



**St Anne's
College**
University of Oxford



**CLARE HALL
CAMBRIDGE**

ESAT Mock Papers

An extract: the first 10 questions of each module, in full

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50 questions across 5 modules (Mathematics 1, Mathematics 2, Physics, Chemistry, Biology), with a question paper, an answer sheet, a mark scheme and full worked solutions for each. 293 pages. The full paper carries 27 questions per module.

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5 September 2026

Dr A. Example
Head of Sixth Form
Example Grammar School

Copy to Mr B. Sample, Head of Physics.

Dear Dr Example,

The ESAT mock papers are enclosed, ready to print. There are five module papers, each with full worked solutions and a mark scheme, plus an answer sheet for each module and one set of invigilator instructions for the room.

The entry numbers you gave me are below, so you can set the print run.

Module	Candidates	Module	Candidates
Mathematics 1	30	Mathematics 2	30
Physics	30	Chemistry	30
Biology	30		

On the questions. This is the free sample pack, so these questions ARE published: they live on lucasrey.com and anyone may download them. Your students may already have seen them. That is the one way this pack differs from a commissioned one, and it is the whole reason a commissioned one exists: its questions are drawn fresh, recorded against your school, and issued to nobody else and published nowhere. Every question here is also now retired from that pool, so a pack you order cannot contain one of them.

Sitting on paper. My recommendation is still to run this on paper on 1 October 2026. Printed papers are more reliable than screens for a room of this size and they avoid every device and login problem on the day.

Notes for the exam room. The format, the syllabus coverage and the marking all follow the published ESAT specification: 27 questions, 40 minutes, one mark each, no penalty for a wrong answer, no calculator. A few things are worth knowing when you run it and when you read the marks.

- Run this on paper, then send the cohort to the official on-screen practice at least once before the window of 12 to 16 October 2026. The real ESAT is sat on screen at a Pearson test centre, so a printed mock trains the mathematics and the pacing while the official platform trains the interface. Doing both is what a full rehearsal looks like.
- Reading the scores: the option count varies question by question as it does on the real paper, 44 at 5 options, 34 at 6 options, 15 at 7 options, 42 at 8 options. A blind guess therefore scores about 0.16 here, against roughly 0.14 across the published ENGAA and NSAA questions and 0.20 on a paper that offered five throughout.
- Questions built around a diagram, by module: Mathematics 1 5, Physics 7, Chemistry 3, Biology 10. The published ENGAA and NSAA papers carry them in very different proportions by module: Biology 45%, Physics 28%, Mathematics 1 16%, Chemistry 9%, Mathematics 2 4%. This pack follows that shape per module rather than spreading diagrams evenly, which is part of what makes a paper feel like its own subject.
- Difficulty is set on what a question demands: the number of worked steps, the length of the working, and the time a fast solver needs, rather than on a label somebody typed. Each module carries a spread from one-step to five-step questions rather than a uniform level, because a paper with no answerable opening questions does not discriminate at the bottom of the range. One honest caveat: the official papers publish an answer key and no worked solutions, so that spread is matched to our own reference papers. Stem length and diagram share, above, are measured against the official papers themselves.
- How it was checked: every question was independently re-derived from its own stem, its key confirmed as uniquely correct, and each wrong option traced to a specific error that produces that exact value. The pack is then checked

mechanically against the specification. That is a question-by-question review rather than one read through end to end, and it is the stronger of the two.

Scope. Five unseen module papers with full worked solutions and mark schemes. Marking and cohort analysis are not included in this engagement. The mark scheme lets Mr B. Sample mark in house and see which topics the cohort is losing marks on. If you would like the marking and the analysis as a separate piece of work, tell me and I will quote.

Cost. Nothing. This pack is free to download and free to use with your own students. It is not for resale or republication.

If anything needs reissuing, a module renumbered or a question you would rather swap out, tell me and I will send a corrected version at no charge.

Yours sincerely,

Lucas Rey

EXAMPLE GRAMMAR SCHOOL

ESAT Mock: Invigilator Instructions

Sitting 1 October 2026

Paper SMP-2026-09 · Version 1.0

Materials for each candidate

- One question paper for the module being sat.
- One answer sheet for the same module.
- Black or blue pen. Candidates may write on the question paper.
- No calculator. No formula sheet. No notes.

Entry numbers by module

Module	Candidates
Mathematics 1	30
Mathematics 2	30
Physics	30
Chemistry	30
Biology	30

Print spare copies of each module. Numbers were provisional when the papers were prepared.

Timing

- 40 minutes per module, timed from the moment you tell candidates to begin.
- If a candidate sits more than one module, give a short break between them.
- Do not grant extra time to recover a late start unless the school's access arrangements say otherwise.

Announce at the start

- Write your name, teaching group and the date on both the question paper and the answer sheet.
- You have 40 minutes. There are 27 questions, each worth 1 mark.
- There is no negative marking. Answer every question, even if you have to guess.
- Only the answer sheet is marked. Anything written on the question paper is ignored.
- Calculators are not permitted.

Announce with 5 minutes remaining

Five minutes remaining. If any answer rows are still blank, fill them in now. There is no penalty for a wrong answer.

Latecomers

- Admit a candidate arriving within the first 10 minutes and let them finish at the same time as the rest of the room.
- After 10 minutes, seat them for a separate sitting rather than a shortened one. A truncated paper gives the department no usable information.

If a paper has a printing fault

- Swap in a spare copy and note the candidate's name and the time lost.
- Give that candidate the lost time back at the end.
- Do not read a missing question aloud, and do not let candidates share a paper.

Collecting in

- Collect answer sheets first, then question papers.
- Check that every answer sheet carries a name before the room is released.

If something is wrong with the paper itself

For example a question that appears to have no correct option: note the module and question number, let the sitting continue, and contact Lucas Rey afterwards at lucas.rey@eng.ox.ac.uk or +39 345 7976656. Do not correct a question in the room, because that changes the paper for one group only.

EXAMPLE GRAMMAR SCHOOL

ESAT Mock Examination

Mathematics 1

Paper SMP-2026-09 · Version 1.0

NAME

TEACHING GROUP

DATE

Instructions

- Time allowed: **15 minutes**.
- There are **10 multiple-choice questions**. Each is worth 1 mark.
- There is no negative marking. Answer every question, even if you have to guess.
- Calculators are not permitted.
- Record your answers on the separate answer sheet. Only the answer sheet is marked.
- Formulae are not provided. The ESAT does not supply a formula sheet.

Note: the real ESAT is sat on screen at a Pearson test centre. This paper trains the mathematics and the pacing, not the on-screen interface.

Written for Example Grammar School by Lucas Rey, Engineering Science, University of Oxford; MRes, University of Cambridge.

Question 1

A winch unwinds 0.75 km of uniform steel cable of linear density 12 g cm^{-1} . Find its mass.

- A. 900 kg
- B. 90 kg
- C. 625 kg
- D. 9 kg
- E. 90000 kg
- F. 9000 kg
- G. 0.9 kg
- H. 0.09 kg

Question 2

A ski tow climbs a uniform slope in a straight line. The cable rises 1.5 m for every 2 m of horizontal distance. Find the height gained along 20 m of cable.

- A. 15 m
- B. 12 m
- C. 6 m
- D. 16 m
- E. 8 m

Question 3

240 g of alloy occupies 30 cm^3 , each to the nearest ten. Find the greatest density.

- A. 9.4 g cm^{-3}
- B. 9.8 g cm^{-3}
- C. 8 g cm^{-3}
- D. 10 g cm^{-3}
- E. 7 g cm^{-3}
- F. 9.6 g cm^{-3}
- G. 12 g cm^{-3}
- H. 12.5 g cm^{-3}

Question 4

A start-up's cash balance C , in thousands of pounds, at the end of month x is modelled by $C = (x - 1)(x - 5)$, valid for months 0 to 6. Treating x as continuous, what is the lowest balance predicted?

- A. $-\$3000$
- B. $\$0$
- C. $-\$9000$
- D. $\$5000$
- E. $-\$4000$
- F. $\$1000$
- G. $-\$14000$
- H. $\$3000$

Question 5

Decrease 80 by 15%.

- A. 92
- B. 68
- C. 65
- D. 94.12
- E. 12

Question 6

A park has a lawn shaped like a sector of a circle. Two straight paths of equal length run from a single corner to the ends of a curved hedge, which forms the lawn's outer boundary. The hedge measures 12π m and the lawn covers $90\pi \text{ m}^2$. Find the angle between the two paths.

- A. 288°
- B. 72°
- C. 15°
- D. 48°
- E. 40°
- F. 36°
- G. 144°
- H. 216°

Question 7

A camera's light meter plots the correct exposure time t , in milliseconds, against the aperture area A , in mm^2 . The plotted curve is a hyperbola in the first quadrant, falling steeply and approaching both axes without ever meeting them. One recorded setting has $A = 8$ and $t = 45$. What exposure time does the curve give when $A = 12$?

- A. 41 ms
- B. 22.5 ms
- C. 20 ms
- D. 101.25 ms
- E. 90 ms
- F. 3.75 ms
- G. 67.5 ms
- H. 30 ms

Question 8

A battery is $\frac{3}{4}$ charged. Using 0.35 of its capacity leaves what percentage?

- A. 40%
- B. 110%
- C. 48.75%
- D. 60%
- E. 74.65%
- F. 71.5%
- G. 65%
- H. 26.25%

Question 9

A vineyard's yield falls by 20% each year while a disease goes untreated. Three harvests after it appeared the yield is 2560 kg. How large was the last healthy harvest?

- A. 6400 kg
- B. 4480 kg
- C. 3200 kg
- D. 4096 kg
- E. 5120 kg
- F. 4000 kg
- G. 6250 kg
- H. 5000 kg

Question 10

A courier walks 4 blocks east and 2 north. How many routes are possible?

- A. 8
- B. 360
- C. 30
- D. 6
- E. 64
- F. 15
- G. 720
- H. 10

END OF PAPER

Questions 11 to 27 are in the full paper

This is an extract. The 10 questions before this page are real questions from a commissioned question paper, shown in full and with nothing withheld, so that the standard can be judged rather than described.

The rest of the paper is not hidden behind this page. It was never in this file. Each commissioned paper is written for one school, recorded against it, and issued to no other, so the questions in it are not published anywhere, including here.

A commissioned paper is NOT this paper with the missing questions restored. Every question in it is different from every question here, the 10 shown above included. Your school's paper is drawn fresh, and no question that has ever been published, on this page or anywhere else, can appear in it.

A full paper runs to 27 questions in this module alone, with a question paper, an answer sheet, a mark scheme and a worked solution for every question, across all five ESAT modules.

Commission a paper for your school

lucasrey.com/teaching/for-schools/

Written by researchers in engineering, physics and mathematics at Oxford and Cambridge, and checked question by question before it is issued.

This page is in the full paper.

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EXAMPLE GRAMMAR SCHOOL

ESAT Mock: Answer Sheet

Mathematics 1

Paper SMP-2026-09 • Version 1.0

NAME

TEACHING GROUP

DATE

Fill in one bubble per question. If you change your mind, cross out the old mark clearly and fill the new bubble. Unfilled or doubly filled rows score zero.

Q	A	B	C	D	E	F	G	H
1	☐	☐	☐	☐	☐	☐	☐	☐
2	☐	☐	☐	☐	☐			
3	☐	☐	☐	☐	☐	☐	☐	☐
4	☐	☐	☐	☐	☐	☐	☐	☐
5	☐	☐	☐	☐	☐			
6	☐	☐	☐	☐	☐	☐	☐	☐
7	☐	☐	☐	☐	☐	☐	☐	☐
8	☐	☐	☐	☐	☐	☐	☐	☐
9	☐	☐	☐	☐	☐	☐	☐	☐
10	☐	☐	☐	☐	☐	☐	☐	☐

MARKER INITIALS

TOTAL OUT OF 10

EXAMPLE GRAMMAR SCHOOL

ESAT Mock: Mark Scheme

Mathematics 1

Paper SMP-2026-09 • Version 1.0

Answer grid

Q	Ans	Q	Ans	Q	Ans
1	A	10	F		
2	B				
3	B				
4	E				
5	B				
6	G				
7	H				
8	A				
9	H				

Totals

10 marks available, 1 mark per question. There is no negative marking, so a wrong answer scores zero and never a negative. Only the answer sheet is marked.

Diagnostic table

Use this to see which topics the cohort is losing marks on.

Q	Ans	Node	Group	Topic	Difficulty
1	A	M1.1	M1	Units and compound measures	★★☆☆☆
2	B	M5.18	M5	Geometry	★★★☆☆
3	B	M2.12	M2	Number	★★★☆☆
4	E	M4.11	M4	Algebra	★★★☆☆
5	B	M3.8	M3	Ratio and proportion	★★☆☆☆
6	G	M5.16	M5	Geometry	★★★☆☆
7	H	M4.12	M4	Algebra	★★★☆☆
8	A	M2.10	M2	Number	★★☆☆☆
9	H	M3.11	M3	Ratio and proportion	★★★☆☆
10	F	M2.5	M2	Number	★★★☆☆

Marks available by topic group

Group	Topic	Marks
M1	Units and compound measures	1
M2	Number	3
M3	Ratio and proportion	2
M4	Algebra	2
M5	Geometry	2
Total		10

Worked solutions for every question are in the separate worked solutions document.

EXAMPLE GRAMMAR SCHOOL

ESAT Mock: Worked Solutions

Mathematics 1

Paper SMP-2026-09 • Version 1.0

Staff copy

This document contains the answers. Keep it away from candidates until the paper has been sat. Marking guidance is in the separate mark scheme.

Question 1

M1.1 • Units and compound measures • ★★☆☆☆ • 85 s

A winch unwinds 0.75 km of uniform steel cable of linear density 12 g cm^{-1} . Find its mass.

