Work along the row as far as magnet 2

End J is a north pole, and the two ends of a single bar magnet are always opposite poles, so K is a south pole.

Magnets 1 and 2 have pulled themselves into contact, and attraction happens only between unlike poles. The end of magnet 2 facing K is L, so L must be unlike K: L is a north pole. Its far end M is therefore a south pole.

2. Cross the open gap to magnet 3

Magnet 3 has been pushed away and is held against the stop, and repulsion happens only between like poles. The end of magnet 3 facing M is V, so V is like M, making V a south pole and the far end W a north pole.

Reading the row from left to right gives N S, then N S, then S N. The north poles are J, L and W, and the only listed pair drawn entirely from those three is J and L.

Sanity check: with that labelling the contact between magnets 1 and 2 is a south face meeting a north face, which does attract, and the open gap is a south face meeting a south face, which does repel. Both observations are reproduced, so the labelling is consistent. Turning magnet 3 end for end would let it close the gap, which is the standard demonstration of the same rule.

Matches Option **D**.

Why the other options are wrong

- **A** (K and M): Attached the compass result to the wrong end of magnet 1, taking K as north so that J is south. The contact then makes L south and M north, and the repulsion makes V north and W south, leaving K, M and V as the north poles and K and M as the listed pair. *(Known pole placed on the wrong end)*

- **B** (J and M): Swapped the two rules, taking attraction as evidence of like poles and repulsion as evidence of unlike poles. Then K is south, the contact makes L south and M north, and the repulsion makes V south and W north, so the north poles come out as J, M and W. *(Attraction and repulsion rules exchanged)*
- **C** (L and V): Applied the rule correctly at the contact but read the open gap as attraction as well, on the idea that magnet 3 had simply not finished moving. That makes V unlike M, so V is north, and the north poles are taken to be J, L and V. *(Gap read as attraction)*
- **E** (J and K): Treated each magnet as though both of its ends were the same pole, so J north forces K north as well. The contact then makes both L and M south and the repulsion makes both V and W south, leaving J and K as the only north poles. *(Both ends of one magnet taken alike)*

Common mistake. Assuming that the magnets which ended up touching must have like poles facing each other, on the loose idea that similar things attract. The rule runs the other way round, and here it decides everything: the faces in contact are unlike, while the faces on either side of the open gap are alike.

Takeaway. **The Cheat Code:** one known pole unlocks a whole row. Alternate N and S across each magnet's own length, put unlike poles at every contact and like poles across every gap that has opened, and the six labels fall out in a single sweep from left to right.

Time-saving tip. Speed Heuristic: write the six letters in a line and fill them in one pass, flipping the pole within each magnet, keeping unlike poles where the magnets touch and like poles across the gap. The chain is finished by the time you reach W, so the options only have to be scanned once.

Question 7

P2.3 · The motor effect · ★★☆☆☆ · 45 s

A wire of length 0.1 m carries a current of 3 A perpendicular to a magnetic field of flux density 1.2 T. Find the force on the wire.

A. 0.25 N
B. 4.30 N
C. 0.04 N
D. 0.36 N
E. 0.12 N
F. 3.60 N
G. 0.30 N
H. 36.00 N

Answer D. Key idea

The motor effect force on a current-carrying wire perpendicular to a field: $F = BIL = 1.2{\times}3{\times}0.1 = 0.36$ N. Direction from Fleming's LEFT-HAND rule (thumb = force, first finger = field, second finger = current).

Worked solution

The motor effect force on a current-carrying wire perpendicular to a field:

$$F = BIL = 1.2 \times 3 \times 0.1 = 0.36 \text{ N}$$

Direction from Fleming's LEFT-HAND rule (thumb = force, first finger = field, second finger = current).

If the wire runs PARALLEL to the field, $F = 0$. In general $F = BIL\sin\theta$, where θ is the angle between the wire and the field, so $F = BIL$ applies only to the perpendicular arrangement.

Matches Option **D**.

Why the other options are wrong

- **A** (0.25 N): Rearranged the formula the wrong way and put the flux density underneath: $IL/B = (3 \times 0.1) \div 1.2 = 0.3 \div 1.2 = 0.25$ N. B is a factor in $F = BIL$, not a divisor, so the product is $0.3 \times 1.2 = 0.36$ N. *(Flux density divided instead of multiplied)*
- **B** (4.30 N): Added the quantities, the formula is a product. *(Conceptual Error)*
- **C** (0.04 N): Divided by the current instead of multiplying by it: $BL/I = (1.2 \times 0.1) \div 3 = 0.12 \div 3 = 0.04$ N. The current is a factor in $F = BIL$, so $0.12 \times 3 = 0.36$ N. *(Arithmetic Error)*
- **E** (0.12 N): Left the current out of the product and multiplied only the flux density by the length: $BL = 1.2 \times 0.1 = 0.12$ N. $F = BIL$ needs all three, so the 3 A must multiply in as well: $0.12 \times 3 = 0.36$ N. *(Current omitted)*
- **F** (3.60 N): Forgot the length, F = BIL needs all three. *(Arithmetic Error)*
- **G** (0.30 N): Omitted the flux density. *(Arithmetic Error)*
- **H** (36.00 N): Divided by the length instead of multiplying by it: $BI/L = (1.2 \times 3) \div 0.1 = 3.6 \div 0.1 = 36.00$ N. Dividing by 0.1 multiplies by ten, so this is one hundred times the true force; multiplying by 0.1 gives $3.6 \times 0.1 = 0.36$ N. *(Length inverted)*

Common mistake. Omitting a factor or adding the quantities.

Takeaway. **The Cheat Code:** $F = BIL$: the force needs all three factors multiplied together. Left hand for motors, right hand for generators.

Time-saving tip. Multiply in the easiest order: $1.2 \times 0.1 = 0.12$, then $0.12 \times 3 = 0.36$ N. Nothing is divided at any stage.

Question 8

P1.1 · Electrostatics · ★★☆☆☆ · 45 s

An acetate rod rubbed with cloth becomes POSITIVE because. . .

A. both objects become positively charged.
B. the rod created new electrons.
C. electrons transferred FROM the rod TO the cloth.
D. positive charges moved onto the cloth.
E. protons moved between the materials.

> **Answer C. Key idea**
> An acetate rod rubbed with cloth becomes POSITIVE: electron transfer decides the sign.

Worked solution

Friction charging facts:

- ONLY electrons move, protons stay bound in nuclei.
- The material with the stronger electron affinity GAINS electrons → negative.
- Equal magnitudes: one body gains exactly what the other loses (charge conservation).
- Identical materials rubbed together transfer nothing.

Applied: an acetate rod rubbed with cloth becomes POSITIVE because electrons transferred from the rod to the cloth.

Matches Option **C**.

Why the other options are wrong

- **A** (both objects become positively charged.): Charge conservation requires one negative, one positive (or neither). *(Conceptual Error)*
- **B** (the rod created new electrons.): Charge is conserved, nothing is created; electrons are TRANSFERRED. *(Conceptual Error)*
- **D** (positive charges moved onto the cloth.): Only negative (electron) transfer occurs in friction charging. *(Conceptual Error)*
- **E** (protons moved between the materials.): Protons are locked in the nucleus, only ELECTRONS move in static electricity. *(Conceptual Error)*

Common mistake. Believing protons transfer or charge is created.

Takeaway. **The Cheat Code:** Friction charging = electron relay: negative charge always moves, one winner one loser. 'The polythene PIRATES electrons.'

Time-saving tip. Electrons transfer; protons never move.

Question 9

P4.4 · Thermal physics · ★★★☆☆ · 90 s

A camping stove's electric plate draws 1200 W from a generator, and 360 W of this is lost to the pan and the surroundings. The pan holds 2.0 litres of water at 20 °C. Water has density 1000 kg m^{-3} and specific heat capacity 4200 J kg^{-1} K^{-1}. Assume the plate reaches its steady output immediately and the loss stays constant. How long does the water take to reach 80 °C?

A. 600 s
B. 1000 s
C. 200 s
D. 800 s
E. 1050 s
F. 294 s
G. 1400 s
H. 420 s

Answer A. Key idea

The energy a body of water must absorb is fixed by its mass, its specific heat capacity and its temperature rise, through $E = mc\Delta T$, and nothing about the heater changes that figure. The heater decides only how fast the energy is handed over. So the calculation splits cleanly in two: find the energy from the water alone, then find the rate at which energy actually arrives in the water, which is the supplied power minus whatever is lost, and divide the first by the second.

Worked solution

1. Mass of water in the pan

2.0 litres is 2.0×10^{-3} m^3, so

$m = \rho V = 1000 \times 2.0 \times 10^{-3} = 2.0$ kg.

2. Energy the water must absorb

The temperature rise is $\Delta T = 80 - 20 = 60$ K, and a rise of one kelvin is the same size as a rise of one degree Celsius, so

$E = mc\Delta T = 2.0 \times 4200 \times 60 = 504000\text{ J} = 504\text{ kJ}.$

3. Power that actually reaches the water

Of the 1200 W drawn from the generator, 360 W never gets into the water, so

$P_{useful} = 1200 - 360 = 840$ W.

4. Time to deliver that energy

$$t = \frac{E}{P_{useful}} = \frac{504000}{840} = 600 \text{ s.}$$

Sanity check: $840 \times 600 = 504000$ J, exactly the energy required. Ten minutes to take two litres from 20 °C to 80 °C on a plate of roughly one kilowatt is the sort of time a camping stove really needs.

Matches Option **A**.

Why the other options are wrong

- **B** (1000 s): Added the two temperatures rather than subtracting them, using $\Delta T = 80 + 20 = 100$ K: $E = 2.0 \times 4200 \times 100 = 840000$ J, then $840000 \div 840 = 1000$ s. The water only has to climb the 60 K between the two readings. *(Temperatures added)*
- **C** (200 s): Took the 20 °C the water starts at as the temperature change: $E = 2.0 \times 4200 \times 20 = 168000$ J, then $168000 \div 840 = 200$ s. ΔT is the difference between the two readings, $80 - 20 = 60$ K. *(Starting temperature used as ΔT)*
- **D** (800 s): Used the final temperature as the rise instead of the change: $E = 2.0 \times 4200 \times 80 = 672000$ J, then $672000 \div 840 = 800$ s. *(Final temperature used as ΔT)*
- **E** (1050 s): Subtracted the loss twice, once from the plate's supply and again from the power reaching the water: $1200 - 360 - 360 = 480$ W, giving $504000 \div 480 = 1050$ s. *(Loss double counted)*
- **F** (294 s): Noticed that 360 W of 1200 W is 30%, then took 30% off the time found from the full power instead of dividing by the 70% that arrives: $504000 \div 1200 = 420$ s, then $420 \times 0.70 = 294$ s. Losing power makes the heating slower, so the time is $420 \div 0.70 = 600$ s. *(Efficiency multiplied instead of divided)*
- **G** (1400 s): Divided by the wasted power rather than the delivered power, $504000 \div 360 = 1400$ s, which is the time the losses alone would take to carry away that much energy. *(Wrong power used)*
- **H** (420 s): Divided by the full power drawn from the generator, $504000 \div 1200 = 420$ s, treating every watt as reaching the water and ignoring the stated 360 W loss. *(Loss ignored)*

Common mistake. Dividing by the 1200 W the plate draws rather than the 840 W that reaches the water. The wasted 360 W is 30% of the supply, so this makes the heating look 180 s quicker than it really is.

Takeaway. **The Cheat Code:** energy first, power second. Work out $mc\Delta T$ for the water on its own, then decide separately how many watts are actually delivering it, and divide only at the very end. Losses never change the energy the water needs, only the time taken to hand it over.

Time-saving tip. Speed Heuristic: cancel before multiplying and 504000 never has to be written. $4200 \times 60 = 252000$, and $252000 \div 840 = 300$, so the time is $2 \times 300 = 600$ s. Spotting that 840 is 2×420 and 420 divides 4200 ten times makes the whole thing mental.

Question 10

P4.2 · Thermal physics · ★★★☆☆ · 90 s

A paraffin lamp burns inside a tall glass chimney that is open at the top and pierced by a ring of small holes around its base; these openings are the only ways in or out. The lamp stands in a draught-free room whose air stays at a steady 20°C, while the flame keeps the air inside the chimney much hotter. Which statement correctly describes how air moves through the chimney, and why?

- **A.** Air enters at the top and leaves through the base holes, because the cooler, denser room air sinks into the chimney and pushes the warm air out below it.
- **B.** Air enters at the top and leaves through the base holes, because warming the air raises its density, so the hot air sinks and spills out through the holes.
- **C.** No air passes through either opening, because the difference in density between the hot and the cool air holds the two bodies of air apart.
- **D.** Air enters through the base holes and leaves at the top, because warming the air lowers its density, so the denser room air displaces it upwards.
- **E.** Air enters through the base holes and leaves at the top, because hot gas always rises, whatever the density of the air surrounding it.

Answer D. Key idea

Heating a fluid makes it expand, so the same mass fills a larger volume and its density falls. Flow then follows from a comparison rather than from the temperature on its own: a less dense body of fluid is displaced upwards by the denser fluid around it, and whatever leaves at one opening has to be replaced through another, which is what turns a single rising column into a continuous circulation.

Worked solution

1. What the flame does to the density of the air

The flame warms the air inside the chimney. A warmed gas expands, so the same mass of air now occupies a larger volume, and density is mass divided by volume. The air in the chimney is therefore less dense than the 20°C air filling the room.

2. What a difference in density does

A body of fluid that is less dense than the fluid surrounding it does not stay where it is. The denser fluid, heavier for the same volume, settles underneath it and displaces it upwards. Here the room air is the denser of the two, so it pushes the warm column of chimney air up and out through the open top. The rising is not something the hot gas does of its own accord: it is what the surrounding air does to it.

3. Close the loop

Air that leaves the top has to be replaced, and the ring of holes at the base is the only other opening, so room air is drawn in there. The result is a steady upward draught: in at the bottom, up past the flame, out at the top, lasting as long as the temperature difference does.

Sanity check: cover the base holes and the replacement air cannot get in, so the draught stops. That is exactly why lamps of this design carry their openings at the bottom rather than at the top.

Matches Option **D**.

Why the other options are wrong

- **A** (Air enters at the top and leaves through the base holes, because the cooler, denser room air sinks into the chimney and pushes the warm air out below it.): Gets the density comparison right and the geometry wrong. The room air is indeed the denser of the two, but denser air entering at the top would have to drive the less dense warm air downwards past the flame and out at the base, which is the wrong way round for the lighter fluid. The cool air does move in, at the bottom. *(Conceptual, flow direction reversed)*

- **B** (Air enters at the top and leaves through the base holes, because warming the air raises its density, so the hot air sinks and spills out through the holes.): Inverts the effect of temperature on density. Warming a gas expands it, so the same mass fills a larger volume and the density falls rather than rises. Sinking and an outflow at the base then follow logically from a false first step. *(Conceptual, temperature and density inverted)*
- **C** (No air passes through either opening, because the difference in density between the hot and the cool air holds the two bodies of air apart.): Treats a density difference as a barrier instead of as the cause of the flow. Two bodies of air of different density in contact do not sit still: the denser one settles beneath the lighter one, and that movement is exactly the convection current the chimney relies on. *(Conceptual, density difference read as static)*
- **E** (Air enters through the base holes and leaves at the top, because hot gas always rises, whatever the density of the air surrounding it.): Gets the direction of the draught right for a reason that does not hold. Hot gas does not rise of its own accord, it rises when the fluid around it is denser. Surround the same hot gas with air that is hotter still, at the same pressure, and it would sink, so the comparison of densities, not the temperature by itself, is what drives the flow. *(Conceptual, right direction from the wrong mechanism)*

Common mistake. Treating "hot air rises" as a property that hot air carries around with it. Rising is a comparison: the air goes up only because the air around it is denser at that moment. Put the same warm air into surroundings that are hotter still, at the same pressure, and it would sink instead.

Takeaway. **The Cheat Code:** In any convection question, name the two bodies of fluid and ask which is denser right now. The denser one moves in underneath, the less dense one is pushed up, and the opening the fluid leaves by fixes the opening it must enter by.

Time-saving tip. Speed Heuristic: split the options by flow direction before reading any of the reasons. The hot, expanded air is the least dense thing in the chimney, so it has to leave at the top, which kills the two options that push air out at the base and the one that says nothing moves. Of the two survivors, keep the reason that names a density difference rather than a rule about hot gas.

END OF SOLUTIONS

Questions 11 to 27 are in the full paper

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Written by researchers in engineering, physics and mathematics at Oxford and Cambridge, and checked question by question before it is issued.

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EXAMPLE GRAMMAR SCHOOL

ESAT Mock Examination

Chemistry

Paper SMP-2026-09 · Version 1.5

NAME

TEACHING GROUP

DATE

Instructions

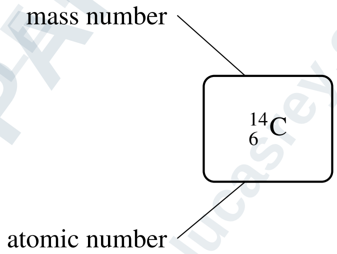
- Time allowed: **15 minutes**.
- There are **10 multiple-choice questions**. Each is worth 1 mark.
- There is no negative marking. Answer every question, even if you have to guess.
- Calculators are not permitted.
- Record your answers on the separate answer sheet. Only the answer sheet is marked.
- Formulae are not provided. The ESAT does not supply a formula sheet.

Note: the real ESAT is sat on screen at a Pearson test centre. This paper trains the mathematics and the pacing, not the on-screen interface.

Written for Example Grammar School by Lucas Rey, Engineering Science, University of Oxford; MRes, University of Cambridge.

Question 1

An atom of a carbon isotope is represented by the standard notation shown. How many neutrons does it contain?



A. 9
B. 7
C. 20
D. 14
E. 8
F. 6

Question 2

A student sets three changes going in identical open glass beakers on a bench in a laboratory held at 19 °C. In the first, magnesium ribbon is dropped into dilute hydrochloric acid and the thermometer climbs to 34 °C. In the second, citric acid solution is stirred into sodium hydrogencarbonate and the thermometer falls to 12 °C. In the third, two solids are ground together with a rod; no thermometer is used, but within a minute a film of ice forms on the outside of that beaker from moisture in the room air. Which statement about the three enthalpy changes is correct?

- **A.** All three have a negative ΔH.
- **B.** The first and second have a negative ΔH; the third is positive.
- **C.** The first has a positive ΔH; the second and third are negative.
- **D.** The first has a negative ΔH; the second and third are positive.
- **E.** All three have a positive ΔH.
- **F.** The first and third have a negative ΔH; the second is positive.

Question 3

A fertiliser is sold as pure ammonium sulfate, $(NH_4)_2SO_4$. Using A_r values $H = 1$, $N = 14$, $O = 16$, $S = 32$, calculate its relative formula mass.

- **A.** 132
- **B.** 118
- **C.** 96
- **D.** 130
- **E.** 128
- **F.** 228
- **G.** 84
- **H.** 114

Question 4

Element Q lies in the same Period as chlorine and two Groups to the left of it. A chlorine atom has the electron configuration 2, 8, 7. Which configuration belongs to an atom of Q?

- **A.** 2, 8, 8
- **B.** 2, 5
- **C.** 2, 8, 2
- **D.** 2, 8, 3
- **E.** 2, 8, 4
- **F.** 2, 8, 6
- **G.** 2, 8, 5
- **H.** 2, 8, 8, 1

Question 5

A technician cuts a cube of sodium from a lump kept under oil and drops it into a trough of cold water. The cube floats, and within a second or two it melts into a ball that darts across the surface, leaving the water alkaline. Why does the cube melt?

A. The oil clinging to the freshly cut surface catches fire on the water, and its flame melts the metal.
B. Friction with the water, as the cube is driven across the surface, heats it up to its melting point.
C. The hydrogen given off burns as it escapes, and that flame supplies the heat needed to melt the metal.
D. Sodium's metallic bonding is weak, so the cold water breaks the lattice apart without any heating.
E. Sodium's melting point is low, and its reaction with the water releases heat fast enough to reach that temperature.
F. The sodium hydroxide formed dissolves into the metal and lowers its melting point below room temperature.

Question 6

Calculate the volume of O_2 gas produced at room temperature and pressure (RTP) when 50.0 g of potassium chlorate decomposes according to the equation: $2KClO_3(s) \rightarrow 2KCl(s) + 3O_2(g)$ (Take molar gas volume at RTP = 24.0 dm^3 mol^{-1}, M_r of potassium chlorate = 122.5).

A. 29.39 dm^3
B. 14.69 dm^3
C. 7.35 dm^3
D. 13.71 dm^3
E. 1200.0 dm^3
F. 9.80 dm^3
G. 6.53 dm^3
H. 146.9 dm^3

Question 7

In a blast furnace, iron(III) oxide (Fe_2O_3) is reduced by carbon monoxide to iron and carbon dioxide. Balanced with the smallest whole numbers, what is the sum of all the coefficients in this equation?

A. 7
B. 18
C. 8
D. 12
E. 11
F. 5
G. 4
H. 9

Question 8

A technician stirs dry sand, which does not dissolve, into a blue solution of copper(II) sulfate in water. Dry crystals of copper(II) sulfate must be recovered from the mixture, with no sand in them. Which pair of steps, carried out in this order, achieves that?

- **A.** Heat the whole mixture until every drop of water has boiled away, then collect the solid left in the dish.
- **B.** Boil the whole mixture and condense the vapour, then keep the clear liquid that collects in the receiver.
- **C.** Cool the whole mixture in ice until crystals appear, then pour it onto filter paper and dry what is caught.
- **D.** Pour the mixture onto filter paper, then heat the liquid that runs through until it is saturated and cool it slowly.
- **E.** Pour the mixture onto filter paper, then wash the solid caught in the paper with warm water and dry it.

Question 9

Calculate the relative molar mass, M_r, of magnesium hydroxide, $Mg(OH)_2$. (A_r(Mg) = 24, A_r(O) = 16, A_r(H) = 1.)

- **A.** 58
- **B.** 42
- **C.** 57
- **D.** 82
- **E.** 41
- **F.** 26
- **G.** 56
- **H.** 29

Question 10

Tellurium has a higher relative atomic mass than iodine, 127.6 against 126.9, yet every modern Periodic Table places tellurium first, with iodine falling in the halogen column beneath bromine. Which rule does the modern table order its elements by?

- **A.** Elements are ordered by increasing number of neutrons, and tellurium's atoms hold fewer neutrons than iodine's atoms.
- **B.** Elements are ordered by increasing relative atomic mass, with awkward pairs swapped to keep chemical families together.
- **C.** Elements are ordered by decreasing atomic radius, and tellurium's atoms are larger than iodine's atoms.
- **D.** Elements are ordered by increasing proton number, and tellurium has 52 protons against iodine's 53.
- **E.** Elements are ordered by increasing number of outer-shell electrons, six for tellurium and seven for iodine.
- **F.** Elements are ordered by increasing number of occupied electron shells, which is greater for iodine than for tellurium.

END OF PAPER

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EXAMPLE GRAMMAR SCHOOL

ESAT Mock: Answer Sheet

Chemistry

Paper SMP-2026-09 · Version 1.5

NAME

TEACHING GROUP

DATE

Fill in one bubble per question. If you change your mind, cross out the old mark clearly and fill the new bubble. Unfilled or doubly filled rows score zero.

Q	A	B	C	D	E	F	G	H
1	○	○	○	○	○	○		
2	○	○	○	○	○	○		
3	○	○	○	○	○	○	○	○
4	○	○	○	○	○	○	○	○
5	○	○	○	○	○	○		
6	○	○	○	○	○	○	○	○
7	○	○	○	○	○	○	○	○
8	○	○	○	○	○			
9	○	○	○	○	○	○	○	○
10	○	○	○	○	○	○		

MARKER INITIALS

TOTAL OUT OF 10

EXAMPLE GRAMMAR SCHOOL

ESAT Mock: Mark Scheme

Chemistry

Paper SMP-2026-09 · Version 1.5

Answer grid

Q	Ans	Q	Ans	Q	Ans
1	**E**	10	**D**		
2	**D**				
3	**A**				
4	**G**				
5	**E**				
6	**B**				
7	**H**				
8	**D**				
9	**A**				

Totals

10 marks available, 1 mark per question. There is no negative marking, so a wrong answer scores zero and never a negative. Only the answer sheet is marked.

Diagnostic table

Use this to see which topics the cohort is losing marks on.

Q	Ans	Node	Group	Topic	Difficulty
1	**E**	C1.3	C1	Know and be able to use the terms atomic number and mass number, together with standard notation (e.g. C 12 6), and so be able to calculate the number of protons, neutrons and electrons in any atom o	★★☆☆☆
2	**D**	C11.1	C11	Energetics	★★☆☆☆
3	**A**	C4.1	C4	Quantitative chemistry	★★☆☆☆
4	**G**	C2.4	C2	The Periodic Table	★★☆☆☆
5	**E**	C7.1	C7	Groups 1, 17 and 18	★★☆☆☆
6	**B**	C4.8	C4	Quantitative chemistry	★★☆☆☆
7	**H**	C3.4	C3	Chemical reactions and equations	★★☆☆☆
8	**D**	C8.2	C8	Separation techniques	★★☆☆☆
9	**A**	C4.1	C4	Quantitative chemistry	★★☆☆☆
10	**D**	C2.2	C2	The Periodic Table	★★★☆☆

Marks available by topic group

Group	Topic	Marks
C1	Atomic structure	1
C2	The Periodic Table	2
C3	Chemical reactions and equations	1
C4	Quantitative chemistry	3
C7	Groups 1, 17 and 18	1
C8	Separation techniques	1
C11	Energetics	1
Total		**10**

Worked solutions for every question are in the separate worked solutions document.

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EXAMPLE GRAMMAR SCHOOL

ESAT Mock: Worked Solutions

Chemistry

Paper SMP-2026-09 · Version 1.5

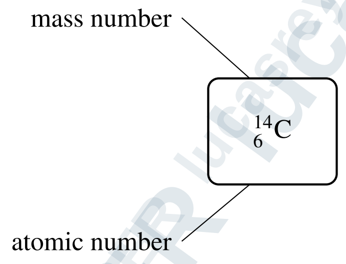
Staff copy

This document contains the answers. Keep it away from candidates until the paper has been sat. Marking guidance is in the separate mark scheme.

Question 1

C1.3 · Know and be able to use the terms atomic number and mass number, together with standard notation (e.g. C 12 6), and so be able to calculate the number of protons, neutrons and electrons in any atom o · ★★☆☆☆ · 45 s

An atom of a carbon isotope is represented by the standard notation shown. How many neutrons does it contain?



A. 9
B. 7
C. 20
D. 14
E. 8
F. 6

Answer E. Key idea

Standard notation $^{A}_{Z}X$: Z (lower) is the atomic/proton number, which defines the element and equals the electron count in a neutral atom, while A (upper) is the mass number, protons plus neutrons, so neutrons $= A - Z$. For carbon-14 ($^{14}_{6}$C): 6 protons, 14 − 6 = 8 neutrons and 6 electrons.

Worked solution

Standard notation $^{A}_{Z}X$:

- Z (lower) = ATOMIC/proton number = number of protons (defines the element; = electrons if neutral).
- A (upper) = MASS number = protons + neutrons.
- Neutrons = A − Z.

For carbon-14 ($^{14}_{6}$C): 6 protons, 8 neutrons, 6 electrons (neutral).

Matches Option **E**.

Why the other options are wrong

- **A** (9): Off by one. *(Arithmetic Error)*
- **B** (7): Assumed a nucleus holds equal numbers of protons and neutrons and halved the mass number: 14 / 2 = 7. That works for carbon-12, not for this isotope; neutrons = A - Z = 14 - 6 = 8. *(Conceptual Error)*
- **C** (20): Added them; subtract instead. *(Arithmetic Error)*
- **D** (14): That's the mass number. *(Conceptual Error)*
- **F** (6): That's the proton number. *(Conceptual Error)*

Common mistake. Swapping A and Z in the notation.

Takeaway. **The Cheat Code:** 'A on top adds, Z at the bottom identifies': A = p + n, Z = identity. Neutrons = A − Z. Always.

Time-saving tip. A adds (p+n); Z identifies.

Question 2

C11.1 · Energetics · ★★☆☆☆ · 90 s

A student sets three changes going in identical open glass beakers on a bench in a laboratory held at 19 °C. In the first, magnesium ribbon is dropped into dilute hydrochloric acid and the thermometer climbs to 34 °C. In the second, citric acid solution is stirred into sodium hydrogencarbonate and the thermometer falls to 12 °C. In the third, two solids are ground together with a rod; no thermometer is used, but within a minute a film of ice forms on the outside of that beaker from moisture in the room air. Which statement about the three enthalpy changes is correct?

A. All three have a negative ΔH.
B. The first and second have a negative ΔH; the third is positive.
C. The first has a positive ΔH; the second and third are negative.
D. The first has a negative ΔH; the second and third are positive.
E. All three have a positive ΔH.
F. The first and third have a negative ΔH; the second is positive.

Answer D. Key idea

The sign of an enthalpy change records the direction energy travelled, not how large the change was. A reaction that pushes energy out into its surroundings leaves the chemicals lower in energy than they started, so ΔH is negative and the surroundings warm. A reaction that pulls energy in from its surroundings leaves the chemicals higher in energy, so ΔH is positive and the surroundings cool. A thermometer is only one way of seeing which way the energy went; anything showing the mixture has dropped below room temperature is the same evidence.

Worked solution

1. The first beaker

The thermometer climbs from the room's 19 °C to 34 °C. Energy has been pushed out of the reacting chemicals into the solution, the glass and the air, which is what exothermic means. The chemicals themselves end lower in energy than they began, so ΔH is negative.