- A. 900 kg
- B. 90 kg
- C. 625 kg
- D. 9 kg
- E. 90000 kg
- F. 9000 kg
- G. 0.9 kg
- H. 0.09 kg

Answer A. Key idea

A compound unit is a recipe for a division, and every unit inside it has to be rescaled separately before the number in front of it means anything. A linear density written in grams per centimetre carries a mass unit on the top and a length unit on the bottom, so moving it to kilograms per metre multiplies by one factor and divides by another. Once the rate is expressed in units that match the length being measured, the mass is a single multiplication, because mass equals rate multiplied by length.

Worked solution

1. Rewrite the linear density in kg m^{-1}

A linear density of 12 g cm^{-1} says that every centimetre of cable weighs 12 g. A metre contains 100 of those centimetres, so one metre of cable weighs

$$12 \times 100 = 1200 \text{ g}$$

and $1200 \text{ g} = 1.2 \text{ kg}$, so the cable is 1.2 kg m^{-1} .

Both unit changes are doing work here and they pull in opposite directions: the length unit contributes a factor of 100 on the bottom, the mass unit a factor of $\frac{1}{1000}$ on the top, and the survivor is a net division by 10. That is why 12 becomes 1.2 and not 1200 or 0.012.

2. Convert the length and multiply

$0.75 \text{ km} = 0.75 \times 1000 = 750 \text{ m}$, and the rate is now per metre, so the two agree:

$$m = 1.2 \times 750 = 750 + 150 = 900 \text{ kg}$$

Check: run it backwards. 900 kg spread over 750 m is 1.2 kg per metre, which is 1200 g per metre, which is 12 g per centimetre, exactly the figure given. A three quarter kilometre of heavy steel cable weighing close to a tonne is also the right order of magnitude for a winch drum.

Matches Option A.

Why the other options are wrong

- **B** (90 kg): Took 0.75 km as 75 m, using a factor of 100 for kilometres to metres instead of 1000: $1.2 \times 75 = 90$. A kilometre is a thousand metres, and the factor of 100 belongs to centimetres. (*Wrong power of ten for km*)
- **C** (625 kg): Reached 1.2 kg m^{-1} and converted the length correctly, then divided by the rate instead of multiplying by it: $750 \div 1.2 = 7500 \div 12 = 625$. A rate in kilograms per metre multiplies a number of metres; dividing by it answers how many metres go with one kilogram, which is not what was asked. (*Rate inverted, divided instead of multiplied*)
- **D** (9 kg): Converted the mass but not the length: $12 \text{ g} = 0.012 \text{ kg}$, then $0.012 \times 750 = 9$. That multiplies a per centimetre rate by a number of metres, so the answer is short by the factor of 100 that the length unit carries. (*Half a compound unit converted*)
- **E** (90000 kg): Knew that a single factor of ten survives between g cm^{-1} and kg m^{-1} but multiplied by it instead of dividing, reading 12 g cm^{-1} as 120 kg m^{-1} , then $120 \times 750 = 90000$. Dividing by 1000 for the mass outweighs multiplying by 100 for the length, so the net move is a division and the rate is 1.2, not 120. (*Net factor applied the wrong way*)
- **F** (9000 kg): Read the numeral 12 as though it were already 12 kg m^{-1} and multiplied: $12 \times 750 = 9000$. Neither the gram nor the centimetre was rescaled, so the figure is ten times the true rate. (*No conversion at all*)
- **G** (0.9 kg): Reached 1.2 kg m^{-1} correctly but multiplied by 0.75 instead of 750, leaving the length in kilometres: $1.2 \times 0.75 = 0.9$. The rate is per metre, so the length must be in metres. (*Length unit not converted*)
- **H** (0.09 kg): Divided for both unit changes: $12 \div 1000 = 0.012 \text{ kg cm}^{-1}$, then divided again by 100 for centimetres to metres to give $0.00012 \text{ kg m}^{-1}$, and $0.00012 \times 750 = 0.09$. The length unit sits on the bottom of the rate, so going from a per centimetre rate to a per metre rate multiplies by 100. (*Both conversions taken as divisions*)

Common mistake. Rescaling the mass unit and leaving the length unit alone, or the other way round. Turning 12 g cm^{-1} into 0.012 kg cm^{-1} and then multiplying straight by 750 treats a per centimetre rate as a per metre rate, and the answer comes out a hundred times too small. Both halves of a compound unit have to be converted before the number is used.

Takeaway. The Cheat Code: $1 \text{ g cm}^{-1} = 0.1 \text{ kg m}^{-1}$, because dividing by 1000 for the mass and multiplying by 100 for the length leaves a single division by 10. Fix the rate first, fix the length second, then multiply once.

Time-saving tip. Speed Heuristic: do not chase the powers of ten one at a time. Grams to kilograms is $\div 1000$ and centimetres to metres is $\times 100$, a net $\div 10$, so 12 g cm^{-1} is 1.2 kg m^{-1} on sight. Then 1.2×750 is 750 plus a fifth of 750, that is $750 + 150 = 900$.

Question 2

M5.18 · Geometry · ★★☆☆☆ · 90 s

A ski tow climbs a uniform slope in a straight line. The cable rises 1.5 m for every 2 m of horizontal distance. Find the height gained along 20 m of cable.

- A. 15 m
- B. 12 m
- C. 6 m
- D. 16 m
- E. 8 m

Answer B. Key idea

For a fixed angle the three ratios sin, cos and tan are fixed numbers, so a slope quoted as a vertical rise per horizontal run hands you tan directly. A distance measured along the slope itself is the hypotenuse, not the adjacent side, so the ratio that connects it to the height is sin. Once the two given lengths are reduced to the 3, 4, 5 side lengths, both ratios are available and the whole conversion is exact arithmetic.

Worked solution**Step 1: Build the right-angled triangle**

The cable, the horizontal ground and the vertical rise form a right-angled triangle. Taking θ as the angle of climb at the bottom of the slope, the rise is the side opposite θ and the horizontal distance is the side adjacent to θ , so opposite = 1.5 m and adjacent = 2 m.

Step 2: Read off the ratio the slope statement gives

$$\text{TOA: } \tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{1.5}{2} = \frac{3}{4}.$$

Step 3: Switch to the ratio that matches the given length

The 20 m is measured along the cable, which is the hypotenuse, so the height needs sin, not tan. From opposite : adjacent = 3 : 4 the hypotenuse is $\sqrt{3^2 + 4^2} = 5$, so the sides are in the ratio 3 : 4 : 5 and $\sin \theta = \frac{3}{5}$.

Step 4: Apply sin to the cable length

$$\text{SOH: height} = 20 \times \frac{3}{5} = 12 \text{ m.}$$

Sanity check: the same 20 m of cable covers $20 \times \frac{4}{5} = 16 \text{ m}$ horizontally, and $\frac{12}{16} = \frac{3}{4}$, which is the $\tan \theta$ we started from. The height is also comfortably less than the cable length, as it must be.

Matches Option B.

Why the other options are wrong

- **A (15 m):** Uses tan on the cable length: $20 \times \frac{3}{4} = 15 \text{ m}$. That is the height only if the 20 m were the horizontal distance, but it is measured along the slope, so it is the hypotenuse. (*Conceptual Error*)
- **C (6 m):** Mixes the ratio numbers with the measured lengths: takes the hypotenuse as 5 from the 3 : 4 : 5 ratio while keeping the rise as the raw 1.5 m, giving $\sin \theta = \frac{1.5}{5} = 0.3$ and $20 \times 0.3 = 6 \text{ m}$. The ratio must be built from two lengths of the same triangle. (*Scaling Error*)
- **D (16 m):** Picks CAH instead of SOH: $20 \times \frac{4}{5} = 16 \text{ m}$. That is the horizontal distance the tow covers, not the height it gains. (*Conceptual Error*)

- **E** (8 m): Stops at the scale factor. The small triangle has hypotenuse 2.5 m, so $20 \div 2.5 = 8$, and the 8 is reported as the answer instead of being used to scale the rise: $8 \times 1.5 = 12$ m. (*Incomplete Solution*)

Common mistake. Treating the 20 m of cable as a horizontal distance and multiplying by $\tan \theta$, giving $20 \times \frac{3}{4} = 15$ m. The 20 m runs along the slope, so it is the hypotenuse and $\sin$ is the ratio that applies.

Takeaway. The Cheat Code: Before choosing a ratio, decide which side of the triangle the given number actually is. A rise per horizontal run fixes $\tan$, but any distance measured along the slope is a hypotenuse, so the ratio you apply to it is $\sin$.

Time-saving tip. Speed Heuristic: 1.5 : 2 scales to 3 : 4, so this is a 3, 4, 5 triangle. For any length measured along the slope, divide by 5 and multiply by 3: $20 \div 5 = 4$, then $4 \times 3 = 12$, with no fractions written down.

Question 3

M2.12 · Number · ★★☆☆ · 90 s

240 g of alloy occupies 30 cm^3 , each to the nearest ten. Find the greatest density.

- A. 9.4 g cm^{-3}
- B. 9.8 g cm^{-3}
- C. 8 g cm^{-3}
- D. 10 g cm^{-3}
- E. 7 g cm^{-3}
- F. 9.6 g cm^{-3}
- G. 12 g cm^{-3}
- H. 12.5 g cm^{-3}

Answer B. Key idea

A figure rounded to the nearest ten is only guaranteed to lie within half of that ten, so each measurement becomes an interval five units wide on either side of the stated value. Density is a quotient, and a quotient is at its largest when the numerator is at its largest and the denominator at its smallest, so the two measurements must be pushed to opposite ends of their intervals rather than to the same end.

Worked solution

1. Turn each rounding into an interval

A value given to the nearest ten carries half of ten, that is 5, of slack on either side. Writing m for the mass and V for the volume,

$$235 \leq m \leq 245 \text{ and } 25 \leq V \leq 35.$$

The two intervals are the same width in absolute terms, but the volume interval is far wider as a proportion of its measurement, which is why the volume controls the answer.

2. Choose the ends that maximise a quotient

Density is $\rho = \frac{m}{V}$. A quotient grows when the numerator grows and when the denominator shrinks, so the greatest possible density pairs the heaviest allowed mass with the smallest allowed volume:

$$\rho_{\max} = \frac{245}{25}.$$

3. Evaluate and check

$$\frac{245}{25} = \frac{490}{50} = \frac{49}{5} = 9.8 \text{ g cm}^{-3}.$$

Sanity check: the stated figures give $240 \div 30 = 8 \text{ g cm}^{-3}$, and the smallest allowed density is $235 \div 35 = 6.71 \dots \text{ g cm}^{-3}$. An upper bound has to sit above the nominal value and it does, since $9.8 > 8$.

Matches Option **B**.

Why the other options are wrong

- **A** (9.4 g cm^{-3}): Paired the two lower bounds: $235 \div 25 = 9.4$. The numerator has to be at its upper bound for the quotient to be greatest. (*Both lower bounds used*)
- **C** (8 g cm^{-3}): Divided the stated figures and ignored the rounding altogether: $240 \div 30 = 8$. That is the nominal density, which sits inside the interval rather than at its top. (*Bounds ignored*)
- **D** (10 g cm^{-3}): Gave the mass the full ten grams of slack instead of half of it: $250 \div 25 = 10$. The nearest ten guarantees only 5 either way, so 250 is outside the possible range. (*Whole quantum instead of half*)
- **E** (7 g cm^{-3}): Pushed both measurements to the top of their intervals, as though the greatest density meant the largest number everywhere: $245 \div 35 = 7$. The denominator has to be at its smallest for a quotient to be greatest, so the volume belongs at 25, not 35. (*Both upper bounds used*)
- **F** (9.6 g cm^{-3}): Took the volume down to its lower bound but left the mass at the figure printed in the stem: $240 \div 25 = 9.6$. The mass is rounded to the nearest ten as well, so it carries the same five grams of slack and must go up to 245. (*Only the denominator moved*)
- **G** (12 g cm^{-3}): Gave the volume the full ten of slack while leaving the mass alone: $240 \div 20 = 12$. Rounding to the nearest ten guarantees only five either way, so the volume cannot fall below 25, and the mass is free to rise to 245. (*Whole quantum on the volume, mass unmoved*)
- **H** (12.5 g cm^{-3}): Used the full ten of slack on both measurements: $250 \div 20 = 12.5$. Both intervals are twice as wide as the rounding allows. (*Whole quantum instead of half*)

Common mistake. Pushing both measurements to the same end of their intervals, as though "largest" meant taking the largest number everywhere. For a quotient the two ends must be opposite: 245 over 25, not 245 over 35 and not 235 over 25.

Takeaway. The Cheat Code: For the maximum of a quotient take the top of the numerator's interval over the bottom of the denominator's, and get each interval from half of the stated rounding unit, so nearest ten means five either way.

Time-saving tip. Speed Heuristic: Only one pairing needs evaluating, so write $245 \div 25$ straight down and double both parts to $490 \div 50 = 9.8$. There is no need to work out the nominal or minimum density first.

Question 4

M4.11 · Algebra · ★★☆☆ · 90 s

A start-up's cash balance C , in thousands of pounds, at the end of month x is modelled by $C = (x - 1)(x - 5)$, valid for months 0 to 6. Treating x as continuous, what is the lowest balance predicted?

- A. $-\$3000$
- B. $\$0$
- C. $-\$9000$
- D. $\$5000$
- E. $-\$4000$
- F. $\$1000$
- G. $-\$14000$
- H. $\$3000$

Answer E. Key idea

A quadratic with a positive squared term has a single minimum, and completing the square puts it in the form $(x - p)^2 + q$, where the bracket cannot be negative, so the least value is q and it occurs at $x = p$. The brackets of a factorised quadratic give the values where the curve is zero, not where it is least; the minimum sits exactly midway between them because the curve is symmetric about its turning point.