2. The second beaker

Here the thermometer falls from 19 °C to 12 °C. Energy has been drawn out of the surroundings and into the reacting chemicals, so the change is endothermic and ΔH is positive.

3. The third beaker

There is no reading, but ice forms on the outside of the glass. Water vapour from the room can only freeze there if the outside of that beaker has been pulled below 0 °C, far below the room's 19 °C. Energy is therefore flowing from the room into the beaker, exactly as in the second one, so this change is endothermic too and ΔH is positive.

Sanity check: one beaker ended warmer than the room and two ended colder, so one ΔH is negative and two are positive, and every beaker has been classified from the direction energy crossed its wall rather than from what was put in it.

Matches Option **D**.

Why the other options are wrong

- **A** (All three have a negative ΔH.): Assumes any change that runs on its own must be giving energy out, so all three are called negative. The second beaker's thermometer drops seven degrees, which cannot happen while the chemicals in it are warming their surroundings. *(All changes assumed exothermic)*
- **B** (The first and second have a negative ΔH; the third is positive.): Classifies the second beaker from what is in it rather than from the thermometer: an acid meeting a hydrogencarbonate is read as a neutralisation, and neutralisation is remembered as exothermic, so its ΔH is called negative. The reading is the evidence and it falls from 19 °C to 12 °C, which is energy drawn into the chemicals and a positive ΔH. The third beaker, with no reaction to recognise, is then read correctly from its frost, so the same candidate treats two beakers that both cool the room in opposite ways. *(Reaction type recalled in place of the reading)*
- **C** (The first has a positive ΔH; the second and third are negative.): Applies the convention the wrong way round throughout, calling the beaker that warmed positive and the two that cooled negative. The sign belongs to the chemicals, whose energy falls when they heat their surroundings. *(Sign convention inverted)*
- **E** (All three have a positive ΔH.): Treats every temperature change as a positive ΔH, reading the size of the change rather than its direction. The first beaker finished 15 °C above the room, which is energy leaving the chemicals, and that carries a negative sign. *(Direction of energy flow ignored)*
- **F** (The first and third have a negative ΔH; the second is positive.): Correct on the two beakers with thermometers, but credits the freezing of the condensed water to the mixture inside the third. The ice forms because that beaker is colder than the room, which is the signature of energy going in, not out. *(Frost misread as energy released)*

Common mistake. Reading the ice on the third beaker as evidence that energy was given out, on the grounds that freezing releases energy. Freezing does release energy, but the water that froze belongs to the room: it gave energy up because the mixture inside was taking energy from it.

Takeaway. **The Cheat Code:** Ask which way energy crossed the glass. Out of the chemicals means the surroundings warm and ΔH is negative; into the chemicals means the surroundings cool, frost included, and ΔH is positive.

Time-saving tip. Speed Heuristic: Sort the beakers by whether each ended warmer or colder than the 19 °C room, ignoring what is in them. One warmer and two colder means one negative and two positive, and only one option carries that split.

Question 3

C4.1 · Quantitative chemistry · ★★☆☆☆ · 85 s

A fertiliser is sold as pure ammonium sulfate, $(NH_4)_2SO_4$. Using A_r values $H = 1$, $N = 14$, $O = 16$, $S = 32$, calculate its relative formula mass.

- **A.** 132
- **B.** 118
- **C.** 96
- **D.** 130
- **E.** 128
- **F.** 228
- **G.** 84
- **H.** 114

Answer A. Key idea

A relative formula mass is the sum of the relative atomic masses of every atom in the formula, each counted as often as it appears. A subscript written outside a bracket multiplies the whole group inside it, every element and every subscript within, so the safe route is to evaluate each bracketed group once as a block, multiply that block by its subscript, and only then add the remaining atoms. Relative masses have no units, being ratios against carbon-12.

Worked solution

1. Price the ammonium block and apply the bracket

One ammonium group is one nitrogen and four hydrogens:

$$M_r(NH_4) = 14 + (4 \times 1) = 18$$

The subscript 2 outside the bracket multiplies everything inside it, the nitrogen and all four hydrogens alike, so the two ammonium groups contribute

$$2 \times 18 = 36$$

2. Price the sulfate block and add

$$M_r(SO_4) = 32 + (4 \times 16) = 32 + 64 = 96$$

$$M_r((NH_4)_2SO_4) = 36 + 96 = 132$$

Cross-check atom by atom rather than by block: 2 nitrogen = 28, 8 hydrogen = 8, 1 sulfur = 32, 4 oxygen = 64, and 28 + 8 + 32 + 64 = 132. The two routes agree, which confirms the bracket was expanded correctly.

Sanity check: the salt must weigh more than the sulfate part alone, 96, and the extra can only be the two light ammonium groups, worth 36 between them. A value below 100 would mean part of the formula had been lost, and a value near 200 would mean something had been double counted.

Matches Option **A**.

Why the other options are wrong

- **B** (118): Applied the outside subscript to the hydrogens and not to the nitrogen, doubling four hydrogens to eight but leaving one nitrogen, so $(NH_4)_2$ was priced as $14 + 8 = 22$ and then $22 + 96 = 118$. The subscript multiplies every atom inside the bracket, giving two nitrogens as well as eight hydrogens, so the block is worth 36. *(Bracket applied to hydrogen only)*
- **C** (96): Quoted the sulfate part alone, $32 + 64 = 96$, leaving out both ammonium groups. Those two groups add a further 36 to the formula mass. *(Cation omitted)*
- **D** (130): Priced the cation as ammonia rather than ammonium, $NH_3 = 14 + 3 = 17$, so the two groups came to $2 \times 17 = 34$ and the total to $34 + 96 = 130$. The group in this salt is NH_4, worth $14 + 4 = 18$, so the pair is worth 36. *(Ammonia used in place of ammonium)*
- **E** (128): Applied the subscript to the nitrogen only and left the hydrogens single, pricing $(NH_4)_2$ as $N_2H_4 = 28 + 4 = 32$, then $32 + 96 = 128$. The 2 outside the bracket multiplies the four hydrogens as well. *(Bracket partially expanded)*
- **F** (228): Read the subscript 2 as multiplying the entire formula instead of the bracketed group alone, so one unit was priced at $18 + 96 = 114$ and then doubled: $2 \times 114 = 228$. Only the ammonium group stands inside the bracket; the sulfate is outside it and is counted once. *(Bracket subscript applied to the whole formula)*
- **G** (84): Dropped the subscript on oxygen and priced the anion as SO: $36 + 32 + 16 = 84$. Sulfate carries four oxygens, worth 64, not one. *(Subscript on oxygen ignored)*
- **H** (114): Left the outside subscript unused and counted a single ammonium group: $18 + 96 = 114$. The bracket subscript doubles the whole group, so 36 is needed, not 18. *(Bracket subscript not applied)*

Common mistake. Multiplying only the first atom inside the bracket by the subscript, so $(NH_4)_2$ is treated as N_2H_4 and priced at 32 instead of 36. The subscript outside a bracket applies to every atom within it, giving two nitrogens and eight hydrogens.

Takeaway. **The Cheat Code:** work in blocks. Find the M_r of each bracketed group first, multiply that whole block by its subscript, then add the rest. A bracket is never distributed to one atom and not the others.

Time-saving tip. Speed Heuristic: memorise the common ions as single numbers and reuse them. Ammonium is 18 and sulfate is 96, so this salt is $96 + 2 \times 18 = 132$ in one line, and those two block values reappear in every ammonium and every sulfate question on the paper.

Question 4

C2.4 · The Periodic Table · ★★☆☆☆ · 85 s

Element Q lies in the same Period as chlorine and two Groups to the left of it. A chlorine atom has the electron configuration 2, 8, 7. Which configuration belongs to an atom of Q?

A. 2, 8, 8
B. 2, 5
C. 2, 8, 2
D. 2, 8, 3
E. 2, 8, 4
F. 2, 8, 6
G. 2, 8, 5
H. 2, 8, 8, 1

Answer G. Key idea

Position in the Periodic Table and electron configuration carry the same information twice over. The number of electrons in the outermost shell is the Group number and the number of occupied shells is the Period, so a move across a Period changes only the last figure of the configuration.

Worked solution

1. Read chlorine's position out of its configuration

The last number is the outer shell. Chlorine has 7 electrons there, so it sits in Group 7. The count of numbers is the number of occupied shells: 2, then 8, then 7 is three shells, so chlorine sits in Period 3.

2. Move two Groups to the left and rebuild the configuration

Q is in the same Period, so it still has three occupied shells, and the two inner ones stay full at 2 and 8. Two Groups to the left of Group 7 is Group $7 - 2 = 5$, so Q has 5 electrons in its outer shell.

Putting the three shells together:

$$2, 8, 5$$

Check: $2 + 8 + 5 = 15$ electrons, so Q has atomic number 15. That is phosphorus, which does sit in Period 3 two columns to the left of chlorine, with sulfur between the two, and its five outer electrons read straight back as Group 5.

Matches Option **G**.

Why the other options are wrong

- **A** (2, 8, 8): Gave the configuration Q reaches once it has reacted rather than the configuration of the atom. A Group 5 atom gains three electrons to fill its outer shell, $5 + 3 = 8$, so Q^{3-} is 2, 8, 8, the same arrangement as argon. The stem asks for an atom of Q, which keeps its own 15 electrons and so is 2, 8, 5. *(Ion given instead of the atom)*
- **B** (2, 5): Counts chlorine's Period as 2 by counting only its full shells, the 2 and the 8, then puts Q's five outer electrons in a second shell. Three shells are occupied in 2, 8, 7, so Q keeps three: 2, 8, 5. *(Period read from the full shells only)*
- **C** (2, 8, 2): Takes the words 'two Groups' as the Group number itself, placing Q in Group 2 of Period 3, which is magnesium, 2, 8, 2. Two Groups to the left of Group 7 is Group 5. *(Group number taken from the wrong words)*
- **D** (2, 8, 3): Carried out the same instruction twice over, once to fix the Group and once again on the electron count: $7 - 2 = 5$, then $5 - 2 = 3$, which is aluminium, 2, 8, 3. The move across is made once, and Group 5 already means five electrons in the outer shell. *(Shift applied twice)*
- **E** (2, 8, 4): Read 'two Groups to the left' as leaving two Groups in between, so sulfur and phosphorus were both stepped over and Q was placed at silicon: $7 - 3 = 4$ outer electrons, 2, 8, 4. Two Groups to the left means two columns of movement, which lands on phosphorus with sulfur alone in between. *(Counted the Groups in between)*
- **F** (2, 8, 6): Moves one column instead of two, landing on Group 6, which is sulfur, 2, 8, 6. The move is $7 - 2 = 5$, so the outer shell holds five electrons, not six. *(Moved one column too few)*
- **H** (2, 8, 8, 1): Moves two places to the right instead of the left, passing argon, 2, 8, 8, and arriving at potassium, 2, 8, 8, 1. That atom has four occupied shells, so it is not even in chlorine's Period. *(Moved the wrong way)*

Common mistake. Reading 'two Groups to the left' as 'Group 2' and answering 2, 8, 2. The phrase counts two columns across from chlorine's own Group 7, which lands on Group 5, not on Group 2.

Takeaway. **The Cheat Code:** The last figure of a configuration is the Group and the count of figures is the Period. Stepping along a Period moves only that last figure, by one for each column crossed.

Time-saving tip. Speed Heuristic: Only the final figure changes. Chlorine ends in 7, so two columns to the left ends in 5, and the leading 2, 8 is untouched, which disposes of B at once; the answer must also keep exactly three shells, which disposes of H.

Question 5

C7.1 · Groups 1, 17 and 18 · ★★☆☆☆ · 90 s

A technician cuts a cube of sodium from a lump kept under oil and drops it into a trough of cold water. The cube floats, and within a second or two it melts into a ball that darts across the surface, leaving the water alkaline. Why does the cube melt?

- **A.** The oil clinging to the freshly cut surface catches fire on the water, and its flame melts the metal.
- **B.** Friction with the water, as the cube is driven across the surface, heats it up to its melting point.
- **C.** The hydrogen given off burns as it escapes, and that flame supplies the heat needed to melt the metal.
- **D.** Sodium's metallic bonding is weak, so the cold water breaks the lattice apart without any heating.
- **E.** Sodium's melting point is low, and its reaction with the water releases heat fast enough to reach that temperature.
- **F.** The sodium hydroxide formed dissolves into the metal and lowers its melting point below room temperature.

Answer E. Key idea

Two properties of the alkali metals meet in this demonstration. The chemical one is that a Group 1 metal reacts with water to give hydrogen and a dissolved hydroxide, releasing a great deal of energy in a very short time. The physical one is that these metals melt at temperatures far below those of ordinary metals. A reaction hot enough and a melting point low enough together explain a solid metal turning liquid in a trough of cold water.

Worked solution

1. Identify the reaction and where its energy goes

Sodium is an alkali metal, so with water it gives hydrogen and a hydroxide:

$$2Na + 2H_2O \rightarrow 2NaOH + H_2$$

The sodium hydroxide dissolving in the trough is why the water is left alkaline. The reaction is strongly exothermic, and it takes place at the surface of the cube, so the energy is delivered straight into a small piece of metal instead of being spread through the whole trough.

2. Bring in the physical property that makes melting possible

Alkali metals are soft and low melting, and sodium melts a little below 100 °C, which is extraordinarily low for a metal. A cube weighing a fraction of a gram needs very little energy to reach that temperature, so a reaction running fast at its surface carries it past the melting point in a second or two, and surface tension then pulls the liquid into a ball.

3. Rule out the other suggested sources of heat

Hydrogen escaping from sodium on cold water often does not ignite at all, and the ball forms whether it does or not, so no flame is needed. The oil is carried off by the knife and by the water. Friction is not the source either: a piece trapped on a floating scrap of filter paper stays where it is put and still melts. And the hydroxide formed is soluble, so it passes into the trough rather than into the metal, and cannot change the metal's melting point.

Sanity check: sodium warmed to 100 °C in a dry tube melts with no water and no flame anywhere near it, which shows the melting point is doing the work.

Matches Option **E**.

Why the other options are wrong

- **A** (The oil clinging to the freshly cut surface catches fire on the water, and its flame melts the metal.): Blames the storage oil. Most of it is wiped away by the cut and the rest floats off on the water, and a film thin enough to cling could not release the energy needed to melt the metal beneath it. *(Heat attributed to the storage oil)*
- **B** (Friction with the water, as the cube is driven across the surface, heats it up to its melting point.): Puts the heat into friction. A piece of sodium trapped on a floating scrap of filter paper stays where it is put and melts just the same, so the darting about cannot be the source; it is a consequence of the escaping gas. *(Heat attributed to friction)*
- **C** (The hydrogen given off burns as it escapes, and that flame supplies the heat needed to melt the metal.): Puts the heat in a flame rather than in the reaction. Hydrogen from sodium on cold water frequently does not ignite, and the ball still forms, so the melting cannot depend on the gas burning. *(Heat attributed to burning hydrogen)*
- **D** (Sodium's metallic bonding is weak, so the cold water breaks the lattice apart without any heating.): Explains the ball as a lattice coming apart in water instead of as metal that has been warmed past its melting point. Weak metallic bonding is the reason sodium's melting point is low, not a way of reaching it: the cube enters the water solid and rounds only after a second or two of reaction, and the same cube melts in a dry tube warmed to about 100 °C with no water present at all. *(Melting attributed to weak bonding rather than to heating)*
- **F** (The sodium hydroxide formed dissolves into the metal and lowers its melting point below room temperature.): Treats the product as acting on the metal. Sodium hydroxide is soluble and passes into the trough, which is why the water is left alkaline, and sodium's melting point is close to 100 °C, nowhere near room temperature. *(Product credited with lowering the melting point)*

Common mistake. Crediting the burning hydrogen with the melting. Sodium on cold water melts before any flame appears, and very often no flame appears at all. The heat comes from the reaction itself, released at the metal's surface faster than the cold water can carry it away.

Takeaway. **The Cheat Code:** when a Group 1 metal melts on water, two facts are working together: the reaction is strongly exothermic and the metal's melting point is unusually low. Neither fact on its own explains it, and no outside flame is needed.

Time-saving tip. Speed Heuristic: ask what each explanation needs before it can work. A flame, a burning oil film, friction and a hydroxide inside the metal are all extra assumptions, while a low melting point and an exothermic reaction are properties sodium has already.

Question 6

C4.8 · Quantitative chemistry · ★★☆☆☆ · 40 s

Calculate the volume of O_2 gas produced at room temperature and pressure (RTP) when 50.0 g of potassium chlorate decomposes according to the equation: $2KClO_3(s) \rightarrow 2KCl(s) + 3O_2(g)$ (Take molar gas volume at RTP = 24.0 dm^3 mol^{-1}, M_r of potassium chlorate = 122.5).

- **A.** 29.39 dm^3
- **B.** 14.69 dm^3
- **C.** 7.35 dm^3
- **D.** 13.71 dm^3
- **E.** 1200.0 dm^3
- **F.** 9.80 dm^3
- **G.** 6.53 dm^3
- **H.** 146.9 dm^3

Answer B. Key idea

Moles connect a mass to a gas volume. Divide the mass by M_r to get moles of $KClO_3$, scale by the equation's ratio of $\frac{3}{2}$ mol of O_2 per mol of $KClO_3$, then multiply by the molar gas volume, 24.0 dm^3 mol^{-1} at RTP.

Worked solution

1. Calculate moles of reactant

$$n = \frac{\text{mass}}{M_r} = \frac{50.0}{122.5} = 0.4082 \text{ mol}$$

2. Apply mole ratio from balanced equation

Moles of $O_2 = \frac{50.0}{122.5} \times 1.5 = 0.6122$ mol

3. Calculate gas volume at RTP

$V = n \times 24.0 \text{ dm}^3 \text{ mol}^{-1} = 0.6122 \times 24.0 = 14.69 \text{ dm}^3$

Matches Option **B**.

Why the other options are wrong

- **A** (29.39 dm^3): Multiplies by the 3 in front of O_2 but forgets the 2 in front of $KClO_3$: $\frac{50.0}{122.5} \times 3 = 1.2245$ mol, then $1.2245 \times 24.0 = 29.39$ dm^3. Using 3 rather than $\frac{3}{2}$ doubles the answer exactly. *(Divisor of the ratio dropped)*
- **C** (7.35 dm^3): Doubles the molar mass because of the 2 in front of $KClO_3$, dividing by $2 \times 122.5 = 245$, and then still applies the $\frac{3}{2}$ ratio: $\frac{50.0}{245} \times 1.5 = 0.3061$ mol, then $0.3061 \times 24.0 = 7.35$ dm^3. The ratio already accounts for the coefficients, so the mass is divided by 122.5 alone. *(Calculation / Conceptual Error)*
- **D** (13.71 dm^3): Finds the moles of oxygen correctly, 0.6122 mol, but multiplies by the molar volume at STP, 22.4 dm^3 mol^{-1}, instead of the 24.0 given for RTP: $0.6122 \times 22.4 = 13.71$ dm^3. *(Calculation / Conceptual Error)*
- **E** (1200.0 dm^3): Multiplies the mass straight by the molar volume without converting to moles: $50.0 \times 24.0 = 1200.0$ dm^3. The molar volume is the volume of one mole of gas, so the mass must first become moles of $KClO_3$ and then moles of O_2. *(Calculation / Conceptual Error)*
- **F** (9.80 dm^3): Converts the mass to moles correctly but reads the equation as one to one, taking one mole of O_2 per mole of chlorate: $50.0/122.5 = 0.4082$ mol, then $0.4082 \times 24.0 = 9.80$ dm^3. The equation gives three O_2 for every two $KClO_3$, so the moles of gas are 1.5 times the moles of solid. *(Mole ratio ignored)*
- **G** (6.53 dm^3): Applies the ratio upside down, $0.4082 \times \frac{2}{3} = 0.2721$ mol, then $0.2721 \times 24.0 = 6.53$ dm^3. The 2 belongs to $KClO_3$ and the 3 to O_2, so moving from chlorate to oxygen multiplies by $\frac{3}{2}$, not $\frac{2}{3}$. *(Mole ratio inverted)*
- **H** (146.9 dm^3): Works in cubic centimetres and converts back with the wrong factor: $0.6122 \times 24000 \approx 14690$ cm^3, then divides by 100 instead of 1000, giving 146.9 dm^3. There are 1000 cm^3 in 1 dm^3, so the volume is 14.69 dm^3. *(Calculation / Conceptual Error)*

Common mistake. Multiplying the mass directly by 24 without converting to moles first.

Takeaway. The Cheat Code: Go from mass to moles, through the coefficient ratio, then to volume, keeping the numbers as fractions: $\frac{50.0}{122.5} = \frac{20}{49}$, so $V = \frac{20}{49} \times \frac{3}{2} \times 24.0 = \frac{720}{49} \approx 14.69$ dm^3.

Time-saving tip. Speed Heuristic: Look at the relative magnitudes and units in the multiple-choice options to eliminate extreme or impossible values immediately.

Question 7

C3.4 · Chemical reactions and equations · ★★☆☆☆ · 85 s

In a blast furnace, iron(III) oxide (Fe_2O_3) is reduced by carbon monoxide to iron and carbon dioxide. Balanced with the smallest whole numbers, what is the sum of all the coefficients in this equation?

- **A.** 7
- **B.** 18
- **C.** 8
- **D.** 12
- **E.** 11
- **F.** 5
- **G.** 4
- **H.** 9

Answer H. Key idea

A balanced equation conserves every element, and once the coefficients are reduced to the smallest whole numbers there is only one set that does it. A formula written with no number in front of it still stands for one formula unit, so it counts when the coefficients are totalled.

Worked solution

1. Balance the equation

The skeleton is $Fe_2O_3 + CO \rightarrow Fe + CO_2$.

Iron first: two Fe atoms on the left need $2Fe$ on the right.

Carbon and oxygen travel together, so let the CO and the CO_2 coefficients both be x. Oxygen on the left is $3 + x$, three from the oxide and one from each CO, and on the right it is $2x$. Setting $3 + x = 2x$ gives $x = 3$.

Balanced: $Fe_2O_3 + 3CO \rightarrow 2Fe + 3CO_2$. Check the atoms: Fe $2 = 2$, C $3 = 3$, O $3 + 3 = 6$ against $3 \times 2 = 6$.

2. Add the coefficients, including the unwritten one

Fe_2O_3 carries no number in front of it, which means one formula unit, not none. The four coefficients are 1, 3, 2 and 3:

$$1 + 3 + 2 + 3 = 9$$

Sanity check: no smaller whole numbers balance this equation, and doubling every coefficient would give $2 + 6 + 4 + 6 = 18$, so 9 is the smallest sum available.

Matches Option **H**.

Why the other options are wrong

- **A** (7): Sets both carbon species to 2 so that they match the two iron atoms, $Fe_2O_3 + 2CO \rightarrow 2Fe + 2CO_2$, giving $1 + 2 + 2 + 2 = 7$. Oxygen is $3 + 2 = 5$ on the left and 4 on the right, so it is still unbalanced. *(Coefficients copied from the iron)*
- **B** (18): Doubles everything to keep the oxygen count even, $2Fe_2O_3 + 6CO \rightarrow 4Fe + 6CO_2$, and totals $2 + 6 + 4 + 6 = 18$. Every atom balances, Fe $4 = 4$, C $6 = 6$, O $6 + 6 = 12$ against 12, but the question asks for the smallest whole numbers and every coefficient here still divides by two. *(Coefficients not reduced)*
- **C** (8): Balances the equation correctly but adds only the coefficients that appear in print, $3 + 2 + 3 = 8$, missing the unwritten 1 in front of Fe_2O_3. *(Invisible coefficient dropped)*

- **D** (12): Reaches for the other blast furnace equation, reduction by carbon rather than by carbon monoxide: $2Fe_2O_3 + 3C \longrightarrow 4Fe + 3CO_2$ totals $2+3+4+3 = 12$. That equation is balanced, but the stem fixes carbon monoxide as the reducing agent, so CO stands on the left and CO_2 on the right. *(Coke equation recalled instead)*
- **E** (11): Totals the atoms rather than the numbers written in front of the formulae. On the left Fe_2O_3 contributes 5 atoms and $3CO$ contributes $3 \times 2 = 6$, giving 11, and since the equation balances the right hand side returns the same, $2 + 3 \times 3 = 11$. A coefficient is the multiplier in front of a formula, not the number of atoms that formula stands for. *(Atoms counted instead of coefficients)*
- **F** (5): Balances the iron and stops there, $Fe_2O_3 + CO \longrightarrow 2Fe + CO_2$, giving $1+1+2+1 = 5$. Oxygen is then $3 + 1 = 4$ on the left against 2 on the right. *(Stopped after the metal)*
- **G** (4): Treats the skeleton $Fe_2O_3 + CO \longrightarrow Fe + CO_2$ as already balanced and adds four invisible ones, $1+1+1+1 = 4$. It is not balanced: two Fe atoms on the left face one on the right. *(Skeleton equation counted)*

Common mistake. Adding only the numbers that are actually printed. Fe_2O_3 carries an invisible coefficient of 1, so totalling $3 + 2 + 3$ reports 8 for an equation that contains four species.

Takeaway. The Cheat Code: Balance the metal first, then the element that appears in only one compound on each side, and leave oxygen until last when it is split between two compounds. Then count every formula, including the ones whose coefficient is an unwritten 1.

Time-saving tip. Speed Heuristic: Carbon and oxygen move together here, so the CO and CO_2 coefficients must be equal, and the three spare oxygen atoms in Fe_2O_3 force both of them to be 3 with no trial and error at all.

Question 8

C8.2 · Separation techniques · ★★☆☆☆ · 90 s

A technician stirs dry sand, which does not dissolve, into a blue solution of copper(II) sulfate in water. Dry crystals of copper(II) sulfate must be recovered from the mixture, with no sand in them. Which pair of steps, carried out in this order, achieves that?

A. Heat the whole mixture until every drop of water has boiled away, then collect the solid left in the dish.
B. Boil the whole mixture and condense the vapour, then keep the clear liquid that collects in the receiver.
C. Cool the whole mixture in ice until crystals appear, then pour it onto filter paper and dry what is caught.
D. Pour the mixture onto filter paper, then heat the liquid that runs through until it is saturated and cool it slowly.
E. Pour the mixture onto filter paper, then wash the solid caught in the paper with warm water and dry it.

> **Answer D. Key idea**
> A mixture is taken apart by physical processes chosen to match how each component is held in it. A solid that never dissolved is still a separate phase and can be strained out on filter paper, while a solid that has dissolved is spread through the solvent and is recovered as crystals by removing solvent until the solution is saturated and then cooling it so crystals grow. The order matters, because anything still undissolved when crystals form is trapped inside them.

Worked solution

1. Sort the mixture into its parts

Three things are present: water, which is the solvent; copper(II) sulfate, a solid that has dissolved in that water; and sand, a solid that has not dissolved at all. The wanted product is the dissolved solid, as dry crystals, so the water is a discard and the sand is an impurity to be removed.

2. Take out the insoluble solid first

Sand grains are far larger than the dissolved copper(II) sulfate, so filter paper holds the sand back and lets the blue solution pass. Every gram of the dissolved salt is carried through in that liquid, and the paper keeps the sand. Doing this first is what guarantees a clean product: solid left in the beaker while crystals grow is trapped inside them.

3. Recover the dissolved solid from the clear liquid

Heat the blue filtrate to drive off part of the water until the solution is saturated, which is when a drop on a cold rod crystallises. Then let it cool slowly. Solubility falls as the temperature falls, so the excess salt comes out of solution as crystals, which are filtered off and dried.

Sanity check: the sand never dissolved, so it can only leave on filter paper, and the salt was dissolved, so it returns as crystals by concentrating and cooling. Boiling everything to dryness would leave a powder still mixed with sand.

Matches Option **D**.

Why the other options are wrong

- **A** (Heat the whole mixture until every drop of water has boiled away, then collect the solid left in the dish.): Boiling every drop away does bring the salt out, but sand cannot evaporate: the dish is left holding copper(II) sulfate powder mixed with the sand, which the question forbids. It also gives a powder rather than the crystals asked for. *(Conceptual Error)*
- **B** (Boil the whole mixture and condense the vapour, then keep the clear liquid that collects in the receiver.): Condensing the vapour keeps the water and throws away the target. The copper(II) sulfate is not volatile, so it stays in the flask with the sand, and the liquid collected in the receiver is pure water. *(Wrong component kept)*
- **C** (Cool the whole mixture in ice until crystals appear, then pour it onto filter paper and dry what is caught.): Cooling does grow crystals, but the sand is still suspended in the liquid, so the paper catches sand and crystals together. With no water removed first the solution is not saturated either, so the crop is tiny. *(Correct steps, wrong order)*
- **E** (Pour the mixture onto filter paper, then wash the solid caught in the paper with warm water and dry it.): This keeps the wrong fraction. The paper traps the sand, while the copper(II) sulfate is in the liquid that runs through, and washing the paper with warm water only dissolves any salt still clinging to the sand. *(Residue and filtrate confused)*

Common mistake. Reaching for the heat first. Boiling the whole mixture to dryness does drive off the water, but the sand is still sitting in the dish, so the solid recovered is a mixture rather than pure copper(II) sulfate, and evaporating to dryness gives a fine powder rather than crystals.

Takeaway. **The Cheat Code:** Take the insoluble solid out before you take the water out. Filter first, then concentrate and cool. Filtering after the crystals have grown just collects the impurity along with them.