Worked solution**1. Put the model in standard form**

$$C = (x - 1)(x - 5) = x^2 - 6x + 5$$

2. Complete the square

$$x^2 - 6x + 5 = (x - 3)^2 - 9 + 5 = (x - 3)^2 - 4$$

3. Read off the turning point

$(x - 3)^2$ is never negative and is zero only at $x = 3$, so the least value of C is -4 , reached in month 3. The same x comes from $-\frac{b}{2a} = \frac{6}{2} = 3$, and substituting back gives $9 - 18 + 5 = -4$.

4. Check the range, then convert the units

Month 3 lies inside the valid span of months 0 to 6, so the model is being used where it applies. Either side, $C(2) = (1)(-3) = -3$ and $C(4) = (3)(-1) = -3$, both above -4 , which confirms $x = 3$ as the trough. Since C is in thousands of pounds, the lowest balance is $-\$4000$.

Matches Option E.

Why the other options are wrong

- **A** ($-\$3000$): The midpoint of the roots was found as half their difference, $(5 - 1) \div 2 = 2$, rather than half their sum. Then $C(2) = (2 - 1)(2 - 5) = -3$, giving $-\$3000$, which is a real balance on the curve but not the least one. (*Midpoint from the difference*)
- **B** ($\$0$): Read the factorised form as handing the answer over: $C = 0$ at $x = 1$ and at $x = 5$, and the balance there was quoted as the lowest. Those are the two months the balance passes through zero, and the curve dips below zero between them, reaching $(3 - 1)(3 - 5) = -4$ thousand in month 3. (*Balance at a root taken as the minimum*)
- **C** ($-\$9000$): The square was completed to $(x - 3)^2 - 9 + 5$ and the working stopped at the -9 , so the $+5$ from the original constant was never carried through. That reports $-\$9000$ instead of -4 thousand. (*Constant not carried through*)
- **D** ($\$5000$): The opening balance was taken as the lowest point: $C(0) = (0 - 1)(0 - 5) = 5$, that is $\$5000$. The curve falls after month 0, so its starting value is the highest point in the first half, not the lowest. (*Endpoint taken as minimum*)
- **F** ($\$1000$): The brackets were expanded as $x^2 - 4x + 5$ by subtracting the roots instead of adding them. That puts the turning point at $x = 2$ with value $4 - 8 + 5 = 1$, giving $\$1000$. Expanding correctly gives $-6x$, since $-1 - 5 = -6$. (*Expansion sign slip*)
- **G** ($-\$14000$): Expanded $(x - 1)(x - 5)$ as $x^2 - 6x - 5$, taking $(-1) \times (-5)$ as -5 . Completing the square on that gives $(x - 3)^2 - 9 - 5 = (x - 3)^2 - 14$, so the minimum reads as -14 thousand. Two negatives multiply to $+5$, and $-9 + 5 = -4$. (*Sign slip on the product of the constants*)

- **H (\$3000):** Located the turning point correctly, since $(x - 3)^2 - 4$ is least at $x = 3$, then reported the 3 instead of the value there. The 3 is the month the trough falls in; substituting it gives $C = (3 - 1)(3 - 5) = -4$, that is $-\$4000$. (*Month of the minimum reported as the balance*)

Common mistake. Quoting a root, or the balance at a root, as the lowest point. The factors give $x = 1$ and $x = 5$, where the balance is zero, and zero is higher than the trough between them. Roots answer where the curve crosses, never how low it goes.

Takeaway. The Cheat Code: For a factorised quadratic the turning point sits midway between the roots, so average them and substitute once. Completing the square says the same thing: the bracket vanishes at the midpoint and the number left outside is the minimum.

Time-saving tip. Speed Heuristic: average the roots, $(1 + 5) \div 2 = 3$, then feed that straight back into the brackets already given: $(3 - 1)(3 - 5) = 2 \times (-2) = -4$. No expansion, no formula and no completing the square are needed.

Question 5

M3.8 · Ratio and proportion · ★★☆☆☆ · 45 s

Decrease 80 by 15%.

- A. 92
- B. 68
- C. 65
- D. 94.12
- E. 12

Answer B. Key idea

Percentage increases and decreases are handled fastest with a single decimal multiplier. A 15% decrease leaves 85% of the original, so the multiplier is 0.85 and $80 \times 0.85 = 68$.

Worked solution

1. Choose the multiplier

A decrease of 15% leaves $100\% - 15\% = 85\%$ of the original amount, so the multiplier is 0.85.

2. Apply it in one step

$$80 \times 0.85 = 68$$

Doing it in two steps (15% of 80 = 12, then $80 - 12 = 68$) gives the same value, but the single multiplier is quicker and leaves nothing to forget.

3. Sense-check the direction

The question asks for a decrease, so the answer must be smaller than 80. That rules out 92 and 94.12 before any arithmetic is checked.

4. Why each of the other options fails

- **A 92:** applied the change the wrong way, 80×1.15 , giving an increase.
- **C 65:** subtracted 15 as a raw number rather than 15% of 80, which is 12.
- **D 94.12:** worked backwards, $80 \div 0.85$, as if 80 were the value after the decrease.
- **E 12:** found 15% of 80 but never took it away from the original.

Matches Option B.

Why the other options are wrong

- **A** (92): Applied the opposite change. (*Conceptual Error*)
- **C** (65): Added/subtracted the percentage as a raw number. (*Conceptual Error*)
- **D** (94.12): Calculated original value before change (reverse percentage). (*Conceptual Error*)
- **E** (12): Calculated the percentage, but didn't apply the change. (*Conceptual Error*)

Common mistake. Forgetting to add/subtract from the original.

Takeaway. The Cheat Code: Do not calculate the percentage and then add/subtract it in two separate steps. Use a single multiplier (e.g., $\text{Original} \times 1.15 = \text{New}$). For reverse percentages, NEVER subtract the percentage from the new amount; instead, divide by the multiplier ($\text{New} / 1.15 = \text{Original}$).

Time-saving tip. Decide first whether the quantity scales up or down, then check your answer against that expectation before you look at the options.

Question 6

M5.16 · Geometry · ★★☆☆ · 90 s

A park has a lawn shaped like a sector of a circle. Two straight paths of equal length run from a single corner to the ends of a curved hedge, which forms the lawn's outer boundary. The hedge measures 12π m and the lawn covers 90π m². Find the angle between the two paths.

- A. 288°
- B. 72°
- C. 15°
- D. 48°
- E. 40°
- F. 36°
- G. 144°
- H. 216°

Answer G. Key idea

A sector is the same fraction of its circle however you measure it: its arc is that fraction of the circumference and its area is that fraction of the whole disc. Eliminating the angle between those two statements gives the compact relation $A = \frac{1}{2}r\ell$, which converts an area and an arc length straight into the radius with no π and no angle involved. Once the radius is known, the angle follows by comparing the arc with the full circumference.

Worked solution

1. Link the area to the curved edge

Write r for the radius, which here is the length of each straight path, and ℓ for the arc, which is the hedge. Since the sector is the fraction $\frac{\theta}{360}$ of the disc,

$$A = \frac{\theta}{360}\pi r^2 = \frac{1}{2}r\left(\frac{\theta}{360} \times 2\pi r\right) = \frac{1}{2}r\ell$$

so area, radius and arc are tied together without the angle appearing.

2. Solve for the radius

$$90\pi = \frac{1}{2} \times r \times 12\pi \Rightarrow 90\pi = 6\pi r \Rightarrow r = 15 \text{ m}$$

Each straight path is 15 m long.

3. Compare the hedge with the whole circumference

A full circle of radius 15 m has circumference $2\pi(15) = 30\pi$ m, so the hedge covers

$$\frac{12\pi}{30\pi} = \frac{2}{5}$$

of the way round.

4. Turn the fraction into an angle, then check

$$\theta = \frac{2}{5} \times 360^\circ = 144^\circ$$

Check against the area, which has not yet been used twice: two fifths of the disc is $\frac{2}{5} \times \pi(15)^2 = \frac{2}{5} \times 225\pi = 90\pi \text{ m}^2$, exactly the area stated. The angle is obtuse but well under a half turn, which suits a lawn wider than a quarter circle and narrower than a semicircle.

Matches Option G.

Why the other options are wrong

- **A** (288°): Dropped the $\frac{1}{2}$ from $A = \frac{1}{2}r\ell$, giving $r = \frac{90\pi}{12\pi} = 7.5$. Then $\frac{\theta}{360} \times 2\pi(7.5) = 12\pi$ gives $\frac{\theta}{360} = \frac{12}{15}$ and $\theta = 288^\circ$. (*Missing factor of one half*)
- **B** (72°): Found $r = 15$ m correctly from $A = \frac{1}{2}r\ell$, then wrote the area of the whole disc as $2\pi r^2 = 2\pi(225) = 450\pi$ instead of $\pi r^2 = 225\pi$. The sector then looks like $\frac{90\pi}{450\pi} = \frac{1}{5}$ of the circle, giving $\theta = \frac{1}{5} \times 360^\circ = 72^\circ$. The $2\pi r$ belongs to the circumference; the area carries no factor of 2. (*Disc area written as $2\pi r^2$*)
- **C** (15°): Stopped at the intermediate value: $A = \frac{1}{2}r\ell$ gives $r = \frac{2 \times 90\pi}{12\pi} = 15$, and 15 was carried straight into the answer. That 15 is the length of each path in metres, not an angle in degrees; the angle still has to be taken from the arc, $\frac{12\pi}{2\pi(15)} \times 360^\circ = 144^\circ$. (*Stopped at the radius*)
- **D** (48°): Divided the arc by the area directly, $\frac{12\pi}{90\pi} = \frac{2}{15}$, and read that as the fraction of a full turn: $\frac{2}{15} \times 360^\circ = 48^\circ$. A length divided by an area is not a fraction of anything. (*Compared a length with an area*)
- **E** (40°): Reached the correct fraction of a turn, $\frac{12\pi}{30\pi} = \frac{2}{5}$, then wrote it as 40 per cent and read 40 off as the angle. A fraction of a circle becomes an angle by multiplying by 360° , so $\frac{2}{5}$ is 144° , whereas 40° is only $\frac{1}{9}$ of a turn and would leave an area of $\frac{1}{9} \times 225\pi = 25\pi \text{ m}^2$. (*Fraction of a turn read as a percentage*)
- **F** (36°): Found $r = 15$ m correctly but then put the diameter into the area formula: $\frac{\theta}{360} \pi(30)^2 = 90\pi$ gives $\frac{\theta}{360} = \frac{90}{900} = \frac{1}{10}$, so $\theta = 36^\circ$. The area of a circle uses the radius, not the diameter. (*Diameter used for radius*)
- **H** (216°): Reached 144° and then gave the rest of the circle, $360^\circ - 144^\circ = 216^\circ$. That reflex angle is the one outside the lawn, on the far side of the two paths. (*Gave the angle outside the sector*)

Common mistake. Dividing the two given quantities into one another, $\frac{12\pi}{90\pi}$, as though the ratio of an arc to an area were the fraction of a turn. An arc is a length and a sector is an area, so the two are only comparable once the radius has been extracted from them, and the fraction of a turn must be measured against a quantity of the same kind.

Takeaway. The Cheat Code: $A = \frac{1}{2}r\ell$ converts an arc and an area into the radius in one line, with no π to cancel and no angle to carry. With r in hand, the angle is just the arc written as a fraction of $2\pi r$, multiplied by 360° .

Time-saving tip. Speed Heuristic: Use the area once and then abandon it. $90\pi = \frac{1}{2}r(12\pi)$ gives $r = 15$ immediately, and from there the hedge is $\frac{12\pi}{30\pi} = \frac{2}{5}$ of the circumference, so the angle is two fifths of 360° .

Question 7

M4.12 · Algebra · ★★☆☆ · 90 s

A camera's light meter plots the correct exposure time t , in milliseconds, against the aperture area A , in mm^2 . The plotted curve is a hyperbola in the first quadrant, falling steeply and approaching both axes without ever meeting them. One recorded setting has $A = 8$ and $t = 45$. What exposure time does the curve give when $A = 12$?

- A. 41 ms
- B. 22.5 ms
- C. 20 ms
- D. 101.25 ms
- E. 90 ms
- F. 3.75 ms
- G. 67.5 ms
- H. 30 ms

Answer H. Key idea

A curve that lies in the first quadrant and has both axes as asymptotes is the graph of the reciprocal function, $t = \frac{k}{A}$. Its defining property is that the product of the two coordinates is the same at every point, so one recorded pair fixes k and every other point then follows by a single division. Because k is not zero, neither coordinate can ever reach zero, which is exactly why the curve closes on the axes without touching them.

Worked solution

1. Name the curve from its description

A hyperbola in the first quadrant with both axes as asymptotes is the reciprocal graph, so the meter is using

$$t = \frac{k}{A}, \quad \text{that is} \quad tA = k$$

2. Fix the constant from the recorded setting

$$k = 8 \times 45 = 360$$

3. Substitute the new area

$$t = \frac{360}{12} = 30 \text{ ms}$$

4. Check it against the shape of the curve

The area rose, and the curve falls, so the time must come out below 45 ms, and 30 does. The product test also passes: $12 \times 30 = 360$, the same as 8×45 . Finally the value is positive and finite, as it must be for a curve that never reaches either axis.

Matches Option **H**.