Time-saving tip. Speed Heuristic: Ask which half you are keeping. Here it is the dissolved solid, so the water must leave and the liquid running through the paper is the valuable half. That single question kills every route that keeps the condensed vapour or the residue on the paper.

Question 9

C4.1 · Quantitative chemistry · ★★☆☆☆ · 45 s

Calculate the relative molar mass, M_r, of magnesium hydroxide, $Mg(OH)_2$. (A_r(Mg) = 24, A_r(O) = 16, A_r(H) = 1.)

- **A.** 58
- **B.** 42
- **C.** 57
- **D.** 82
- **E.** 41
- **F.** 26
- **G.** 56
- **H.** 29

Answer A. Key idea

M_r is the sum of the A_r values, each multiplied by its subscript: $24 + (2 \times 16) + (2 \times 1) = 58$.

Worked solution

1. Read the subscripts off the formula

$Mg(OH)_2$ contains $1 \times$ Mg, $2 \times$ O, $2 \times$ H.

2. Multiply each A_r by how many of that atom there are

$M_r = 24 + (2 \times 16) + (2 \times 1)$

3. Add the contributions

$M_r = 58$

M_r is a *relative* mass, so it carries no units.

Matches Option **A**.

Why the other options are wrong

- **B** (42): Reads the 2 as belonging to the H it stands next to, giving $24 + 16 + (2 \times 1) = 42$. The bracket closes after the H, so the 2 also applies to the O, which contributes $2 \times 16 = 32$. *(Bracket applied to the hydrogen only)*
- **C** (57): Doubles the O but leaves the H single: $24 + (2 \times 16) + 1 = 57$. The subscript sits outside the bracket, so it multiplies everything inside it and the hydrogen contributes $2 \times 1 = 2$. *(Bracket applied to the oxygen only)*
- **D** (82): Mg was counted twice. Each element in the formula contributes exactly once, multiplied by its subscript. *(Double counting)*
- **E** (41): Every subscript was read as 1, so the repeated atoms were counted only once. Each subscript multiplies the A_r of the atom it follows. *(Ignored subscripts)*
- **F** (26): The contribution of O was left out altogether. Work through the formula atom by atom so nothing is missed. *(Omitted a term)*
- **G** (56): Drops the hydrogen because A_r(H) = 1 looks too small to matter: $24 + (2 \times 16) = 56$. Every atom in the formula contributes, and the two hydrogen atoms add 2. *(Hydrogen discarded as negligible)*
- **H** (29): Exactly half the true value, $58 \div 2 = 29$, the result of halving the total somewhere. M_r is a plain sum, with no division anywhere. *(Spurious division)*

Common mistake. Ignoring a subscript that sits outside a bracket, such as the 2 in $Mg(OH)_2$, which applies to *both* the O and the H.

Takeaway. The Cheat Code: work left to right through the formula and write the running total as you go. A subscript outside a bracket multiplies everything inside it, so expand the bracket before you add anything.

Time-saving tip. Read what the question actually asks for before you calculate: a large share of lost marks here come from producing a correct intermediate value and stopping one step early.

Question 10

C2.2 · The Periodic Table · ★★★☆☆ · 85 s

Tellurium has a higher relative atomic mass than iodine, 127.6 against 126.9, yet every modern Periodic Table places tellurium first, with iodine falling in the halogen column beneath bromine. Which rule does the modern table order its elements by?

- **A.** Elements are ordered by increasing number of neutrons, and tellurium's atoms hold fewer neutrons than iodine's atoms.
- **B.** Elements are ordered by increasing relative atomic mass, with awkward pairs swapped to keep chemical families together.
- **C.** Elements are ordered by decreasing atomic radius, and tellurium's atoms are larger than iodine's atoms.
- **D.** Elements are ordered by increasing proton number, and tellurium has 52 protons against iodine's 53.
- **E.** Elements are ordered by increasing number of outer-shell electrons, six for tellurium and seven for iodine.
- **F.** Elements are ordered by increasing number of occupied electron shells, which is greater for iodine than for tellurium.

Answer D. Key idea

The modern Periodic Table is ordered by proton number, and every rival ordering is either a consequence of that rule or an approximation to it. Proton number is fixed for an element, since all of its isotopes share it, so it hands each element exactly one position and increases by exactly one from each element to the next. Relative atomic mass is an average taken over an isotope mixture, so it can run backwards between neighbours, which is why the tellurium and iodine pair is the standard test of which rule the table actually obeys.

Worked solution

1. Test the relative atomic mass rule against what the table does

If relative atomic mass set the order, the lighter element would come first. Iodine at 126.9 is the lighter of the pair, so a mass ordered table would read iodine and then tellurium. Every modern table reads tellurium and then iodine, so the observed order contradicts the mass rule outright. The chemistry contradicts it as well: mass ordering would drop iodine into the column headed by oxygen and sulfur, where it would be expected to form compounds of the type H_2X and to have no halogen behaviour, whereas iodine in fact forms HI, NaI and I_2 exactly as chlorine and bromine do.

2. Apply the proton number rule and check both facts at once

Tellurium has 52 protons and iodine has 53. Counting up in proton number puts tellurium at position 52 and iodine at position 53, which is the order the table uses, with no exception made and nothing moved by hand. The same rule lands iodine in Group 7 beneath bromine, which is where its chemistry says it belongs.

Sanity check: every isotope of one element shares its proton number but differs in mass, so only proton number gives each element a single unambiguous place in the sequence. Relative atomic mass cannot, and the tellurium and iodine pair is precisely where that failure shows.

Matches Option **D**.

Why the other options are wrong

- **A** (Elements are ordered by increasing number of neutrons, and tellurium's atoms hold fewer neutrons than iodine's atoms.): Substitutes the neutron count for the proton count, and states the fact backwards as well. Iodine's only stable isotope holds $127 - 53 = 74$ neutrons, while tellurium's common isotopes hold $128 - 52 = 76$ and $130 - 52 = 78$, so tellurium carries more neutrons rather than fewer, and those extra neutrons are exactly why it is the heavier element despite having one proton less. Neutron count also differs from one isotope of an element to the next, so it cannot fix a single place for each element. *(Neutron count used in place of proton count)*
- **B** (Elements are ordered by increasing relative atomic mass, with awkward pairs swapped to keep chemical families together.): This is Mendeleev's rule with a patch stitched on. Mass ordering gives iodine at 126.9 before tellurium at 127.6, so the pair has to be swapped deliberately to keep iodine with the halogens. The modern table needs no such patch, because proton number already gives the observed order. *(Superseded rule kept with an exception)*
- **C** (Elements are ordered by decreasing atomic radius, and tellurium's atoms are larger than iodine's atoms.): Uses a trend in place of a rule. Atomic radius does fall from tellurium to iodine across Period 5, but it jumps back up at the start of the next Period, so it cannot generate one continuously increasing sequence for the whole table. *(Periodic trend mistaken for the ordering)*
- **E** (Elements are ordered by increasing number of outer-shell electrons, six for tellurium and seven for iodine.): Works inside a single Period and fails everywhere else. Tellurium has six outer electrons and iodine seven, which happens to give the right order for this pair, but the count drops back to one at the start of the next Period, so the sequence breaks at every row boundary. *(Local pattern generalised)*
- **F** (Elements are ordered by increasing number of occupied electron shells, which is greater for iodine than for tellurium.): Occupied shells label the Periods, not the sequence within one. Tellurium and iodine both lie in Period 5 and so have the same number of occupied shells, which cannot separate them at all, let alone place tellurium first. *(Period label used as the ordering rule)*

Common mistake. Accepting relative atomic mass as the ordering rule and treating tellurium and iodine as an exception that was moved by hand to tidy the columns. Nothing was moved: proton number produces that order automatically, and it is mass ordering, not the table, that fails.

Takeaway. **The Cheat Code:** when a table position and a relative atomic mass disagree, back the proton number. Mass is an average over isotopes and can invert between neighbours, while proton number counts up by one every single time.

Time-saving tip. Speed Heuristic: an ordering rule has to give a whole number that rises by exactly one from each element to the next, right across the table. Radius rises and falls, outer electrons reset at the start of every Period, shell count is constant along a Period, and mass inverts here. Only proton number survives that filter.

END OF SOLUTIONS

Questions 11 to 27 are in the full paper

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Written by researchers in engineering, physics and mathematics at Oxford and Cambridge, and checked question by question before it is issued.

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EXAMPLE GRAMMAR SCHOOL

ESAT Mock Examination

Biology

Paper SMP-2026-09 · Version 1.5

NAME TEACHING GROUP DATE

Instructions

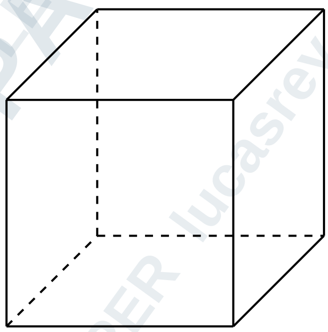
- Time allowed: **15 minutes**.
- There are **10 multiple-choice questions**. Each is worth 1 mark.
- There is no negative marking. Answer every question, even if you have to guess.
- Calculators are not permitted.
- Record your answers on the separate answer sheet. Only the answer sheet is marked.
- Formulae are not provided. The ESAT does not supply a formula sheet.

Note: the real ESAT is sat on screen at a Pearson test centre. This paper trains the mathematics and the pacing, not the on-screen interface.

Written for Example Grammar School by Lucas Rey, Engineering Science, University of Oxford; MRes, University of Cambridge.

Question 1

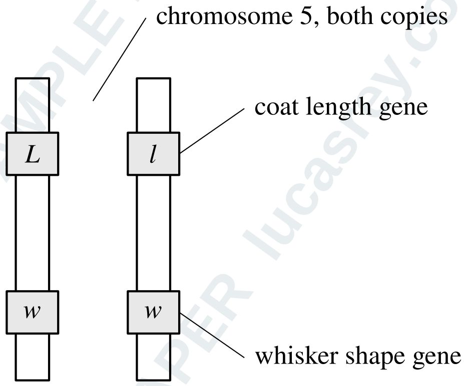
The diagram shows a cubic cell, with the length of one side marked. What is its surface area to volume ratio in its simplest integer form?



A. 6 : 1
B. 1 : 10
C. 3 : 5
D. 5 : 3
E. 3 : 50

Question 2

In domestic cats, one gene on chromosome 5 controls coat length, with the short-fur allele L completely dominant over the long-fur allele l. A second gene on the same chromosome controls whisker shape, with the straight allele W completely dominant over the curled allele w. The diagram shows the alleles carried on one cat's two copies of chromosome 5. Which statement describes this cat correctly?



- **A.** It is homozygous for coat length and heterozygous for whisker shape, so it has short fur and straight whiskers.
- **B.** It is homozygous for coat length and homozygous for whisker shape, so it has short fur and curled whiskers.
- **C.** It is heterozygous for coat length and heterozygous for whisker shape, so it has short fur and straight whiskers.
- **D.** It is heterozygous for coat length and homozygous for whisker shape, so it has short fur and curled whiskers.
- **E.** It is heterozygous for coat length and homozygous for whisker shape, so it has long fur and curled whiskers.
- **F.** It is homozygous for coat length and homozygous for whisker shape, so it has long fur and curled whiskers.

Question 3

Gene expression is controlled from the nucleus by

- **A.** selecting which genes are transcribed to mRNA.
- **B.** deleting unused genes permanently.
- **C.** moving genes to the ribosomes.
- **D.** choosing which proteins are eaten.
- **E.** dissolving the nuclear membrane.

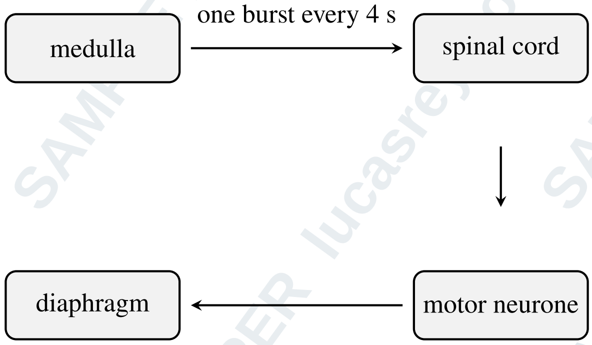
Question 4

A sperm carrying a Y chromosome fertilises an X egg. The child will be

- **A.** female (XX).
- **B.** female (X).
- **C.** male (YY).
- **D.** male (XX).
- **E.** male (XXY).
- **F.** male (XY).
- **G.** female (XY).
- **H.** cannot be determined.

Question 5

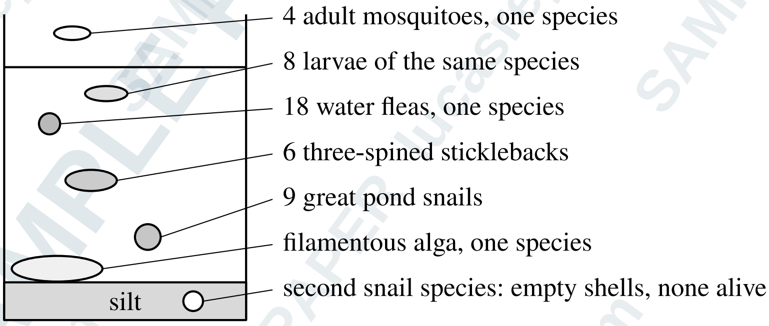
The diagram shows the pathway and the timing of the bursts of impulses that drive each inspiration in a resting adult. Every burst contains 30 impulses, and each whole burst produces one contraction of the diaphragm, drawing 450 cm^3 of air into the lungs. Breathing stays quiet and regular. Calculate the volume of air drawn into the lungs each minute.



A. 6750 $dm^3\ min^{-1}$
B. 13.5 $dm^3\ min^{-1}$
C. 27.0 $dm^3\ min^{-1}$
D. 0.45 $dm^3\ min^{-1}$
E. 1.8 $dm^3\ min^{-1}$
F. 202.5 $dm^3\ min^{-1}$
G. 67.5 $dm^3\ min^{-1}$
H. 6.75 $dm^3\ min^{-1}$

Question 6

A drainage ditch is surveyed as part of a study of freshwater ecology. The diagram shows the ditch in cross-section, and what the survey recorded in it. The surveyors also record the temperature of the water, the depth of the silt and the concentration of dissolved oxygen. Assume the survey found every species that is present as a living organism, and that the ditch is isolated from other water bodies. How many populations make up the community in this ditch?



A. 5
B. 4
C. 6
D. 7
E. 45

Question 7

A single cell undergoes 4 consecutive rounds of mitosis. Assuming all daughter cells survive and divide, how many cells will be present at the end?

A. 8
B. 256
C. 16
D. 15
E. 5

Question 8

Which list goes from simplest to most complex?

A. muscle cell → muscle tissue → heart → circulatory system.
B. circulatory system → heart → muscle tissue → muscle cell.
C. muscle tissue → muscle cell → circulatory system → heart.
D. muscle cell → heart → muscle tissue → circulatory system.
E. heart → muscle cell → muscle tissue → circulatory system.
F. muscle cell → muscle tissue → circulatory system → heart.