Why the other options are wrong

- **A** (41 ms): The sum was held constant instead of the product: $8 + 45 = 53$, then $53 - 12 = 41$. A constant sum is a straight line of gradient -1 , which would cut the A axis at 53 rather than run alongside it. (*Constant sum instead of product*)
- **B** (22.5 ms): The area rose by half, from 8 to 12, so the time was cut by half: $45 \times 0.5 = 22.5$. Under $tA = k$ the time is scaled by the ratio of the areas, $\frac{8}{12} = \frac{2}{3}$, which is a cut of a third and not a half. The product test also fails, since $12 \times 22.5 = 270$ rather than 360. (*Percentage change mirrored*)
- **C** (20 ms): An inverse square rule was used instead of the reciprocal: $45 \times \left(\frac{8}{12}\right)^2 = 45 \times \frac{4}{9} = 20$. That curve does approach both axes, but it is not a hyperbola and it fails the constant product test, since $12 \times 20 = 240$. (*Inverse square used*)
- **D** (101.25 ms): The area ratio was squared and applied the way the area grows: $45 \times \left(\frac{12}{8}\right)^2 = 45 \times 2.25 = 101.25$. That is the curve $t \propto A^2$, which climbs away from both axes rather than closing on them, and it fails the constant product test since $12 \times 101.25 = 1215$. (*Direct square proportion*)
- **E** (90 ms): The constant was found correctly, $k = 8 \times 45 = 360$, but then divided by the increase in area instead of the area itself: $\frac{360}{12-8} = 90$. The curve gives the time at $A = 12$, so the division is $\frac{360}{12} = 30$, and a wider aperture cannot need a longer exposure than the 45 ms already recorded. (*Divided by the change in area*)
- **F** (3.75 ms): The recorded time was divided by the new area, $45 \div 12 = 3.75$. The division must be applied to the constant 360, not to a time; dividing a time by an area does not even leave a time. (*Divided the wrong quantity*)
- **G** (67.5 ms): The time was scaled up with the area, $45 \times \frac{12}{8} = 67.5$, which is direct proportion. That is a straight line through the origin, and it contradicts a curve that falls as the area grows. (*Direct proportion assumed*)

Common mistake. Treating a falling curve as a falling straight line. A straight line through the recorded point would cross the A axis at some finite area, which would mean a real setting needing no exposure time at all. The stated asymptotes rule that out and force the reciprocal model.

Takeaway. The Cheat Code: Asymptotes on both axes means the product of the coordinates is constant. Multiply the pair you are given, divide by the new value, and you are done in two operations.

Time-saving tip. Speed Heuristic: skip the constant. The area is multiplied by $\frac{12}{8} = \frac{3}{2}$, so on a reciprocal curve the time is multiplied by $\frac{2}{3}$, and two thirds of 45 is 30.

Question 8

M2.10 • Number • ★★☆☆☆ • 90 s

A battery is $\frac{3}{4}$ charged. Using 0.35 of its capacity leaves what percentage?

- A. 40%
- B. 110%
- C. 48.75%
- D. 60%
- E. 74.65%
- F. 71.5%
- G. 65%
- H. 26.25%

Answer A. Key idea

A fraction, a decimal and a percentage are three notations for the same kind of number, and they can only be combined once they are all written against the same reference quantity. Here both the charge held and the amount used are measured against the battery's full capacity, so converting each to a percentage of that capacity makes the calculation a single subtraction rather than a mixture of unlike quantities.

Worked solution**1. Write the starting charge as a percentage**

A percentage is a number of parts per hundred, so multiply the fraction by 100:

$$\frac{3}{4} = \frac{3}{4} \times 100\% = 75\%.$$

The quarter is 25%, and three of them give 75%, so the battery starts at 75% of capacity.

2. Write the amount used as a percentage of the same thing

$$0.35 = \frac{35}{100} = 35\%.$$

This is stated as a share of the capacity, the same reference as the 75%, so the two figures can now be compared directly. Had it been a share of the charge present it would have had to be multiplied by the 75% instead.

3. Subtract and check

$$75\% - 35\% = 40\%.$$

Sanity check in decimals: $0.75 - 0.35 = 0.40$, which is 40% of capacity. The answer is below the starting 75% and above zero, as it must be, and the battery is left a little under half charged.

Matches Option A.

Why the other options are wrong

- **B** (110%): Added the amount used instead of subtracting it: $75\% + 35\% = 110\%$, a charge above full capacity, which is impossible. (*Wrong operation*)
- **C** (48.75%): Read the 0.35 as a share of the charge present rather than of the capacity, so 35% of 75% was removed: $75 \times 0.65 = 48.75$, giving 48.75%. (*Wrong reference quantity*)
- **D** (60%): Answered how much of the capacity is empty rather than how much charge remains: the battery starts $100\% - 75\% = 25\%$ empty and a further 35% is used, so $25 + 35 = 60$. What is left is the other part, $100\% - 60\% = 40\%$. (*Gave the empty share*)
- **E** (74.65%): Subtracted the raw decimal from the percentage: $75 - 0.35 = 74.65$. The two figures have to be written in the same notation before they can be combined, and 0.35 of the capacity is 35% of it, so the subtraction is $75\% - 35\% = 40\%$. (*Decimal subtracted without converting*)
- **F** (71.5%): Converted 0.35 to 3.5% by multiplying by 10 rather than 100: $75\% - 3.5\% = 71.5\%$. (*Decimal to percentage slip*)
- **G** (65%): Subtracted from a full battery instead of a three quarters full one: $100\% - 35\% = 65\%$, ignoring the starting charge entirely. (*Starting value ignored*)
- **H** (26.25%): Combined the two figures with a product rather than a difference: $0.75 \times 0.35 = 0.2625$, that is 26.25%. That number is the charge used if the 0.35 were a share of the charge present, so it is an amount removed and not an amount left. (*Multiplied instead of subtracting*)

Common mistake. Subtracting 0.35 from 75 without converting, giving 74.65%, or converting 0.35 to 3.5% and giving 71.5%. A decimal becomes a percentage by multiplying by 100, so 0.35 is 35%, not 3.5% and not 0.35%.

Takeaway. The Cheat Code: Put every quantity into one notation before combining them: fraction to percentage multiply by 100, decimal to percentage multiply by 100, and check that both figures are measured against the same whole before you add or subtract.

Time-saving tip. Speed Heuristic: Stay in decimals for the whole item. $\frac{3}{4}$ is 0.75, so the answer is $0.75 - 0.35 = 0.4$, and only the final number needs the single conversion to 40%.

Question 9

M3.11 · Ratio and proportion · ★★☆☆ · 90 s

A vineyard's yield falls by 20% each year while a disease goes untreated. Three harvests after it appeared the yield is 2560 kg. How large was the last healthy harvest?

- A. 6400 kg
- B. 4480 kg
- C. 3200 kg
- D. 4096 kg
- E. 5120 kg
- F. 4000 kg
- G. 6250 kg
- H. 5000 kg

Answer H. Key idea

A quantity that falls by a fixed percentage each year is multiplied by the same factor every year, so three years of decay amount to one multiplication by that factor cubed. Undoing the decay therefore means dividing by the cube, not adding the percentage back on. The multiplier for a 20% fall is $1 - 0.20 = 0.8$, and because each year's loss is taken from a smaller amount than the year before, repeated falls never combine into a single larger percentage.

Worked solution

1. Write down the yearly multiplier

A fall of 20% leaves 80% of the yield standing, so each year multiplies the yield by

$$1 - 0.20 = 0.8$$

2. Compound it over the three harvests

Three successive falls multiply the healthy yield Y by

$$0.8^3 = 0.8 \times 0.8 \times 0.8 = 0.512$$

so $0.512 Y = 2560$.

3. Reverse the multiplication

$$Y = \frac{2560}{0.512} = 5000 \text{ kg}$$

Dividing by 0.512 is awkward on paper, so divide by 0.8 three times instead, which is the same as multiplying by 1.25 three times: $2560 \rightarrow 3200 \rightarrow 4000 \rightarrow 5000$.

Sanity check by running the model forwards from 5000 kg: the three harvests come out as 4000 kg, then 3200 kg, then 2560 kg, each one a fifth down on the one before, and the last matches the figure given. The total loss is $1 - 0.512 = 0.488$, that is 48.8%, so the healthy harvest should be a little under twice the present one, and 5000 sits just below $2 \times 2560 = 5120$.

Matches Option **H**.

Why the other options are wrong

- **A** (6400 kg): Three falls of 20% were added into one fall of 60%, giving $2560 \div 0.4 = 6400$. Compounded over three years a 20% annual fall removes 48.8% in total, not 60%. (*Percentages added*)
- **B** (4480 kg): The correct reverse multiplier $\frac{1}{0.8} = 1.25$ was used, but its effect was added as a fixed amount three times: 25% of 2560 is 640, so $2560 + 3 \times 640 = 4480$. Compounding takes 25% of a larger figure each time, so the true steps up are 640, then 800, then 1000, reaching 5000 kg. (*Reverse rise added as a flat amount*)
- **C** (3200 kg): Only one year of decay was reversed: $2560 \div 0.8 = 3200$. That is the yield at the second harvest of the outbreak, not the healthy one, because two further falls still stand between it and the start. (*Only one year reversed*)
- **D** (4096 kg): The fall was treated as a fixed amount rather than a fixed percentage, using 20% of the final yield each time: 20% of 2560 is 512, and $2560 + 3 \times 512 = 4096$. Percentage decay strips a shrinking amount each year, so the earlier losses were larger than 512 kg. (*Fixed amount instead of fixed percentage*)
- **E** (5120 kg): The three year factor $0.8^3 = 0.512$ was rounded to 0.5, so the final yield was simply doubled: $2560 \div 0.5 = 5120$. The 0.012 is worth 120 kg here, and dividing by 0.512 exactly gives 5000 kg. (*Compound factor rounded to a half*)
- **F** (4000 kg): Only two of the three falls were undone: $2560 \div 0.8^2 = 2560 \div 0.64 = 4000$. That is the yield at the first harvest after the disease appeared, and one further division by 0.8 is needed to get back to the healthy one, giving 5000 kg. (*Two years reversed*)
- **G** (6250 kg): Four years of decay were reversed instead of three: $2560 \div 0.8^4 = 2560 \div 0.4096 = 6250$. The disease appeared three harvests before the one quoted, so the factor to undo is $0.8^3 = 0.512$. (*One year too many*)

Common mistake. Treating three years of 20% as a single fall of 60% and dividing by 0.4 to reach 6400 kg. Each year's 20% is taken from a smaller yield than the year before, so the losses cannot be added: their combined effect is $1 - 0.512 = 0.488$, a fall of 48.8%.

Takeaway. The Cheat Code: Repeated percentage change is repeated multiplication, so undoing it is repeated division by the same factor. Dividing by 0.8 is the same as multiplying by 1.25, and running the chain forwards at the end costs seconds and catches every slip.

Time-saving tip. Speed Heuristic: never divide by 0.512. Multiply by 1.25 three times instead, which is just adding a quarter each time: $2560 \rightarrow 3200 \rightarrow 4000 \rightarrow 5000$. Every step is a quarter of a round number, so the whole reversal is mental arithmetic.

Question 10

M2.5 · Number · ★★☆☆ · 90 s

A courier walks 4 blocks east and 2 north. How many routes are possible?

- A. 8
- B. 360
- C. 30
- D. 6
- E. 64
- F. 15
- G. 720
- H. 10

Answer F. Key idea

A route is nothing more than the order in which the individual block moves are made, so the walk is a list of six moves of which four are east and two are north. Because the four east moves are indistinguishable from one another and so are the two north moves, a route is fixed as soon as you say which positions in the list the north moves occupy. Counting positions rather than listing whole routes turns an awkward enumeration into a short, systematic count.

Worked solution

1. Describe a route as a list of moves

Every route uses exactly 4 east moves and 2 north moves, so it is a list of 6 single block moves. No route can be longer or shorter, since the courier must gain four blocks east and two blocks north in total.

2. Fix a route by the positions of the north moves

The east moves are identical to each other and so are the north moves, so the whole route is determined once you say which two of the six places in the list hold the north moves. Listing those pairs of positions systematically, starting with the first north move in place 1:

(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), then (2, 3), (2, 4), (2, 5), (2, 6), then (3, 4), (3, 5), (3, 6), then (4, 5), (4, 6), then (5, 6).

3. Count the pairs and check

That is $5 + 4 + 3 + 2 + 1 = 15$ pairs. The same total comes from $\frac{6 \times 5}{2} = 15$, where the division by 2 removes the double counting caused by the two north moves being interchangeable.

Sanity check by building the count junction by junction. Along the bottom row of junctions the counts are 1, 1, 1, 1, 1; the middle row is 1, 2, 3, 4, 5; the top row is 1, 3, 6, 10, 15, since each junction is reached from the one to its left plus the one below it. The far corner reads 15, which agrees.

Matches Option F.

Why the other options are wrong

- **A** (8): Multiplied the leg lengths, $4 \times 2 = 8$, as though a route were one choice of an east block paired with one choice of a north block. The two legs are not independent single choices. (*Product rule misapplied*)
- **B** (360): All six moves were ordered, $6! = 720$, and the count was then divided by the $2!$ for the two identical north moves but not by the $4!$ for the four identical east moves: $\frac{720}{2} = 360$. Both sets of repeats have to come out, and $\frac{720}{4! \times 2!} = \frac{720}{48} = 15$. (*Only one repeat divided out*)

- **C** (30): Chose the two north positions in order: $6 \times 5 = 30$. Every route is then counted twice, once for each order of the two identical north moves, so this is exactly double the answer. (*Double counting identical moves*)
- **D** (6): Added the two leg lengths, $4 + 2 = 6$. That counts the number of blocks walked, not the number of orders in which they can be walked. (*Counted steps, not routes*)
- **E** (64): Treated each of the 6 moves as a free choice between east and north: $2^6 = 64$. Most of those sequences do not contain exactly four easts and two norths, so they do not end at the right corner. (*Constraint ignored*)
- **G** (720): The six moves were counted as six distinct objects and their orderings totalled: $6! = 720$. Swapping two east moves does not change the route walked, so each route has been counted $4! \times 2! = 48$ times over, and $\frac{720}{48} = 15$. (*Moves treated as distinguishable*)
- **H** (10): The two north moves were placed in different gaps around the four east moves. There are 5 such gaps, one before each east move and one after the last, so this gives $\frac{5 \times 4}{2} = 10$ ways. It discards the 5 routes in which both north moves are taken one straight after the other, both sitting in the same gap, and $10 + 5 = 15$. (*Adjacent north moves excluded*)

Common mistake. Treating the six moves as six distinguishable objects and counting orderings of them, or at the other extreme multiplying the two leg lengths together. The four east moves cannot be told apart, so swapping two of them does not create a new route.

Takeaway. The Cheat Code: A journey made of two kinds of identical steps is counted by choosing which places in the sequence the rarer step occupies: with 6 steps of which 2 are north, that is $\frac{6 \times 5}{2}$, and the junction by junction addition grid gives the same number as a free check.

Time-saving tip. Speed Heuristic: Count the rarer move, not the common one. Two north moves among six places gives $\frac{6 \times 5}{2} = 15$ in one line, whereas working with the four east moves needs $\frac{6 \times 5 \times 4 \times 3}{4 \times 3 \times 2}$ for the same answer.

END OF SOLUTIONS

Questions 11 to 27 are in the full paper

This is an extract. The 10 questions before this page are real questions from a commissioned worked solutions, shown in full and with nothing withheld, so that the standard can be judged rather than described.

The rest of the paper is not hidden behind this page. It was never in this file. Each commissioned paper is written for one school, recorded against it, and issued to no other, so the questions in it are not published anywhere, including here.

A commissioned paper is NOT this paper with the missing questions restored. Every question in it is different from every question here, the 10 shown above included. Your school's paper is drawn fresh, and no question that has ever been published, on this page or anywhere else, can appear in it.

A full paper runs to 27 questions in this module alone, with a question paper, an answer sheet, a mark scheme and a worked solution for every question, across all five ESAT modules.

Commission a paper for your school

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Written by researchers in engineering, physics and mathematics at Oxford and Cambridge, and checked question by question before it is issued.

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EXAMPLE GRAMMAR SCHOOL

ESAT Mock Examination

Mathematics 2

Paper SMP-2026-09 · Version 1.0

NAME

TEACHING GROUP

DATE

Instructions

- Time allowed: **15 minutes**.
- There are **10 multiple-choice questions**. Each is worth 1 mark.
- There is no negative marking. Answer every question, even if you have to guess.
- Calculators are not permitted.
- Record your answers on the separate answer sheet. Only the answer sheet is marked.
- Formulae are not provided. The ESAT does not supply a formula sheet.

Note: the real ESAT is sat on screen at a Pearson test centre. This paper trains the mathematics and the pacing, not the on-screen interface.

Written for Example Grammar School by Lucas Rey, Engineering Science, University of Oxford; MRes, University of Cambridge.

Question 1

Simplify $\frac{x^{5/2} \cdot x^{-1/2}}{x^{3/2}}$.

- A. 1
- B. x^2
- C. $x^{3/2}$
- D. x
- E. $x^{1/2}$
- F. $x^{7/2}$
- G. $x^{9/2}$
- H. $x^{-1/2}$

Question 2

A pressure sensor's calibration line is $y = 2x + 5$. After the sensor is re-tuned its calibration line has a gradient 50% larger and an intercept on the y-axis 3 units lower. Give the coordinates of the point where the two calibration lines meet.

- A. (2, 9)
- B. (-3, -1)
- C. (-7, -9)
- D. (5, 15)
- E. (-3, -4)
- F. (3, 11)
- G. (0, 5)
- H. (6, 17)

Question 3

A lens surface is machined so that its cross-section satisfies $y = \frac{3x^6 + 5x^2\sqrt{x} - 2}{x^2}$ for $x > 0$, with x and y in millimetres. Find the gradient of the cross-section at $x = 1$.

- A. $\frac{49}{2}$
- B. 21
- C. 11
- D. $\frac{61}{4}$
- E. $\frac{21}{2}$
- F. $\frac{37}{2}$
- G. $\frac{29}{2}$
- H. $\frac{77}{4}$

Question 4

Rationalise the denominator of $\frac{\sqrt{7} - \sqrt{3}}{\sqrt{7} + \sqrt{3}}$ and simplify fully.

- A. $\frac{5 + \sqrt{21}}{2}$
- B. $\frac{5 - \sqrt{21}}{2}$
- C. $\frac{10 - \sqrt{21}}{2}$
- D. $\frac{5 - \sqrt{21}}{5}$
- E. $\frac{2 - \sqrt{21}}{2}$

Question 5

Functions f and g are integrable on $1 \leq x \leq 9$. The integral of f is 14 over $1 \leq x \leq 5$ and 22 over $5 \leq x \leq 9$.

Given $\int_1^9 (f(x) + g(x)) dx = 92$, find $\int_1^9 g(x) dx$.

- A. 74
- B. 84
- C. 56
- D. 70
- E. 128
- F. 100
- G. 78
- H. 36

Question 6

A curve $y = a^x$ has $a > 1$. The chord joining its points at $x = 1$ and $x = 4$ has gradient 26. Find y when $x = -2$.

- A. 9
- B. $\frac{1}{6}$
- C. $\frac{1}{3}$
- D. $\frac{1}{81}$
- E. $-\frac{1}{9}$
- F. $\frac{1}{27}$
- G. $-\frac{1}{9}$
- H. $\frac{1}{9}$

Question 7

A horizontal line $y = k$ never meets the curve $y = x^2 - 2x - 7$. Find all possible values of k .

- A. $k < -8$
- B. $k < -6$
- C. $k > -8$
- D. $k > 8$
- E. $k < -7$
- F. $k > -6$
- G. $k < -11$
- H. $k \leq -8$

Question 8

Each of the five functions below is defined for all real x . Which is one-to-one?

- A. $f(x) = \sin x$
- B. $f(x) = x^4 + x$
- C. $f(x) = |x| - 4$
- D. $f(x) = x^3 - 3x$
- E. $f(x) = x^3 + 2x$
- F. $f(x) = 2^x - x$
- G. $f(x) = 2^{x^2}$
- H. $f(x) = x^2 - 6x$

Question 9

Let $\log_a 2 = p$ and $\log_a 3 = q$, where $a > 1$ is a fixed base. Express $\log_a \left(\frac{9\sqrt{a}}{8} \right)$ in terms of p and q .

- A. $3q - 2p + \frac{1}{2}$
- B. $2q - 3p + 2$
- C. $q^2 - p^3 + \frac{1}{2}$
- D. $\frac{2q}{3p} + \frac{1}{2}$
- E. $2q + 3p + \frac{1}{2}$
- F. $2q - 3p + \frac{1}{2}$
- G. $2q - 3p + 1$
- H. $3p - 2q - \frac{1}{2}$

Question 10

For an acute angle θ , $\frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} = \frac{25}{12}$. Find the exact value of $\tan \theta$.

- A. $\frac{3}{4}$
- B. $\frac{24}{25}$
- C. $\frac{4}{3}$
- D. $\frac{24}{7}$
- E. $\frac{7}{24}$

END OF PAPER

Questions 11 to 27 are in the full paper

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EXAMPLE GRAMMAR SCHOOL

ESAT Mock: Answer Sheet

Mathematics 2

Paper SMP-2026-09 • Version 1.0

NAME

TEACHING GROUP

DATE

Fill in one bubble per question. If you change your mind, cross out the old mark clearly and fill the new bubble. Unfilled or doubly filled rows score zero.

Q	A	B	C	D	E	F	G	H
1	☐	☐	☐	☐	☐	☐	☐	☐
2	☐	☐	☐	☐	☐	☐	☐	☐
3	☐	☐	☐	☐	☐	☐	☐	☐
4	☐	☐	☐	☐	☐			
5	☐	☐	☐	☐	☐	☐	☐	☐
6	☐	☐	☐	☐	☐	☐	☐	☐
7	☐	☐	☐	☐	☐	☐	☐	☐
8	☐	☐	☐	☐	☐	☐	☐	☐
9	☐	☐	☐	☐	☐	☐	☐	☐
10	☐	☐	☐	☐	☐			

MARKER INITIALS

TOTAL OUT OF 10

EXAMPLE GRAMMAR SCHOOL

ESAT Mock: Mark Scheme

Mathematics 2

Paper SMP-2026-09 • Version 1.0

Answer grid

Q	Ans	Q	Ans	Q	Ans
1	E	10	D		
2	F				
3	F				
4	B				
5	C				
6	H				
7	A				
8	E				
9	F				

Totals

10 marks available, 1 mark per question. There is no negative marking, so a wrong answer scores zero and never a negative. Only the answer sheet is marked.

Diagnostic table

Use this to see which topics the cohort is losing marks on.

Q	Ans	Node	Group	Topic	Difficulty
1	E	MM1.1	MM1	Algebra	★★☆☆☆
2	F	MM8.3	MM8	Graphs and transformations	★★☆☆☆
3	F	MM6.2	MM6	Differentiation	★★☆☆☆
4	B	MM1.2	MM1	Algebra and functions	★★☆☆☆
5	C	MM7.4	MM7	Integration	★★☆☆☆
6	H	MM5.1	MM5	Exponentials and logarithms	★★☆☆☆
7	A	MM1.3	MM1	Algebra and functions	★★☆☆☆
8	E	MM1.7	MM1	Algebra and functions	★★☆☆☆
9	F	MM5.2	MM5	Exponentials and logarithms	★★☆☆☆
10	D	MM4.5	MM4	Trigonometry	★★★★☆

Marks available by topic group

Group	Topic	Marks
MM1	Algebra and functions	4
MM4	Trigonometry	1
MM5	Exponentials and logarithms	2
MM6	Differentiation	1
MM7	Integration	1
MM8	Graphs and transformations	1
Total		10

Worked solutions for every question are in the separate worked solutions document.

EXAMPLE GRAMMAR SCHOOL

ESAT Mock: Worked Solutions

Mathematics 2

Paper SMP-2026-09 • Version 1.0

Staff copy

This document contains the answers. Keep it away from candidates until the paper has been sat. Marking guidance is in the separate mark scheme.

Question 1

MM1.1 • Algebra • ★★☆☆☆ • 30 s

Simplify $\frac{x^{5/2} \cdot x^{-1/2}}{x^{3/2}}$.

- A. 1
- B. x^2
- C. $x^{3/2}$
- D. x
- E. $x^{1/2}$
- F. $x^{7/2}$
- G. $x^{9/2}$
- H. $x^{-1/2}$

Answer E. Key idea

Numerator: $x^{5/2+(-1/2)} = x^2$. Then $x^2/x^{3/2} = x^{1/2}$.

Worked solution

1. Simplify the numerator

$$x^{5/2} \cdot x^{-1/2} = x^{5/2-1/2} = x^{4/2} = x^2$$

2. Divide by the denominator

$$\frac{x^2}{x^{3/2}} = x^{2-3/2} = x^{1/2}$$

Matches Option E.

Why the other options are wrong

- **A** (1): Arises from a common inversion, algebraic sign error, or omitting an intermediate conversion factor.
(Calculation / Conceptual Error)
- **B** (x^2): Arises from a common inversion, algebraic sign error, or omitting an intermediate conversion factor.
(Calculation / Conceptual Error)
- **C** ($x^{3/2}$): Arises from a common inversion, algebraic sign error, or omitting an intermediate conversion factor.
(Calculation / Conceptual Error)

- **D** (x): The $x^{-1/2}$ factor is lost when the fraction is copied, leaving $x^{5/2}/x^{3/2} = x^{5/2-3/2} = x^1$, printing x . The missing factor is worth $-1/2$ on the index, which is exactly the difference between x and the key $x^{1/2}$. (*Factor dropped from the numerator*)
- **F** ($x^{7/2}$): Added the denominator exponent instead of subtracting it. (*Sign Error*)
- **G** ($x^{9/2}$): Every index is added and both negatives are dropped, the $-1/2$ in the numerator and the subtraction owed to the denominator: $5/2 + 1/2 + 3/2 = 9/2$, printing $x^{9/2}$. This is one step past the $x^{7/2}$ slip, which at least keeps the sign on the $-1/2$. (*Both minus signs ignored*)
- **H** ($x^{-1/2}$): The numerator is simplified correctly to $x^{5/2-1/2} = x^2$, then the last subtraction is taken the wrong way round: the numerator index is subtracted from the denominator index, $3/2 - 2 = -1/2$, printing $x^{-1/2}$. Dividing takes numerator index minus denominator index, $2 - 3/2 = 1/2$. (*Subtraction reversed at the division step*)

Common mistake. Adding all the exponents (including the denominator): $5/2 - 1/2 + 3/2 = 7/2$.

Takeaway. The Cheat Code: Always inspect units and algebraic structure before calculating. Simplifying fractions and factoring terms early reduces computation time by more than 50%.

Time-saving tip. Speed Heuristic: Look at the relative magnitudes and units in the multiple-choice options to eliminate extreme or impossible values immediately.

Question 2

MM8.3 • Graphs and transformations • ★★☆☆ • 90 s

A pressure sensor's calibration line is $y = 2x + 5$. After the sensor is re-tuned its calibration line has a gradient 50% larger and an intercept on the y -axis 3 units lower. Give the coordinates of the point where the two calibration lines meet.

- A. (2, 9)
- B. (-3, -1)
- C. (-7, -9)
- D. (5, 15)
- E. (-3, -4)
- F. (3, 11)
- G. (0, 5)
- H. (6, 17)

Answer F. Key idea

Altering m and altering c do two different things to a straight line. The gradient m controls steepness, so changing it pivots the line about the point where it crosses the y -axis, while c is that crossing itself, so changing it slides every point of the line vertically by the same amount. Once both parameters of the second line are known, the point the two lines have in common is found by setting their expressions for y equal.

Worked solution

Step 1: Read the two parameters of the original line.

Written in the form $y = mx + c$, the line $y = 2x + 5$ has $m = 2$ and $c = 5$, so its gradient is 2 and it crosses the y -axis at (0, 5).

Step 2: Alter m .

A gradient 50% larger means multiplying by $\frac{3}{2}$, so $m' = \frac{3}{2} \times 2 = 3$. This change pivots the line about its y-axis crossing, so the crossing is still at 5 after this step alone.

Step 3: Alter c .

Lowering the intercept by 3 gives $c' = 5 - 3 = 2$, a pure vertical translation. The re-tuned calibration line is therefore $y = 3x + 2$.