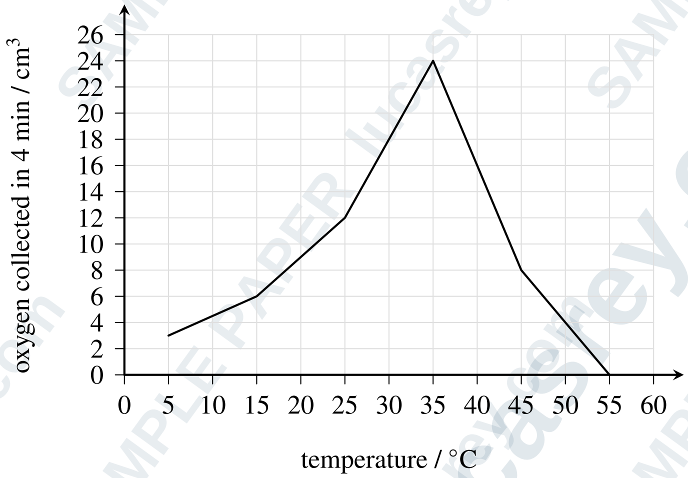
Question 9

A gardener takes a single healthy potato plant and grows 200 new plants from its tubers, planting no seed. She then crosses two different potato varieties and raises 200 seedlings from the seed produced. Assuming no mutations occur, which statement about the two groups of 200 plants is correct?

A. The tuber-grown plants are all genetically identical; the seedlings are not.
B. The seedlings are all genetically identical; the tuber-grown plants are not.
C. All 400 plants are genetically identical to one another.
D. Neither group contains any two plants that are genetically identical.
E. Each seedling is genetically identical to one tuber-grown plant, though neither group is uniform.

Question 10

A student investigates catalase, the enzyme in potato tissue that breaks hydrogen peroxide down into water and oxygen. Identical potato discs are dropped into hydrogen peroxide held in a water bath, and the volume of oxygen collected during the first four minutes is recorded at six temperatures. The graph shows the results. Below its optimum temperature this enzyme obeys a temperature coefficient $Q_{10} = 2.0$, meaning that the rate is multiplied by 2.0 for every 10°C rise. Assume hydrogen peroxide is in excess throughout and that all the gas is collected at room temperature and pressure. Using the 15°C reading, calculate the rate of oxygen production at 45°C that this relationship predicts.



A. 9 cm^3 min^{-1}
B. 24 cm^3 min^{-1}
C. 3 cm^3 min^{-1}
D. 48 cm^3 min^{-1}
E. 6 cm^3 min^{-1}
F. 2 cm^3 min^{-1}
G. 8 cm^3 min^{-1}
H. 12 cm^3 min^{-1}

END OF PAPER

Questions 11 to 27 are in the full paper

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EXAMPLE GRAMMAR SCHOOL

ESAT Mock: Answer Sheet

Biology

Paper SMP-2026-09 · Version 1.5

NAME

TEACHING GROUP

DATE

Fill in one bubble per question. If you change your mind, cross out the old mark clearly and fill the new bubble. Unfilled or doubly filled rows score zero.

Q	A	B	C	D	E	F	G	H
1	○	○	○	○	○			
2	○	○	○	○	○	○		
3	○	○	○	○	○			
4	○	○	○	○	○	○	○	○
5	○	○	○	○	○	○	○	○
6	○	○	○	○	○			
7	○	○	○	○	○			
8	○	○	○	○	○	○		
9	○	○	○	○	○			
10	○	○	○	○	○	○	○	○

MARKER INITIALS

TOTAL OUT OF 10

EXAMPLE GRAMMAR SCHOOL

ESAT Mock: Mark Scheme

Biology

Paper SMP-2026-09 · Version 1.5

Answer grid

Q	Ans	Q	Ans	Q	Ans
1	**C**	10	**H**		
2	**D**				
3	**A**				
4	**F**				
5	**H**				
6	**A**				
7	**C**				
8	**A**				
9	**A**				

Totals

10 marks available, 1 mark per question. There is no negative marking, so a wrong answer scores zero and never a negative. Only the answer sheet is marked.

Diagnostic table

Use this to see which topics the cohort is losing marks on.

Q	Ans	Node	Group	Topic	Difficulty
1	**C**	B2.1	B2	Cells	★★☆☆☆
2	**D**	B4.2	B4	Genetics	★★☆☆☆
3	**A**	B4.1	B4	Know and understand the nucleus as a site of genetic material in eukaryotic cells	★★☆☆☆
4	**F**	B3.4	B3	Sex determination	★★☆☆☆
5	**H**	B9.2	B9	Human physiology	★★☆☆☆
6	**A**	B10.1	B10	Ecology	★★☆☆☆
7	**C**	B3.1	B3	Cell division	★☆☆☆☆
8	**A**	B1.3	B1	Know and understand the levels of organisation within organisms as	★★☆☆☆
9	**A**	B3.3	B3	Cell division and reproduction	★★☆☆☆
10	**H**	B8.3	B8	Enzymes and digestion	★★★☆☆

Marks available by topic group

Group	Topic	Marks
B1	Cells and organisation	1
B2	Transport across membranes	1
B3	Cell division and reproduction	3
B4	Genetics	2
B8	Enzymes and digestion	1
B9	Human physiology	1
B10	Ecology	1
Total		**10**

Worked solutions for every question are in the separate worked solutions document.

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EXAMPLE GRAMMAR SCHOOL

ESAT Mock: Worked Solutions

Biology

Paper SMP-2026-09 · Version 1.5

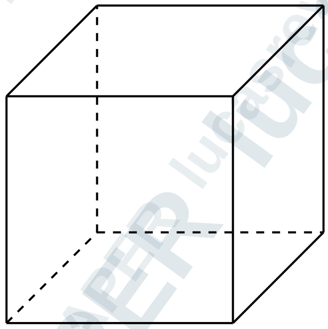
Staff copy

This document contains the answers. Keep it away from candidates until the paper has been sat. Marking guidance is in the separate mark scheme.

Question 1

B2.1 · Cells · ★★☆☆☆ · 40 s

The diagram shows a cubic cell, with the length of one side marked. What is its surface area to volume ratio in its simplest integer form?



A. 6 : 1
B. 1 : 10
C. 3 : 5
D. 5 : 3
E. 3 : 50

Answer C. Key idea

Surface area grows with the square of a length while volume grows with the cube, so the ratio of the two is not a fixed property of a shape: it depends on size. For a cube of side L the surface area is $6L^2$ and the volume is L^3, so the ratio is $6L^2 : L^3$, which simplifies to $6 : L$. The larger the cell, the smaller its surface area to volume ratio, which is why a large cell cannot supply its interior by diffusion across its surface alone.

Worked solution

1. Calculate Surface Area

$\text{SA} = 6 \times L^2 = 6 \times (10)^2 = 6 \times 100 = 600\,\mu\text{m}^2$

2. Calculate Volume

$V = L^3 = (10)^3 = 1000\,\mu\text{m}^3$

3. Simplify ratio

Ratio = 600 : 1000 Divide both sides by their greatest common divisor (200): 3 : 5

Matches Option **C**.

Why the other options are wrong

- **A** (6 : 1): Squared the side instead of cubing it when finding the volume, so V = 100 rather than 1000. That gives 600 : 100, which simplifies to 6 : 1. *(Calculation Error)*
- **B** (1 : 10): Counted the area of one face only, taking SA = 10 x 10 = 100 instead of 6 x 100 = 600. That gives 100 : 1000, which simplifies to 1 : 10. *(Conceptual Error)*
- **D** (5 : 3): Reversed the ratio to Volume : Surface Area, giving 1000 : 600, which simplifies to 5 : 3. *(Reading Error)*
- **E** (3 : 50): Forgot to square the side in the surface area, taking SA = 6 x 10 = 60 instead of 6 x 100 = 600. That gives 60 : 1000, which simplifies to 3 : 50. *(Calculation Error)*

Common mistake. Reversing the ratio to Volume:Surface Area.

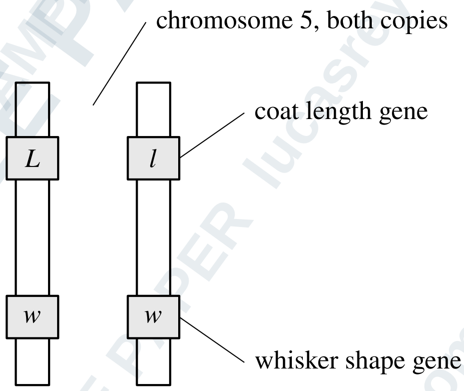
Takeaway. The Cheat Code: for a cube of side L the ratio is always 6 : L before simplifying. Compute the two quantities separately, then cancel: never quote the ratio the other way round, because surface area comes first in the name.

Time-saving tip. Speed Heuristic: for a cube the ratio is 6 : L before simplifying, so a side of 10 gives 6 : 10, which cancels to 3 : 5 without working out either quantity.

Question 2

B4.2 · Genetics · ★★☆☆☆ · 85 s

In domestic cats, one gene on chromosome 5 controls coat length, with the short-fur allele L completely dominant over the long-fur allele l. A second gene on the same chromosome controls whisker shape, with the straight allele W completely dominant over the curled allele w. The diagram shows the alleles carried on one cat's two copies of chromosome 5. Which statement describes this cat correctly?



- **A.** It is homozygous for coat length and heterozygous for whisker shape, so it has short fur and straight whiskers.
- **B.** It is homozygous for coat length and homozygous for whisker shape, so it has short fur and curled whiskers.
- **C.** It is heterozygous for coat length and heterozygous for whisker shape, so it has short fur and straight whiskers.
- **D.** It is heterozygous for coat length and homozygous for whisker shape, so it has short fur and curled whiskers.
- **E.** It is heterozygous for coat length and homozygous for whisker shape, so it has long fur and curled whiskers.
- **F.** It is homozygous for coat length and homozygous for whisker shape, so it has long fur and curled whiskers.

Answer D. Key idea

Homozygous and heterozygous describe one pair of alleles, not an organism as a whole, so the same cat can be homozygous at one gene and heterozygous at another. Whether the two alleles at that gene are identical is the only thing that decides the term. Which characteristic then shows is a separate question, settled by dominance: a completely dominant allele shows whenever it is present, so the recessive characteristic appears only where both alleles at that gene are the recessive one.

Worked solution

1. Compare the two alleles at each gene

Take the genes one at a time, reading across the pair of chromosomes.

At the coat-length gene the cat carries L on one copy and l on the other. Two different versions of one gene means heterozygous.

At the whisker gene it carries w on one copy and w on the other. Two identical versions means homozygous, and here homozygous for the recessive allele.

2. Apply dominance to get the characteristics

L is completely dominant, so it shows whenever it is present: the Ll cat has short fur. The whisker pair is ww, with no W anywhere on either chromosome, so there is nothing to mask the recessive allele and the whiskers are curled.

The cat is therefore heterozygous for coat length, homozygous for whisker shape, short-furred and curl-whiskered.

Sanity check: the two options offering straight whiskers both need a W that the diagram does not show, and the two options offering long fur both need the L on the left-hand copy to be masked by l, which contradicts L being completely dominant. Two statements survive both tests, B and D, and the terms separate them: the coat-length pair Ll is heterozygous, which leaves D.

Matches Option **D**.

Why the other options are wrong

- **A** (It is homozygous for coat length and heterozygous for whisker shape, so it has short fur and straight whiskers.): Swaps the two words, calling the mixed pair Ll homozygous and the identical pair ww heterozygous. Following that through, the supposedly heterozygous whisker pair is then given the dominant characteristic, so the whiskers come out straight. *(Terms Swapped)*
- **B** (It is homozygous for coat length and homozygous for whisker shape, so it has short fur and curled whiskers.): Reads both characteristics correctly, short fur because L is completely dominant in the Ll pair and curled whiskers because the ww pair contains no W to mask the recessive allele, but calls both genes homozygous on the grounds that each gene is present on both copies of chromosome 5. Homozygous asks whether the two alleles at one gene are the same letter, and at the coat-length gene they are L and l, so that pair is heterozygous. This differs from the reading that makes the same mistake with the terms and then also lets the recessive l show and reports long fur: here dominance is applied correctly and only the terminology is wrong, which is why it needs the definition to eliminate. *(Homozygous read as carrying two copies of the gene)*
- **C** (It is heterozygous for coat length and heterozygous for whisker shape, so it has short fur and straight whiskers.): Assumes an individual must be heterozygous at every gene because its two chromosome copies came from different parents. Both pairs are then called heterozygous, and a heterozygote shows the dominant characteristic, giving straight whiskers as well as short fur. *(Conceptual Error)*
- **E** (It is heterozygous for coat length and homozygous for whisker shape, so it has long fur and curled whiskers.): Reads both terms correctly but lets the lower-case l show in the Ll pair. That treats the long-fur

allele as the one expressed in a heterozygote, which reverses the dominance stated, and gives long fur. *(Dominance Reversed)*

- **F** (It is homozygous for coat length and homozygous for whisker shape, so it has long fur and curled whiskers.): Takes homozygous to mean nothing more than carrying two alleles of one gene, which is true of every pair, so both genes are called homozygous. The recessive letter is then read off at each, giving long fur and curled whiskers. *(Conceptual Error)*

Common mistake. Calling the whole cat heterozygous because its two chromosomes came from different parents. The terms apply gene by gene, and at the whisker gene the two copies happen to carry the same allele.

Takeaway. **The Cheat Code:** Read each gene's pair on its own. Two letters the same is homozygous, two letters different is heterozygous. Then, with complete dominance, the small letter shows only where there is no capital in that pair.

Time-saving tip. Speed Heuristic: Go to the whisker gene first. Both copies carry w, so curled whiskers are certain and the two options promising straight whiskers go at once. Only the coat-length pair on the four survivors then has to be read.

Question 3

B4.1 · Know and understand the nucleus as a site of genetic material in eukaryotic cells · ★★☆☆☆ · 45 s

Gene expression is controlled from the nucleus by

A. selecting which genes are transcribed to mRNA.
B. deleting unused genes permanently.
C. moving genes to the ribosomes.
D. choosing which proteins are eaten.
E. dissolving the nuclear membrane.

Answer A. Key idea

The nucleus controls gene expression by selecting which of its genes are transcribed into mRNA at any given time. The DNA stays inside the nucleus; only the mRNA copy leaves, so control is exercised over transcription rather than over the genes themselves.

Worked solution

The nucleus: stores chromosomes (DNA+protein), transcribes selected genes to mRNA, controls the cell's activities. Prokaryotes keep DNA free in the cytoplasm.

Applied: gene expression is controlled from the nucleus by. . . $\rightarrow$ selecting which genes are transcribed to mRNA.

Matches Option **A**.

Why the other options are wrong

- **B** (deleting unused genes permanently.): Cells regulate TRANSCRIPTION, not genome content. *(Conceptual Error)*
- **C** (moving genes to the ribosomes.): mRNA travels; genes stay. *(Conceptual Error)*
- **D** (choosing which proteins are eaten.): Nutrition is irrelevant to gene expression control. *(Conceptual Error)*
- **E** (dissolving the nuclear membrane.): That happens only in division. *(Conceptual Error)*

Common mistake. Thinking the nucleus controls expression by removing genes or sending them out to the ribosomes. The genes stay in the nucleus, intact, and control lies in which of them are transcribed to mRNA.

Takeaway. Treat the nucleus as both library and head office: it stores the chromosomes intact and decides which genes get transcribed. Regulation changes which mRNA is made, never which genes the cell possesses.

Time-saving tip. Speed Heuristic: the genes never leave the nucleus and are never discarded, so the only control left is choosing which of them are transcribed to mRNA.

Question 4

B3.4 · Sex determination · ★★☆☆☆ · 45 s

A sperm carrying a Y chromosome fertilises an X egg. The child will be

- **A.** female (XX).
- **B.** female (X).
- **C.** male (YY).
- **D.** male (XX).
- **E.** male (XXY).
- **F.** male (XY).
- **G.** female (XY).
- **H.** cannot be determined.

> **Answer F. Key idea**
>
> Every egg carries an X, so the sperm's sex chromosome decides the outcome: an X sperm gives a female (XX), a Y sperm gives a male (XY). Here a Y sperm fertilises the X egg, so the zygote is XY and the child is male.

Worked solution

Females are XX (all eggs carry X). Males are XY (sperm split 50/50 X:Y). Punnett square of XX $\times$ XY: 2 XX, 2 XY $\rightarrow$ 1:1 sex ratio, decided at fertilisation.

Applied: a sperm carrying a Y chromosome fertilises an X egg. The child will be... $\rightarrow$ male (XY).

Matches Option **F**.

Why the other options are wrong

- **A** (female (XX).): Two X's make female; this zygote has X and Y. *(Conceptual Error)*
- **B** (female (X).): Records only the mother's contribution, the single X of the egg, and calls the zygote female because no Y has been written down. The sperm's Y enters the zygote as well, giving XY. *(Conceptual Error)*
- **C** (male (YY).): Assumes the male pair matches, by analogy with the female XX: the sperm brought a Y, so the pair is written YY. The egg still contributes its X, so the zygote is XY. *(Conceptual Error)*
- **D** (male (XX).): XX is the female genotype, and this zygote carries the sperm's Y. *(Conceptual Error)*
- **E** (male (XXY).): Forgets that meiosis halves the chromosome number and treats the mother as passing both her X chromosomes: XX from the egg plus Y from the sperm gives XXY, called male because a Y is present. An egg carries one X, not two, so the zygote is XY. *(Conceptual Error)*
- **G** (female (XY).): XY determines male. *(Conceptual Error)*
- **H** (cannot be determined.): The chromosomes given settle it. *(Conceptual Error)*

Common mistake. Defaulting to 'female' because the egg contributes an X: the egg always carries X, so it is the sperm's chromosome (here Y) that decides the sex.

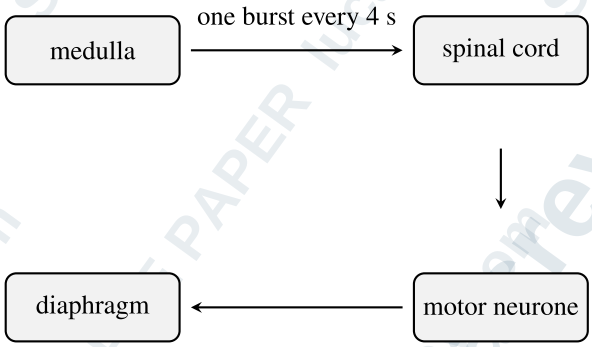
Takeaway. The Cheat Code: Mum can only give an X, so Dad's sperm flips the coin: an X sperm makes XX (female) and a Y sperm makes XY (male), which is why the ratio is 1:1.

Time-saving tip. Speed Heuristic: the egg always supplies an X, so add the sperm's chromosome to it: X plus Y is XY, which is male.

Question 5

B9.2 · Human physiology · ★★☆☆☆ · 90 s

The diagram shows the pathway and the timing of the bursts of impulses that drive each inspiration in a resting adult. Every burst contains 30 impulses, and each whole burst produces one contraction of the diaphragm, drawing 450 cm^3 of air into the lungs. Breathing stays quiet and regular. Calculate the volume of air drawn into the lungs each minute.



- **A.** 6750 $dm^3\ min^{-1}$
- **B.** 13.5 $dm^3\ min^{-1}$
- **C.** 27.0 $dm^3\ min^{-1}$
- **D.** 0.45 $dm^3\ min^{-1}$
- **E.** 1.8 $dm^3\ min^{-1}$
- **F.** 202.5 $dm^3\ min^{-1}$
- **G.** 67.5 $dm^3\ min^{-1}$
- **H.** 6.75 $dm^3\ min^{-1}$

Answer H. Key idea

A motor neurone carries impulses out from the central nervous system to an effector, and the whole burst that arrives drives one contraction of that effector, however many impulses the burst contains. The volume of air moved per minute is then the volume drawn in by one contraction multiplied by the number of contractions in a minute, with both quantities put into the units the answer is wanted in.

Worked solution

1. Turn the burst interval into a breathing rate

One whole burst produces one contraction of the diaphragm, so one burst is one inspiration whatever the number of impulses inside it. A burst every 4 seconds gives

$$\frac{60}{4} = 15 \text{ inspirations per minute.}$$

The 30 impulses describe the traffic along the motor neurone that drives a single contraction; they are not a count of breaths.

2. Put the volume into the units of the answer

$$450\ cm^3 = \frac{450}{1000} = 0.45\ dm^3 \text{ drawn in per inspiration.}$$

3. Combine volume per breath with breaths per minute

$0.45 \times 15 = 6.75\ \text{dm}^3\ \text{min}^{-1}$.

Sanity check: a resting adult moves roughly half a cubic decimetre of air about fifteen times a minute, so an answer near 7 $\text{dm}^3\ \text{min}^{-1}$ is the size expected. An answer of 6750 $\text{dm}^3\ \text{min}^{-1}$ would be over six cubic metres of air every minute.

Matches Option **H**.

Why the other options are wrong

- **A** (6750 $\text{dm}^3\ \text{min}^{-1}$): Multiplies 450 by 15 and labels the result in cubic decimetres: $450 \times 15 = 6750$. The tidal volume is measured in cm^3 and $1000\ \text{cm}^3 = 1\ \text{dm}^3$, so it must be divided by 1000 before the multiplication. *(Volume conversion omitted)*
- **B** (13.5 $\text{dm}^3\ \text{min}^{-1}$): Takes the 30 impulses in a burst as thirty inspirations: $0.45 \times 30 = 13.5$. Every impulse in one burst serves the same single contraction, so the burst counts once, not thirty times. *(Impulses counted as breaths)*
- **C** (27.0 $\text{dm}^3\ \text{min}^{-1}$): Assumes one inspiration each second, giving sixty per minute: $0.45 \times 60 = 27.0$. The diagram gives one burst every 4 seconds, so the sixty seconds must be divided by four first. *(Interval dropped from the conversion)*
- **D** (0.45 $\text{dm}^3\ \text{min}^{-1}$): Converts the tidal volume and stops there: $450 \div 1000 = 0.45$. That is the volume drawn in by one contraction, and it still has to be multiplied by the 15 inspirations a minute contains. *(Volume per breath reported as volume per minute)*
- **E** (1.8 $\text{dm}^3\ \text{min}^{-1}$): Reads 'a burst every 4 seconds' as four inspirations per minute: $0.45 \times 4 = 1.8$. An interval has to be inverted before it becomes a rate, and $60 \div 4$ gives 15 inspirations per minute. *(Interval used directly as a rate)*
- **F** (202.5 $\text{dm}^3\ \text{min}^{-1}$): Gets the burst rate right but gives every impulse its own contraction: $30 \times 15 = 450$ contractions per minute, then $0.45 \times 450 = 202.5$. The stem states that one whole burst, all 30 impulses of it, produces a single contraction. *(Each impulse counted as a contraction)*
- **G** (67.5 $\text{dm}^3\ \text{min}^{-1}$): Divides by 100 instead of 1000: $450 \div 100 = 4.5$, then $4.5 \times 15 = 67.5$. There are 1000 cm^3 in 1 dm^3, so each inspiration is 0.45 dm^3. *(Wrong factor in the volume conversion)*

Common mistake. Using the number of impulses in a burst as the breathing rate. The impulses are the signal that produces one contraction; how often breathing happens is set by how often a burst arrives, which here is once every 4 seconds.

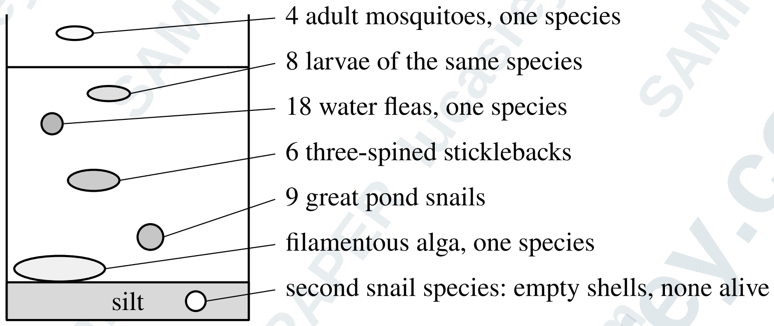
Takeaway. The Cheat Code: Turn an interval into a per-minute rate before anything else, and convert the volume into the units the options use at the same time. Any per-minute total is then volume per event multiplied by events per minute.

Time-saving tip. Speed Heuristic: $60 \div 4 = 15$, then split the multiplication: $0.45 \times 10 = 4.5$ and $0.45 \times 5 = 2.25$, which add to 6.75. The count of 30 impulses never enters the arithmetic.

Question 6

B10.1 · Ecology · ★★☆☆☆ · 90 s

A drainage ditch is surveyed as part of a study of freshwater ecology. The diagram shows the ditch in cross-section, and what the survey recorded in it. The surveyors also record the temperature of the water, the depth of the silt and the concentration of dissolved oxygen. Assume the survey found every species that is present as a living organism, and that the ditch is isolated from other water bodies. How many populations make up the community in this ditch?



A. 5
B. 4
C. 6
D. 7
E. 45

Answer A. Key idea

The levels of organisation in an ecosystem are nested: an individual is one organism, a population is all the individuals of one species living in the same place, a community is all the populations living there together, and the ecosystem is that community together with the non-living surroundings it interacts with. Counting populations therefore means counting living species rather than individuals, and a larva and an adult of one species belong to a single population, while a species left only as dead remains has no living members and so no population in that place.

Worked solution

1. Separate the living organisms from everything else recorded

A community is made up of living organisms only, so the temperature of the water, the depth of the silt and the concentration of dissolved oxygen are set aside straight away. They are the non-living part of the ecosystem, not members of the community.

2. Group the living individuals into species

Water fleas, great pond snails, three-spined sticklebacks, mosquitoes and the filamentous alga make five species. The alga is counted: a population is a population whether the organism photosynthesises or not, and the producers belong to the community as much as the consumers do. The 8 larvae and the 4 adult mosquitoes are one species at two stages of a single life cycle, so they are 12 individuals forming one population, not two populations.

3. Rule out the species with no living members

The second snail species appears only as empty shells in the silt. A population is the living individuals of a species in a place, so with none alive in the ditch that species contributes no population to this community. The community is therefore made up of 5 populations.

Sanity check: every number in the survey, 18, 9, 6, 8 and 4, counts individuals, and individuals sit one level below populations, so the answer must be far smaller than their total of 45.

Matches Option **A**.

Why the other options are wrong

- **B** (4): Took a community to mean the animals in it and left the filamentous alga out, counting only water fleas, snails, sticklebacks and mosquitoes for 4. A community is every population in the habitat, and the producers are part of it just as the consumers are. *(Producers excluded)*
- **C** (6): Added the second snail species on the strength of its empty shells, giving $5 + 1 = 6$. Shells are dead remains, and with no living individual of that species in the ditch there is no population of it to count. *(Dead remains counted)*
- **D** (7): Counted the empty shells as a population and also split the mosquitoes into a larval population and an adult population, giving $5 + 1 + 1 = 7$. The shells are not alive, and the larvae and adults of one species make one population. *(Dead remains counted and life stages split)*
- **E** (45): Added the individuals recorded, $18 + 9 + 6 + 8 + 4 = 45$, and gave that as the answer. Individuals are the level below populations: those 45 organisms are grouped into populations, and the question asks how many of those groups there are. *(Individuals counted instead of populations)*

Common mistake. Splitting one species across its life stages. The 8 mosquito larvae and the 4 adult mosquitoes look like two different kinds of animal and get counted as two populations, but they are the same species living in the same ditch, so they are 12 individuals of one population.

Takeaway. The Cheat Code: Go down the list once and ask two questions of every entry: is it alive, and is it a species already counted? Anything non-living belongs to the ecosystem rather than the community, and any second life stage, second sex or second colour form of a species already on the list adds nothing to the total.

Time-saving tip. Speed Heuristic: Count species names, not entries. Strike out the three physical measurements, merge the larvae with the adults, and delete the shells, and the survey collapses to five living species before a single number is used.

Question 7

B3.1 · Cell division · ★☆☆☆☆ · 20 s

A single cell undergoes 4 consecutive rounds of mitosis. Assuming all daughter cells survive and divide, how many cells will be present at the end?

A. 8
B. 256
C. 16
D. 15
E. 5

Answer C. Key idea

Mitosis turns each cell into two genetically identical daughter cells, so when every cell divides the number of cells doubles in each round: after n rounds a single cell has become 2^n cells.

Worked solution

1. Understand mitosis cell division

Each round of mitosis doubles the number of cells. The formula for the number of cells after n divisions is 2^n.

2. Calculate

$2^4 = 16$

Matches Option **C**.

Why the other options are wrong

- **A** (8): Multiplied by 2 instead of calculating 2^n. *(Mathematical Error)*
- **B** (256): Treated each round as meiosis, which makes four cells from one, giving $4^4 = 256$. Mitosis makes two daughter cells, so four rounds give $2^4 = 16$. *(Calculation / Conceptual Error)*
- **D** (15): Counted the divisions rather than the cells: $1 + 2 + 4 + 8 = 15$ cells divide over the four rounds, but each division leaves two cells, so $2^4 = 16$ cells are present at the end. *(Calculation / Conceptual Error)*
- **E** (5): Added one cell per round, $1 + 4 = 5$, as though only one cell divided each time. Every cell divides in every round, so the number doubles: $2^4 = 16$. *(Calculation / Conceptual Error)*

Common mistake. Multiplying the number of divisions by 2 instead of taking 2 to the power of n.

Takeaway. **The Cheat Code:** Each round doubles the count, so n rounds from one cell give 2^n cells: list the doublings, 2, 4, 8, 16, one per round.