Step 4: Set the two expressions for y equal.

$2x + 5 = 3x + 2$, so $5 - 2 = 3x - 2x$, giving $x = 3$.

Step 5: Find y and check on both lines.

The original line gives $y = 2(3) + 5 = 11$ and the re-tuned line gives $y = 3(3) + 2 = 11$, so the meeting point is $(3, 11)$. Sanity check: at $x = 0$ the new line sits 3 below the old one, and it gains $3 - 2 = 1$ unit on it for every unit of x , so it needs exactly 3 units of x to catch up, which agrees with $x = 3$.

Matches Option F.

Why the other options are wrong

- **A** $((2, 9))$: Reading '3 units lower' as an intercept of 3 rather than $5 - 3 = 2$ gives $y = 3x + 3$. Then $2x + 5 = 3x + 3$ gives $x = 2$, and $y = 2(2) + 5 = 9$, printing $(2, 9)$. (*Misread of the intercept change*)
- **B** $((-3, -1))$: Lifting the intercept instead of lowering it gives $c' = 5 + 3 = 8$ and the line $y = 3x + 8$. Then $2x + 5 = 3x + 8$ gives $-3 = x$, and $y = 2(-3) + 5 = -1$, printing $(-3, -1)$. (*Wrong direction for the change in c*)
- **C** $((-7, -9))$: A sign slip when collecting terms: writing $2x - 3x = 2 + 5$ instead of $2x - 3x = 2 - 5$ gives $-x = 7$, so $x = -7$ and $y = 2(-7) + 5 = -9$, printing $(-7, -9)$. (*Sign error moving a constant across the equals sign*)
- **D** $((5, 15))$: Steepening is taken to swing the line about $(0, 0)$ rather than about its y-axis crossing, so the re-tuned line is written as $y = 3x$ with no intercept at all. Then $2x + 5 = 3x$ gives $x = 5$, and $y = 2(5) + 5 = 15$, printing $(5, 15)$. Changing m pivots about $(0, 5)$, and the intercept only moves when c is altered. (*Line pivoted about the origin*)
- **E** $((-3, -4))$: The intercept drop is applied to the original line and only the steepening to the re-tuned one, giving $y = 2x + 2$ and $y = 3x + 5$. Then $2x + 2 = 3x + 5$ gives $x = -3$, and $y = 2(-3) + 2 = -4$, printing $(-3, -4)$. Both changes describe the line after re-tuning, so both belong to the same equation. (*The two changes applied to opposite lines*)
- **G** $((0, 5))$: Applying only the gradient change leaves c at 5, giving $y = 3x + 5$. Two lines sharing a y-intercept meet on the y-axis, so $2x + 5 = 3x + 5$ gives $x = 0$ and $y = 5$, printing $(0, 5)$. (*Second alteration omitted*)
- **H** $((6, 17))$: The gradient 50% larger is read as adding 0.5 to it rather than multiplying by $\frac{3}{2}$, giving $m' = 2 + 0.5 = 2.5$ and the line $y = 2.5x + 2$. Then $2x + 5 = 2.5x + 2$ gives $0.5x = 3$, so $x = 6$ and $y = 2(6) + 5 = 17$, printing $(6, 17)$. Fifty per cent of 2 is 1, so the new gradient is 3. (*Percentage added instead of multiplied*)

Common mistake. Assuming that steepening a line also moves where it crosses the y-axis. Changing m pivots the line about $(0, c)$, so the crossing stays at 5 until c itself is altered. Candidates who instead pivot about the origin write the new line as $y = 3x$, solve $2x + 5 = 3x$ and report $(5, 15)$.

Takeaway. The Cheat Code: In $y = mx + c$, m is a pivot and c is a lift: change m and the line turns about $(0, c)$, change c and every point slides vertically. Two lines then close their intercept gap at a rate equal to the difference of their gradients.

Time-saving tip. Speed Heuristic: Skip the simultaneous equations. The gradients differ by 1 and the intercepts differ by 3, so the lower line closes the gap at 1 per unit of x and they meet at $x = 3$; one substitution gives $y = 11$.

Question 3

MM6.2 · Differentiation · ★★☆☆ · 90 s

A lens surface is machined so that its cross-section satisfies $y = \frac{3x^6 + 5x^2\sqrt{x} - 2}{x^2}$ for $x > 0$, with x and y in millimetres. Find the gradient of the cross-section at $x = 1$.

- A. $\frac{49}{2}$
- B. 21
- C. 11
- D. $\frac{61}{4}$
- E. $\frac{21}{2}$
- F. $\frac{37}{2}$
- G. $\frac{29}{2}$
- H. $\frac{77}{4}$

Answer F. Key idea

The power rule differentiates x^n to nx^{n-1} for any rational n , positive, negative or fractional, but it applies only to a sum of separate powers. An expression written as one fraction is not in that form, so the first move is to divide the denominator into each term and rewrite roots and reciprocals as indices. Only then does term by term differentiation apply, and a gradient at a particular point is the value of that derivative there.

Worked solution**1. Simplify into a sum of powers**

Dividing each term of the numerator by x^2 , and using $x^2\sqrt{x} = x^{5/2}$:

$$y = \frac{3x^6}{x^2} + \frac{5x^{5/2}}{x^2} - \frac{2}{x^2} = 3x^4 + 5x^{1/2} - 2x^{-2}$$

In plainer form this is $y = 3x^4 - \frac{2}{x^2} + 5\sqrt{x}$.

2. Differentiate term by term

$$\frac{dy}{dx} = 3(4)x^3 + 5\left(\frac{1}{2}\right)x^{-1/2} - 2(-2)x^{-3}$$

$$\frac{dy}{dx} = 12x^3 + 4x^{-3} + \frac{5}{2}x^{-1/2} = 12x^3 + \frac{4}{x^3} + \frac{5}{2\sqrt{x}}$$

Note the middle sign: a negative coefficient multiplied by a negative index gives a positive term.

3. Substitute the required value

At $x = 1$ every power of x is 1, so

$$\frac{dy}{dx} = 12(1) + \frac{4}{1} + \frac{5}{2 \times 1} = 12 + 4 + \frac{5}{2}$$

4. Combine

$$12 + 4 + \frac{5}{2} = 16 + \frac{5}{2} = \frac{32 + 5}{2} = \frac{37}{2}$$

Sanity check on the simplification: multiplying back, $x^2(3x^4 + 5x^{1/2} - 2x^{-2}) = 3x^6 + 5x^{5/2} - 2$, which is the numerator given. All three derivative terms are positive at $x = 1$, so the surface is rising steeply there, and a gradient of $\frac{37}{2}$, that is 18.5, is consistent with the dominant $12x^3$ term.

Matches Option F.

Why the other options are wrong

- **A** ($\frac{49}{2}$): The x^2 is cancelled against $3x^6$ and the rest of the numerator is left as it stands, giving $y = 3x^4 + 5x^{5/2} - 2$. Differentiating that gives $12x^3 + \frac{25}{2}x^{3/2}$ with the constant vanishing, and at $x = 1$ the value is $12 + \frac{25}{2} = \frac{49}{2}$. Every term of the numerator must be divided by x^2 . (*Denominator divided into the first term only*)
- **B** (21): Writes the derivative of $5x^{1/2}$ as $5x^{-1/2}$, lowering the index without multiplying by it, so the last term contributes 5 instead of $\frac{5}{2}$ and the total is $12 + 4 + 5 = 21$. The rule multiplies by the old index as well as reducing it. (*Index reduced but coefficient not multiplied*)
- **C** (11): $x^2\sqrt{x}$ is read as $x^{2 \times 1/2} = x$ instead of $x^{2+1/2} = x^{5/2}$, so the middle term becomes $\frac{5x}{x^2} = 5x^{-1}$, whose derivative $-5x^{-2}$ is worth -5 at $x = 1$. The total is $12 - 5 + 4 = 11$. Multiplying powers of the same base adds the indices. (*Indices multiplied when powers are multiplied*)
- **D** ($\frac{61}{4}$): Differentiates top and bottom separately: the numerator gives $18x^5 + \frac{25}{2}x^{3/2}$ and the denominator gives $2x$, and at $x = 1$ that is $\frac{18 + 12.5}{2} = \frac{61}{4}$. Differentiation does not distribute over a division, so the fraction must be simplified into separate powers first. (*Quotient differentiated piecewise*)
- **E** ($\frac{21}{2}$): Differentiates $-2x^{-2}$ to $-4x^{-3}$, keeping the minus sign, so the value becomes $12 - 4 + \frac{5}{2} = \frac{21}{2}$. Bringing the index -2 down onto the coefficient -2 multiplies two negatives, so the term is $+4x^{-3}$ and contributes $+4$. (*Sign error on the reciprocal term*)
- **G** ($\frac{29}{2}$): Reads $-\frac{2}{x^2}$ as a constant because its numerator carries no x , so the middle term is dropped and the total is $12 + 0 + \frac{5}{2} = \frac{29}{2}$. The variable sits in the denominator, so the term is $-2x^{-2}$ and its derivative is $+4x^{-3}$, worth 4 at $x = 1$. (*Constant-looking term differentiated to zero*)
- **H** ($\frac{77}{4}$): Dividing by x^2 is carried out by halving each index rather than subtracting 2: x^6 becomes x^3 and $x^{5/2}$ becomes $x^{5/4}$, giving $y = 3x^3 + 5x^{5/4} - 2x^{-2}$. Its derivative $9x^2 + \frac{25}{4}x^{1/4} + 4x^{-3}$ is $9 + \frac{25}{4} + 4 = \frac{77}{4}$ at $x = 1$. Division subtracts indices, so the terms are x^4 and $x^{1/2}$. (*Indices divided instead of subtracted*)

Common mistake. Differentiating the numerator and the denominator separately, as though a quotient could be handled a piece at a time. Dividing through first is what makes the power rule legal here, and it costs one line because every term of the numerator divides cleanly by x^2 .

Takeaway. The Cheat Code: Get the expression into the form sum of kx^n before differentiating anything. Divide out the denominator, write roots as fractional indices and reciprocals as negative indices, and the whole problem reduces to applying nx^{n-1} three times.

Time-saving tip. Speed Heuristic: The gradient is wanted at $x = 1$, where every power equals 1, so only the coefficients matter. Simplify to $3x^4 + 5x^{1/2} - 2x^{-2}$ and read off 3×4 , $5 \times \frac{1}{2}$ and $(-2) \times (-2)$, then add: no powers need evaluating at all.

Question 4

MM1.2 · Algebra and functions · ★★☆☆ · 85 s

Rationalise the denominator of $\frac{\sqrt{7} - \sqrt{3}}{\sqrt{7} + \sqrt{3}}$ and simplify fully.

- A. $\frac{5 + \sqrt{21}}{2}$
B. $\frac{5 - \sqrt{21}}{2}$
C. $\frac{10 - \sqrt{21}}{2}$
D. $\frac{5 - \sqrt{21}}{5}$
E. $\frac{2 - \sqrt{21}}{2}$

Answer B. Key idea

A denominator containing a sum of two surds is cleared by multiplying above and below by its conjugate, because a conjugate pair multiplies out as a difference of two squares and leaves a rational number behind. When the numerator happens to be that same conjugate, the multiplication squares it, so the cross term of the expansion survives and the simplified answer still carries a surd on top.

Worked solution**1. Multiply by the conjugate of the denominator**

The denominator is $\sqrt{7} + \sqrt{3}$, whose conjugate is $\sqrt{7} - \sqrt{3}$. Multiplying numerator and denominator by the same quantity leaves the value unchanged:

$$\frac{\sqrt{7} - \sqrt{3}}{\sqrt{7} + \sqrt{3}} = \frac{(\sqrt{7} - \sqrt{3})(\sqrt{7} - \sqrt{3})}{(\sqrt{7} + \sqrt{3})(\sqrt{7} - \sqrt{3})} = \frac{(\sqrt{7} - \sqrt{3})^2}{(\sqrt{7})^2 - (\sqrt{3})^2} = \frac{(\sqrt{7} - \sqrt{3})^2}{7 - 3}$$

The denominator is now the rational number 4.

2. Expand the numerator and divide through

$$(\sqrt{7} - \sqrt{3})^2 = 7 - 2\sqrt{7}\sqrt{3} + 3 = 10 - 2\sqrt{21}$$

so the fraction becomes

$$\frac{10 - 2\sqrt{21}}{4} = \frac{2(5 - \sqrt{21})}{4} = \frac{5 - \sqrt{21}}{2}$$

Both terms of the numerator carry the factor 2, so the factor cancels out of the whole bracket, not out of one term only.

Sanity check

With $\sqrt{7} \approx 2.646$ and $\sqrt{3} \approx 1.732$, the original quotient is about $0.914 \div 4.378 \approx 0.209$. Using $\sqrt{21} \approx 4.583$, the answer gives $(5 - 4.583) \div 2 \approx 0.209$, which agrees. The value must also be positive and smaller than 1, since the numerator is the smaller of the two surd expressions, and it is.

Matches Option B.