Time-saving tip. Speed Heuristic: count the doublings in your head, one per round: 2, 4, 8, 16.

Question 8

B1.3 · Know and understand the levels of organisation within organisms as · ★★☆☆☆ · 45 s

Which list goes from simplest to most complex?

A. muscle cell → muscle tissue → heart → circulatory system.
B. circulatory system → heart → muscle tissue → muscle cell.
C. muscle tissue → muscle cell → circulatory system → heart.
D. muscle cell → heart → muscle tissue → circulatory system.
E. heart → muscle cell → muscle tissue → circulatory system.
F. muscle cell → muscle tissue → circulatory system → heart.

Answer A. Key idea

Organisms are organised in levels of increasing complexity: similar cells form a tissue, different tissues form an organ, and organs working together form an organ system. The heart is an organ, and the circulatory system is the organ system that contains it.

Worked solution

The hierarchy: CELL (basic unit) → TISSUE (similar cells, one job) → ORGAN (different tissues, one structure) → ORGAN SYSTEM (organs cooperating) → ORGANISM.

Applied: which list goes from simplest to most complex. . . → muscle cell → muscle tissue → heart → circulatory system.

Matches Option **A**.

Why the other options are wrong

- **B** (circulatory system → heart → muscle tissue → muscle cell.): Reversed. *(Conceptual Error)*
- **C** (muscle tissue → muscle cell → circulatory system → heart.): Swaps both pairs: it places muscle tissue before the muscle cell (cells build tissues) and the circulatory system before the heart (organs build organ systems). *(Conceptual Error)*
- **D** (muscle cell → heart → muscle tissue → circulatory system.): Tissues build organs, not the reverse. *(Conceptual Error)*

- **E** (heart → muscle cell → muscle tissue → circulatory system.): Puts the heart first, but an organ is made of tissues, which are made of cells, so the heart belongs after the muscle tissue and before the circulatory system. *(Conceptual Error)*
- **F** (muscle cell → muscle tissue → circulatory system → heart.): Gets the bottom of the hierarchy right, cell then tissue, then swaps the top two by placing the heart last as the largest single structure named. The circulatory system contains the heart together with the blood vessels, so the organ sits below the organ system: cell, tissue, organ, organ system. *(Organ ranked above the system containing it)*

Common mistake. Placing the heart above the circulatory system because it is the largest single structure named. The circulatory system contains the heart and the blood vessels, so the organ comes before the organ system.

Takeaway. cells→tissues→organs→systems→organism

Time-saving tip. Speed Heuristic: the heart must come before the circulatory system that contains it, which leaves A, D and E, and only A puts the muscle tissue before the heart.

Question 9

B3.3 · Cell division and reproduction · ★★☆☆☆ · 85 s

A gardener takes a single healthy potato plant and grows 200 new plants from its tubers, planting no seed. She then crosses two different potato varieties and raises 200 seedlings from the seed produced. Assuming no mutations occur, which statement about the two groups of 200 plants is correct?

- **A.** The tuber-grown plants are all genetically identical; the seedlings are not.
- **B.** The seedlings are all genetically identical; the tuber-grown plants are not.
- **C.** All 400 plants are genetically identical to one another.
- **D.** Neither group contains any two plants that are genetically identical.
- **E.** Each seedling is genetically identical to one tuber-grown plant, though neither group is uniform.

Answer A. Key idea

The number of parents decides whether offspring are copies. Asexual reproduction involves one parent and no fusion of gametes, so when no mutations occur the offspring carry exactly that parent's genes and are therefore genetically identical to the parent and to each other. Sexual reproduction involves two parents, and each offspring receives its own combination of genes drawn from both, so when each parent is heterozygous at many genes, as potato varieties are, the offspring differ from their parents and from one another. Growing plants from the tubers of one plant is the first case; raising plants from seed made by crossing two varieties is the second.

Worked solution

1. Classify each of the gardener's two methods

Tubers are part of the parent plant itself. Growing 200 plants from the tubers of one potato plant involves that single plant and no other, and no gametes fuse, so this is asexual reproduction with one parent.

The seed was produced by crossing two different varieties. Two parents contributed and each seed carries a combination of genes drawn from both, so this is sexual reproduction with two parents.

2. Apply the genetic consequence to each group

Asexual reproduction copies one parent's genes, and the stem rules out mutations, so nothing at all introduces a difference. All 200 tuber-grown plants therefore carry the same genes as the original plant and so as each other: that group is uniform.

Sexual reproduction combines two different sets of genes, and every seed receives its own combination. Each potato variety is heterozygous at many of its genes, so meiosis gives every pollen grain and every egg its own mixture of alleles, and the 200 seedlings differ from one another.

Putting the two together: the tuber-grown group is genetically identical throughout and the seedlings are not.

Sanity check: the phrase assuming no mutations occur is what makes the first group exactly uniform, since mutation is the only thing that could disturb a copy. It does nothing to the second group, because the seedlings' variation comes from sexual reproduction between two heterozygous parents rather than from mutation.

Matches Option **A**.

Why the other options are wrong

- **B** (The seedlings are all genetically identical; the tuber-grown plants are not.): This swaps the two methods over. It follows from thinking that a seed, which is shed by one plant, must be a copy of it, while tubers dug up and replanted in separate places must give rise to different plants. The direction is backwards: the seed here carries genes from two varieties, and the tubers carry the genes of one plant only. *(Asexual and sexual routes reversed)*
- **C** (All 400 plants are genetically identical to one another.): This takes assuming no mutations occur to mean that nothing can differ anywhere, so all 400 plants are counted as copies. Removing mutation does fix the tuber-grown group as identical, but the seedlings' variation was never caused by mutation. It comes from meiosis and random fertilisation in a cross between two parents that are each heterozygous at many genes, and the assumption does not touch that. *(No-mutation assumption over-extended)*
- **D** (Neither group contains any two plants that are genetically identical.): This applies the rule for sexual reproduction to both batches, assuming that reproduction always produces offspring that differ. It overlooks the defining feature of the one-parent route: with no gametes fusing and no mutations, there is nothing to make one tuber-grown plant differ from the next, so that group contains 200 plants that are identical. *(Variation assumed to be universal)*
- **E** (Each seedling is genetically identical to one tuber-grown plant, though neither group is uniform.): This assumes the two batches were drawn from one shared stock, so that the seedlings mirror the tuber plants one for one. Nothing in the scenario links them that way. The tuber plants copy a single original plant, while each seedling is a fresh combination from two different varieties, so pairing them off across the groups has no basis, and it also denies that the tuber group is uniform. *(Invents a pairing between the two groups)*

Common mistake. Reading assuming no mutations occur as a promise that nothing anywhere can differ, and concluding that all 400 plants must match. That assumption only removes the one source of difference that could disturb copies made from a single parent. Variation among the seedlings has an entirely different origin: meiosis and random fertilisation, which give each seed its own mix of the two parents' alleles.

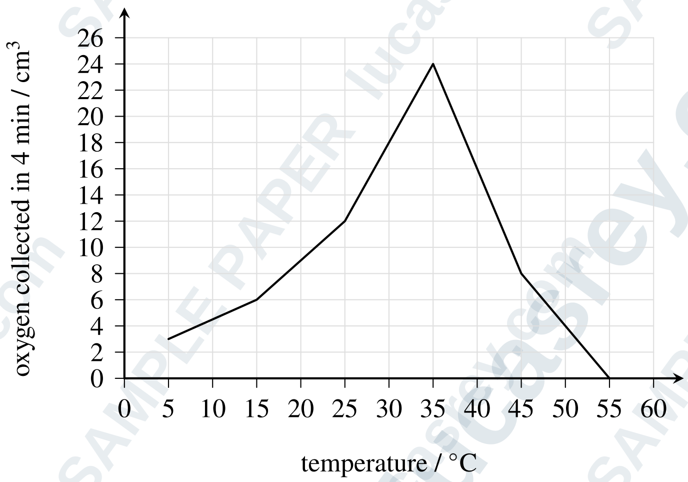
Takeaway. The Cheat Code: Count the parents before anything else. One parent with no mutations means the offspring are copies of it and of each other; two parents means every offspring is a fresh combination and no two need match. Tubers, runners, bulbs and cuttings are all the one-parent route, even though the plants they give end up growing separately.

Time-saving tip. Speed Heuristic: Label the two batches one parent and two parents before you read a single option, then look for the option that pairs identical with the one-parent batch. Four of the five can be dismissed without thinking about potatoes at all.

Question 10

B8.3 · Enzymes and digestion · ★★★☆☆ · 90 s

A student investigates catalase, the enzyme in potato tissue that breaks hydrogen peroxide down into water and oxygen. Identical potato discs are dropped into hydrogen peroxide held in a water bath, and the volume of oxygen collected during the first four minutes is recorded at six temperatures. The graph shows the results. Below its optimum temperature this enzyme obeys a temperature coefficient $Q_{10} = 2.0$, meaning that the rate is multiplied by 2.0 for every 10°C rise. Assume hydrogen peroxide is in excess throughout and that all the gas is collected at room temperature and pressure. Using the 15°C reading, calculate the rate of oxygen production at 45°C that this relationship predicts.



A. 9 cm^3 min^{-1}
B. 24 cm^3 min^{-1}
C. 3 cm^3 min^{-1}
D. 48 cm^3 min^{-1}
E. 6 cm^3 min^{-1}
F. 2 cm^3 min^{-1}
G. 8 cm^3 min^{-1}
H. 12 cm^3 min^{-1}

Answer H. Key idea

Below the temperature at which it denatures, an enzyme works faster as the temperature rises, because substrate and enzyme molecules collide more often and with more energy. A temperature coefficient of 2.0 states that this speeding up is a doubling for every 10°C, so a rise made up of n ten-degree steps multiplies the rate by 2.0^n, not by $2.0n$. Above the optimum the relationship fails, because the active site is losing its shape and the measured rate falls. The answer is wanted as a rate, so a volume collected over a fixed interval has to be divided by that interval, and because the scaling is a multiplication, that division can come before or after it.

Worked solution

1. Turn the graph reading into a rate

At 15°C the apparatus collects 6 cm^3 of oxygen in four minutes. A rate is a volume per unit time, so this reading is divided by the collection time, here before it is scaled:

rate at 15°C $= \dfrac{6}{4} = 1.5$ cm^3 min^{-1}

2. Find the multiplying factor for the temperature rise

From 15°C to 45°C is a rise of 30°C, which is three separate 10°C steps. Each step multiplies the rate by 2.0, so the steps compound:

factor = $2.0^3 = 8$

3. Apply the factor

predicted rate = $1.5 \times 8 = 12$ cm^3 min^{-1}

Sanity check: the plotted points below the optimum do double every ten degrees, running 3 → 6 → 12 → 24 cm^3 per four minutes at 5, 15, 25 and 35°C. One further step gives 48 cm^3 per four minutes at 45°C, and dividing by four again returns 12 cm^3 min^{-1}. The graph shows only 8 cm^3 per four minutes actually collected at 45°C, because above the optimum the enzyme is denaturing and the relationship no longer holds; the question asks for the predicted value, not the measured one.

Matches Option **H**.

Why the other options are wrong

- **A** (9 cm^3 min^{-1}): The number of steps is used as a multiplier rather than a power: $2.0 \times 3 = 6$, then $1.5 \times 6 = 9$ cm^3 min^{-1}. Repeated doubling compounds, so the correct factor is $2.0^3 = 8$. *(Multiplying by the coefficient instead of raising it to a power)*
- **B** (24 cm^3 min^{-1}): Counts the four temperatures 15, 25, 35 and 45°C as four steps rather than the three gaps between them: $2.0^4 = 16$, then $1.5 \times 16 = 24$. A rise of 30°C contains three steps of 10°C, so the factor is $2.0^3 = 8$. *(Ten-degree steps counted one too many)*
- **C** (3 cm^3 min^{-1}): The coefficient is applied once, whatever the size of the rise: $1.5 \times 2.0 = 3$ cm^3 min^{-1}. This treats Q_{10} as a single doubling rather than a doubling per 10°C, and it ignores two of the three steps between 15°C and 45°C. *(Coefficient applied once for the whole rise)*
- **D** (48 cm^3 min^{-1}): The four-minute volume is used as though it were already a rate: $6 \times 8 = 48$. That is the volume the apparatus would collect in four minutes at 45°C, so it is a factor of four too large for an answer in cm^3 min^{-1}. *(Collection time never divided out)*
- **E** (6 cm^3 min^{-1}): Counts only the two readings lying between the start and the target, at 25 and 35°C, as the steps: $2.0^2 = 4$, then $1.5 \times 4 = 6$. The rise from 15°C to 45°C is three ten-degree steps, not two. *(Ten-degree steps counted one too few)*
- **F** (2 cm^3 min^{-1}): This is the measured point read straight off the graph at 45°C: 8 cm^3 in four minutes, so $8 \div 4 = 2$ cm^3 min^{-1}. It is what the enzyme really does, but the enzyme is denaturing there, so it is not what the Q_{10} relationship predicts. *(Reading the observed value instead of extrapolating)*
- **G** (8 cm^3 min^{-1}): Reads the plotted point at 45°C, 8 cm^3 collected in four minutes, and quotes the 8 as though it were already a rate; the multiplying factor $2.0^3 = 8$ lands on the same number and makes it look confirmed. Dividing by the four minutes gives 2 cm^3 min^{-1}, and that point is the denatured measurement rather than the prediction asked for. *(Graph value quoted without dividing by the collection time)*

Common mistake. Multiplying the coefficient by the number of steps instead of raising it to that power: taking $2.0 \times 3 = 6$ and reporting $1.5 \times 6 = 9$ cm^3 min^{-1}. Three successive doublings give $2 \times 2 \times 2 = 8$, not 6.

Takeaway. The Cheat Code: A Q_{10} is a multiplier per 10°C, so count the whole ten-degree steps the temperature moves and make that count the power, never the multiplier. Then check what the axis of the graph actually measures: a volume collected over a fixed interval is not yet a rate, and one division separates the two.

Time-saving tip. Speed Heuristic: Do the division by four last. $6 \times 8 = 48$, then $48 \div 4 = 12$, and both steps are mental arithmetic. The intermediate 48 is itself printed as an option, so reaching 48 means the division by the collection time is still to come, not that the working is finished.

END OF SOLUTIONS

Questions 11 to 27 are in the full paper

This is an extract. The 10 questions before this page are real questions from a commissioned worked solutions, shown in full and with nothing withheld, so that the standard can be judged rather than described.

The rest of the paper is not hidden behind this page. It was never in this file. Each commissioned paper is written for one school, recorded against it, and issued to no other, so the questions in it are not published anywhere, including here.

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A full paper runs to 27 questions in this module alone, with a question paper, an answer sheet, a mark scheme and a worked solution for every question, across all five ESAT modules.

Written by researchers in engineering, physics and mathematics at Oxford and Cambridge, and checked question by question before it is issued.

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