Why the other options are wrong

- **A** ($\frac{5 + \sqrt{21}}{2}$): The cross term was expanded with the wrong sign: $(\sqrt{7} - \sqrt{3})^2$ was written as $7 + 2\sqrt{21} + 3 = 10 + 2\sqrt{21}$. Dividing by the correct denominator 4 then gives $\frac{10 + 2\sqrt{21}}{4} = \frac{5 + \sqrt{21}}{2} \approx 4.79$, which is far larger than 1 even though the original fraction has the smaller expression on top. (*Cross-term sign slip*)
- **C** ($\frac{10 - \sqrt{21}}{2}$): From the correct $\frac{10 - 2\sqrt{21}}{4}$, the factor 2 was cancelled against the 4 but applied only to the surd term, leaving $\frac{10 - \sqrt{21}}{2}$. The 10 must be halved as well, since the whole numerator is being divided. (*Partial cancellation*)
- **D** ($\frac{5 - \sqrt{21}}{5}$): The denominator was multiplied out as $\sqrt{7} \times \sqrt{7} + \sqrt{3} \times \sqrt{3} = 7 + 3 = 10$ rather than as the difference of two squares $7 - 3 = 4$. That gives $\frac{10 - 2\sqrt{21}}{10} = \frac{5 - \sqrt{21}}{5} \approx 0.083$, which is well below the true value of about 0.209. (*Denominator added instead of differenced*)
- **E** ($\frac{2 - \sqrt{21}}{2}$): The second square was subtracted rather than added when expanding the numerator: $7 - 2\sqrt{21} - 3 = 4 - 2\sqrt{21}$. Dividing by 4 gives $\frac{4 - 2\sqrt{21}}{4} = \frac{2 - \sqrt{21}}{2} \approx -1.29$, a negative value, although both $\sqrt{7} - \sqrt{3}$ and $\sqrt{7} + \sqrt{3}$ are positive. (*Second square subtracted in the expansion*)

Common mistake. Multiplying numerator and denominator by the denominator itself instead of by its conjugate. That produces $\frac{(\sqrt{7} - \sqrt{3})(\sqrt{7} + \sqrt{3})}{(\sqrt{7} + \sqrt{3})^2} = \frac{4}{10 + 2\sqrt{21}}$, which still has a surd underneath, so nothing has been rationalised and the work has to be started again.

Takeaway. The Cheat Code: When the top and bottom of a surd fraction are conjugates of one another, one multiplication does everything: the denominator collapses to a plain integer and the numerator becomes a perfect square. Expect the answer in the form $\frac{p - q\sqrt{r}}{s}$, and divide every term of the numerator by that integer.

Time-saving tip. Speed Heuristic: compute the denominator first. Here $(\sqrt{7})^2 - (\sqrt{3})^2 = 4$, and the expanded numerator $10 - 2\sqrt{21}$ is even, so the printed answer must sit over 2, which discards the option written over 5 at once. A single decimal estimate near 0.21 then separates the four remaining forms without any further algebra.

Question 5

MM7.4 • Integration • ★★☆☆☆ • 90 s

Functions f and g are integrable on $1 \leq x \leq 9$. The integral of f is 14 over $1 \leq x \leq 5$ and 22 over $5 \leq x \leq 9$. Given $\int_1^9 (f(x) + g(x)) dx = 92$, find $\int_1^9 g(x) dx$.

- A. 74
- B. 84
- C. 56
- D. 70
- E. 128
- F. 100
- G. 78
- H. 36

Answer C. Key idea

Definite integrals over ranges that meet end to end add to give the integral over the joined range, and over one shared range the integral of a sum of functions equals the sum of the two integrals. Both facts come from areas stacking, so a question built on them needs no formula for either function.

Worked solution

1. Join the two ranges for f

$[1, 5]$ and $[5, 9]$ are contiguous: they meet at $x = 5$ and between them cover $[1, 9]$, so their integrals add.

2. Total the integral of f

$$\int_1^9 f(x) dx = \int_1^5 f(x) dx + \int_5^9 f(x) dx = 14 + 22 = 36$$

3. Split the combined integrand over the equal range

Both integrals now run over the same range $[1, 9]$, so

$$\int_1^9 (f(x) + g(x)) dx = \int_1^9 f(x) dx + \int_1^9 g(x) dx$$

$$\text{giving } 92 = 36 + \int_1^9 g(x) dx.$$

4. Solve for the unknown integral

$$\int_1^9 g(x) dx = 92 - 36 = 56$$

Sanity check

Rebuilding the combined value from the two parts: $36 + 56 = 92$, the figure given in the question.

Matches Option C.

Why the other options are wrong

- **A** (74): Because $[1, 5]$ and $[5, 9]$ each cover half of $[1, 9]$, the two values are averaged rather than added: $\frac{14 + 22}{2} = 18$ is taken as $\int_1^9 f$, and $92 - 18 = 74$. Each figure is already the whole integral over its own sub-range, so ranges that meet end to end add: $14 + 22 = 36$. (*Contiguous pieces averaged instead of added*)
- **B** (84): The contiguous pieces were subtracted rather than added, $22 - 14 = 8$, and then $92 - 8 = 84$. Ranges that join end to end add. (*Contiguous ranges subtracted*)
- **D** (70): Only the second piece was removed, $92 - 22 = 70$, leaving out the 14 that f contributes over $[1, 5]$. (*First sub-range dropped*)
- **E** (128): The two pieces of f are joined correctly to $14 + 22 = 36$, then $92 = 36 + \int_1^9 g$ is rearranged by adding rather than subtracting: $92 + 36 = 128$. The integral of g is what is left of the combined 92 after f 's share is removed, so it must be smaller than 92, not larger. (*Sign slip rearranging for the unknown*)
- **F** (100): Written out in full the equation is $92 = 14 + 22 + \int_1^9 g$. Moving the 14 across correctly but carrying the 22 over with its sign unchanged gives $\int_1^9 g = 92 - 14 + 22 = 100$. Both pieces belong to f , so both are subtracted. (*Only one piece of f changed sign*)
- **G** (78): Only the first piece was removed, $92 - 14 = 78$, leaving out the 22 that f contributes over $[5, 9]$. (*Second sub-range dropped*)
- **H** (36): The work stopped after joining the contiguous pieces: $14 + 22 = 36$ is $\int_1^9 f(x) dx$, and the question asks for the integral of g . (*Answered the wrong quantity*)

Common mistake. Subtracting the two pieces of f instead of adding them. The ranges $[1, 5]$ and $[5, 9]$ follow one after the other rather than one sitting inside the other, so their values combine as $14 + 22$.

Takeaway. The Cheat Code: Two rules cover this whole family: ranges that meet end to end add their integrals, and over one shared range the integral of a sum is the sum of the integrals. Neither rule asks what f or g actually is.

Time-saving tip. Speed Heuristic: Collapse the two pieces of f to 36 before reading anything else, and the question reduces to the single subtraction $92 - 36$.

Question 6

MM5.1 · Exponentials and logarithms · ★★☆☆ · 90 s

A curve $y = a^x$ has $a > 1$. The chord joining its points at $x = 1$ and $x = 4$ has gradient 26. Find y when $x = -2$.

- A. 9
- B. $\frac{1}{6}$
- C. $\frac{1}{3}$
- D. $\frac{1}{81}$
- E. $-\frac{1}{9}$
- F. $\frac{1}{27}$
- G. $-\frac{1}{9}$
- H. $\frac{1}{9}$

Answer H. Key idea

A curve $y = a^x$ with $a > 1$ is completely determined by its base: it passes through $(0, 1)$, multiplies its height by a at every unit step to the right, and stays strictly positive because a positive base raised to any power is positive. That means the coordinates of any point on it are (x, a^x) , so a geometric condition such as the gradient of a chord becomes an ordinary algebraic equation in a . Negative values of x are handled by the reciprocal rule $a^{-n} = \frac{1}{a^n}$, which is why the curve falls towards the axis on the left without ever reaching it.

Worked solution

1. Write down the two points

Every point of the curve has the form (x, a^x) , so the chord joins

$(1, a)$ and $(4, a^4)$

2. Turn the gradient into an equation in the base

$$\text{gradient} = \frac{a^4 - a}{4 - 1} = 26$$

$$a^4 - a = 78$$

3. Solve for the base

Factorise the left side as $a(a^3 - 1) = 78$. Testing simple bases greater than one: $a = 2$ gives $2(8 - 1) = 14$, which is too small, and $a = 3$ gives $3(27 - 1) = 3 \times 26 = 78$, which is exact. So $a = 3$.

This is the only solution with $a > 1$, because both a and $a^3 - 1$ increase as a increases, so their product is strictly increasing and can hit 78 only once.

4. Evaluate the curve at the required point

$$y = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

Sanity check: the curve is $y = 3^x$, which passes through (1, 3) and (4, 81), and the chord between them has gradient $\frac{81-3}{3} = \frac{78}{3} = 26$ as required. At $x = -2$ the curve lies to the left of (0, 1), so the height must be positive and smaller than 1, and $\frac{1}{9}$ is.

Matches Option H.

Why the other options are wrong

- **A (9)**: Evaluates $3^2 = 9$, ignoring the minus sign in $x = -2$. That is the height of the curve at $x = +2$, and the two are reciprocals of one another, so the required value is $\frac{1}{9}$. (*Sign of the exponent dropped*)
- **B ($\frac{1}{6}$)**: Finds $a = 3$ correctly, then reads 3^{-2} as $\frac{1}{3 \times 2} = \frac{1}{6}$, multiplying the base by the index instead of raising it. The index counts repeated multiplication, so $3^2 = 9$ and the height is $\frac{1}{9}$. (*Index treated as a multiplier*)
- **C ($\frac{1}{3}$)**: Works back from the known point (1, 3), where each unit step to the left divides the height by 3. The gap from $x = 1$ to $x = -2$ is three steps, giving $3 \div 3^3 = \frac{1}{9}$, but counting it as $|-2| = 2$ steps gives $3 \div 3^2 = \frac{1}{3}$. The number of steps is $1 - (-2) = 3$, not the size of the target x . (*Steps back along the curve miscounted*)
- **D ($\frac{1}{81}$)**: Carries the value $3^4 = 81$ across from the trial in the previous step and writes $\frac{1}{81}$. That is the height at $x = -4$, not $x = -2$; the required power is $3^2 = 9$. (*Wrong power reciprocated*)
- **E ($-\frac{1}{9}$)**: Finds $a = 3$ correctly, then writes $3^{-2} = -\frac{1}{3^2} = -\frac{1}{9}$, applying the reciprocal rule and carrying the minus sign into the value on top of it. The negative index does one job only, inverting the power, and $y = 3^x$ is positive at every x , so the height is $+\frac{1}{9}$. (*Reciprocal taken and the minus kept as well*)
- **F ($\frac{1}{27}$)**: Finds $a = 3$, then evaluates with the run of the chord, $4 - 1 = 3$, in place of the required x , giving $3^{-3} = \frac{1}{27}$. The 3 was used up in forming the gradient equation $a^4 - a = 78$; the point asked for is $x = -2$, so the power needed is $3^2 = 9$. (*Chord run substituted for the required x*)
- **G (-9)**: Interprets 3^{-2} as $-3^2 = -9$. A negative index inverts, it does not negate: $y = 3^x$ is positive for every x , so no point of the curve can lie below the horizontal axis. (*Negative index read as a negative value*)

Common mistake. Treating the chord gradient as though the curve were a straight line and writing $a = 26$, or dividing by the wrong run. The rise is the difference of two powers, $a^4 - a$, and the run is $4 - 1 = 3$, so the base satisfies a quartic condition rather than reading off directly.

Takeaway. The Cheat Code: Any point on $y = a^x$ is (x, a^x) , so geometric statements about the graph become index equations in a . Once the base is known, a negative exponent is only a reciprocal: $a^{-n} = \frac{1}{a^n}$, never a negative height.

Time-saving tip. Speed Heuristic: Write $a^4 - a = 78$ as $a(a^3 - 1) = 78$ and split $78 = 3 \times 26$. Since $26 = 27 - 1 = 3^3 - 1$, the base $a = 3$ falls out with no trial at all, and the answer is then just $\frac{1}{3^2}$.

Question 7

MM1.3 · Algebra and functions · ★★☆☆☆ · 90 s

A horizontal line $y = k$ never meets the curve $y = x^2 - 2x - 7$. Find all possible values of k .

- A. $k < -8$
- B. $k < -6$
- C. $k > -8$
- D. $k > 8$
- E. $k < -7$
- F. $k > -6$
- G. $k < -11$
- H. $k \leq -8$

Answer A. Key idea

A horizontal line $y = k$ crosses a curve exactly where the curve's expression equals k , so "never meets" is the statement that $x^2 - 2x - 7 = k$ has no real solution. There are two equivalent ways to pin that down. Completing the square rewrites an upward parabola as a square plus a constant, and since a square is never negative, that constant is the lowest height the curve ever reaches; heights below it are the ones the curve misses. The discriminant says the same thing formally: rearrange to the form $ax^2 + bx + c = 0$ and demand $b^2 - 4ac < 0$. Both routes must give the same inequality, and using one to check the other is the safest way to get the direction right.

Worked solution**1. Complete the square**

$$y = x^2 - 2x - 7 \quad y = (x - 1)^2 - (-1)^2 - 7 \quad y = (x - 1)^2 - 1 - 7 \quad y = (x - 1)^2 - 8$$

The bracket $(x - 1)^2$ is never negative and equals 0 at $x = 1$, so the lowest point of the curve is $(1, -8)$ and its least y -value is -8 .

2. Turn "never meets" into a condition on the roots

The line $y = k$ meets the curve wherever $x^2 - 2x - 7 = k$, so "never meets" means that equation has no real solution. Written in standard form,

$$x^2 - 2x - (7 + k) = 0$$

so the discriminant is

$$D = (-2)^2 - 4(1)(-(7 + k)) = 4 + 4(7 + k) = 32 + 4k$$

No real roots requires $D < 0$:

$$32 + 4k < 0 \implies 4k < -32 \implies k < -8$$

3. Read it off the graph and check

The completed square gives the same test in one line: $(x - 1)^2 = k + 8$ can be solved only when $k + 8 \geq 0$, so the line misses the curve exactly when $k < -8$.

Sanity check: $k = -9$ lies below the lowest point of the curve, and $D = 32 - 36 = -4 < 0$, so that line misses. $k = -8$ is the height of the lowest point itself, where the line touches at $(1, -8)$, so -8 is correctly excluded by

the strict inequality. $k = 0$ gives $x^2 - 2x - 7 = 0$ with $D = 32 > 0$, two crossings, as expected for a height above the minimum.

Matches Option A.

Why the other options are wrong

- **B** ($k < -6$): A sign slip on the correction term: adding instead of subtracting gives $y = (x - 1)^2 + 1 - 7 = (x - 1)^2 - 6$, a least value of -6 and the range $k < -6$. Substituting $x = 1$ into the original expression settles it, since $1 - 2 - 7 = -8$ and not -6 . (*Sign error completing the square*)
- **C** ($k > -8$): The minimum -8 is found correctly and then the inequality is pointed the wrong way, treating heights above the lowest point as the ones the curve avoids. In fact every $k > -8$ gives $D = 32 + 4k > 0$ and two crossings, so this range describes the lines that cut the curve, not the lines that miss it. (*Inequality reversed*)
- **D** ($k > 8$): Moving k to the wrong side before forming the discriminant: from $x^2 - 2x - 7 + k = 0$ the discriminant becomes $D = 4 - 4(k - 7) = 32 - 4k$, and $32 - 4k < 0$ delivers $k > 8$. Testing $k = 9$ against the curve exposes it, since a line nine units above the axis clearly cuts an upward parabola whose lowest point is at -8 . (*Sign error rearranging before the discriminant*)
- **E** ($k < -7$): Completing the square but never subtracting the square of the halved coefficient: writing $y = (x - 1)^2 - 7$ leaves the constant untouched at -7 , so the least value looks like -7 and the range comes out as $k < -7$. The check kills it: at $k = -7.5$ the discriminant is $32 + 4(-7.5) = 2 > 0$, so that line does cross the curve twice. (*Incomplete completing the square*)
- **F** ($k > -6$): Quotes the discriminant as $b^2 + 4ac$ instead of $b^2 - 4ac$. From $x^2 - 2x - (7 + k) = 0$, with $a = 1$, $b = -2$ and $c = -7 - k$, that gives $4 + 4(-7 - k) = -24 - 4k$, and $-24 - 4k < 0$ rearranges to $k > -6$. The correct form gives $4 + 28 + 4k = 32 + 4k$, and $k = 0$ exposes the slip, since $x^2 - 2x - 7 = 0$ plainly has two roots and so needs a positive discriminant. (*Sign error in the discriminant formula*)
- **G** ($k < -11$): Completes the square with the whole coefficient instead of half of it, writing $y = (x - 2)^2 - 2^2 - 7 = (x - 2)^2 - 11$, so the least value looks like -11 and the range prints as $k < -11$. Halving is what makes the bracket reproduce the $-2x$ term, since $(x - 2)^2$ expands to $x^2 - 4x + 4$ and not $x^2 - 2x + 4$. The discriminant kills it: at $k = -10$, $D = 32 + 4(-10) = -8 < 0$, so that line does miss the curve yet this range excludes it. (*Coefficient of x not halved*)
- **H** ($k \leq -8$): Writes the no-real-roots condition as $D \leq 0$ rather than $D < 0$, so $32 + 4k \leq 0$ gives $k \leq -8$. The equality case is exactly the one that has to go: $k = -8$ makes $D = 0$, a repeated root at $x = 1$, and the line $y = -8$ touches the curve at its vertex $(1, -8)$. Touching is meeting, so -8 itself is not a value the line may take. (*Tangency wrongly included*)

Common mistake. Locating the lowest point correctly at height -8 and then choosing the wrong side of it. Values of k above -8 are heights the curve does reach, and each such line cuts the curve twice; it is the heights below the minimum that are never attained. Reading the answer straight off the vertex without asking which side of it the line has to sit is what turns a correct -8 into a wrong inequality.

Takeaway. The Cheat Code: Once a curve is written as $y = (x - p)^2 + q$, its y -values are exactly $y \geq q$. A horizontal line $y = k$ then cuts it twice when $k > q$, touches it once when $k = q$, and misses it when $k < q$. Sorting the three cases by comparing k with q is faster and safer than forming the discriminant from scratch, and it tells you at once whether the inequality should be strict.

Time-saving tip. Speed Heuristic: You never need the roots, and you never need the full discriminant here. Halve the coefficient of x to get the bracket $(x - 1)^2$, then subtract the square of that half from the constant: $-7 - 1 = -8$ is the vertex height. The answer is everything strictly below it, so scan the options for the one reading $k < -8$.

Question 8

MM1.7 · Algebra and functions · ★★☆☆☆ · 85 s

Each of the five functions below is defined for all real x . Which is one-to-one?

- A. $f(x) = \sin x$
- B. $f(x) = x^4 + x$
- C. $f(x) = |x| - 4$
- D. $f(x) = x^3 - 3x$
- E. $f(x) = x^3 + 2x$
- F. $f(x) = 2^x - x$
- G. $f(x) = 2^{x^2}$
- H. $f(x) = x^2 - 6x$

Answer E. Key idea

A mapping is one-to-one only when no two different inputs share an output, so a single pair of distinct inputs giving equal outputs settles the matter against a function immediately. Proving the positive case takes more care: a function that increases strictly across its whole domain can never repeat an output, because a larger input always produces a larger output.

Worked solution**1. Hunt for a repeated output**

A single pair of distinct inputs with the same output is enough to rule a function out, so test the candidates rather than trying to prove anything first.

Option A: $f(3) = |3| - 4 = -1$ and $f(-3) = |-3| - 4 = -1$.

Option C: $f(0) = 0 - 0 = 0$ and $f(6) = 36 - 36 = 0$.

Option D: $\sin 0^\circ = 0$ and $\sin 180^\circ = 0$.

Option E: $f(1) = 1 - 3 = -2$ and $f(-2) = -8 + 6 = -2$.

Each of those four sends two different inputs to one output, so each is many-to-one.

2. Confirm the survivor is one-to-one

$f(x) = x^3 + 2x$ is the sum of x^3 and $2x$, and each of those grows strictly as x grows. A sum of strictly increasing functions is strictly increasing, so $x_1 < x_2$ forces $f(x_1) < f(x_2)$ and two inputs can never share an output.

Sanity check. Suppose $x_1^3 + 2x_1 = x_2^3 + 2x_2$. Subtracting and factorising gives $(x_1 - x_2)(x_1^2 + x_1x_2 + x_2^2 + 2) = 0$.

The second bracket equals $(x_1 + \frac{1}{2}x_2)^2 + \frac{3}{4}x_2^2 + 2$, which is never smaller than 2, so the first bracket must vanish and $x_1 = x_2$. The mapping is one-to-one.

Matches Option E.

Why the other options are wrong

- **A** ($f(x) = \sin x$): The sine function repeats every 360° , and even inside one period $\sin 0^\circ = 0$ and $\sin 180^\circ = 0$. Infinitely many inputs therefore share each output value. (*Overlooked periodicity*)
- **B** ($f(x) = x^4 + x$): Reasons that x^4 on its own repeats outputs but the added x destroys the symmetry, so the sum must be one-to-one. The fold survives, merely shifted off the axis: $f(0) = 0 + 0 = 0$ and $f(-1) = (-1)^4 + (-1) = 1 - 1 = 0$, two different inputs with the same output. An even leading power forces a minimum, and any graph with a minimum turns back on itself. (*Assumed a linear term removes the fold*)

- **C** ($f(x) = |x| - 4$): The modulus folds the negative half of the number line onto the positive half, so inputs of equal size give identical outputs: $f(3) = 3 - 4 = -1$ and $f(-3) = 3 - 4 = -1$. Two inputs, one output. (*Ignored the reflection built into the modulus*)
- **D** ($f(x) = x^3 - 3x$): Treating every cubic as one-to-one because x^3 is. The $-3x$ term creates two turning points: $f(1) = 1 - 3 = -2$ and $f(-2) = -8 + 6 = -2$, so this graph repeats outputs. (*Over-generalised from $y = x^3$*)
- **F** ($f(x) = 2^x - x$): Takes it that anything built from 2^x inherits its strict increase and therefore cannot repeat an output. Close to the origin the $-x$ term wins: $f(0) = 2^0 - 0 = 1$ and $f(1) = 2^1 - 1 = 1$. One output, two inputs, so the mapping is many-to-one; the graph dips between $x = 0$ and $x = 1$ before the exponential takes over. (*Assumed the exponential forces strict increase*)
- **G** ($f(x) = 2^{x^2}$): Applies the rule that $y = a^x$ is one-to-one without checking what sits in the index. The index here is x^2 , which already sends inputs of equal size to the same number: $f(2) = 2^4 = 16$ and $f(-2) = 2^4 = 16$. The exponential step is one-to-one, but the squaring that feeds it is not, and a many-to-one first stage is enough to make the whole mapping many-to-one. (*Even index inside the exponent overlooked*)
- **H** ($f(x) = x^2 - 6x$): Every quadratic is symmetric about its vertex. Factorising, $x^2 - 6x = x(x - 6)$, so $f(0) = 0$ and $f(6) = 0$: the outputs pair up about $x = 3$. (*Overlooked quadratic symmetry*)

Common mistake. Assuming that every cubic is one-to-one because $y = x^3$ is. A cubic carrying a negative multiple of x doubles back on itself: for $x^3 - 3x$, $f(1) = -2$ and $f(-2) = -2$, so it repeats outputs exactly as the quadratic and the modulus do.

Takeaway. The Cheat Code: One counterexample kills injectivity, so go looking for a repeated output before attempting any proof. Symmetry (a modulus, an even power) and repetition (the trigonometric functions) hand you the pair on sight, and for $x^3 - kx$ with $k > 0$ the pair $f(0) = f(\sqrt{k}) = 0$ ends it in one line.

Time-saving tip. Speed Heuristic: Sort the options by shape before substituting anything. The modulus, the quadratic and the sine all repeat outputs by inspection, which leaves only the two cubics, and the one with the negative x term is the one that turns back. Total work: a single pair of substitutions.

Question 9

MM5.2 · Exponentials and logarithms · ★★☆☆ · 90 s

Let $\log_a 2 = p$ and $\log_a 3 = q$, where $a > 1$ is a fixed base. Express $\log_a \left(\frac{9\sqrt{a}}{8} \right)$ in terms of p and q .

- A. $3q - 2p + \frac{1}{2}$
- B. $2q - 3p + 2$
- C. $q^2 - p^3 + \frac{1}{2}$
- D. $\frac{2q}{3p} + \frac{1}{2}$
- E. $2q + 3p + \frac{1}{2}$
- F. $2q - 3p + \frac{1}{2}$
- G. $2q - 3p + 1$
- H. $3p - 2q - \frac{1}{2}$

Answer F. Key idea

The laws of logarithms convert the arithmetic inside a logarithm into simpler arithmetic outside it: a product becomes a sum, a quotient becomes a difference, and an index becomes a multiplying factor. So

an argument that is built from powers of a few known numbers can always be dismantled into multiples of the logarithms of those numbers. A surd is only a fractional index, and a logarithm of the base itself is 1, which together fix the contribution of any root of the base without needing its value.

Worked solution

1. Split the quotient

The subtraction law turns the division inside the logarithm into a subtraction outside it:

$$\log_a \left(\frac{9\sqrt{a}}{8} \right) = \log_a (9\sqrt{a}) - \log_a 8$$

2. Split the remaining product

$$\log_a (9\sqrt{a}) = \log_a 9 + \log_a \sqrt{a}$$

3. Bring every index down as a factor

$$\log_a 9 = \log_a 3^2 = 2 \log_a 3 = 2q$$

$$\log_a 8 = \log_a 2^3 = 3 \log_a 2 = 3p$$

$$\log_a \sqrt{a} = \log_a a^{1/2} = \frac{1}{2} \log_a a = \frac{1}{2}$$

4. Reassemble

$$\log_a \left(\frac{9\sqrt{a}}{8} \right) = 2q + \frac{1}{2} - 3p = 2q - 3p + \frac{1}{2}$$

Sanity check with a concrete base. Take $a = 2$, so $p = \log_2 2 = 1$ and the expression predicts $2q - 3 + \frac{1}{2} = 2q - \frac{5}{2}$.

Working directly, $\log_2 \left(\frac{9\sqrt{2}}{8} \right) = \log_2 9 + \log_2 \sqrt{2} - \log_2 8 = 2 \log_2 3 + \frac{1}{2} - 3 = 2q - \frac{5}{2}$, the same expression, so the dismantling is consistent.

Matches Option F.

Why the other options are wrong

- **A** ($3q - 2p + \frac{1}{2}$): Recognises the indices 2 and 3 but pairs them with the wrong logarithms, writing $\log_a 9 = 3q$ and $\log_a 8 = 2p$ to give $3q - 2p + \frac{1}{2}$. The multiplier comes from the index of the number itself: $9 = 3^2$ contributes $2q$ and $8 = 2^3$ contributes $3p$. (*Coefficients attached to the wrong letters*)
- **B** ($2q - 3p + 2$): Converts the surd with the index the wrong way up, taking $\log_a \sqrt{a} = 2 \log_a a = 2$ instead of $\frac{1}{2} \log_a a = \frac{1}{2}$, so the total becomes $2q - 3p + 2$. A square root is the index $\frac{1}{2}$: $\sqrt{a} \times \sqrt{a} = a$ forces the two indices to add to 1, so each is one half, and the index 2 belongs to a^2 . (*Root index inverted*)
- **C** ($q^2 - p^3 + \frac{1}{2}$): Raises the logarithm to the index instead of multiplying by it, taking $\log_a 3^2 = (\log_a 3)^2 = q^2$ and $\log_a 2^3 = (\log_a 2)^3 = p^3$, for $q^2 - p^3 + \frac{1}{2}$. The law reads $\log_a x^k = k \log_a x$, so the index becomes a factor in front, not an index on the answer. (*Power law applied to the logarithm*)
- **D** ($\frac{2q}{3p} + \frac{1}{2}$): Handles $\sqrt{a}$ correctly but writes $\log_a \frac{9}{8} = \frac{\log_a 9}{\log_a 8} = \frac{2q}{3p}$. Division inside a logarithm becomes subtraction outside it, so that part is $2q - 3p$; a ratio of two logarithms is a change of base, which is a different statement entirely. (*Quotient inside turned into a quotient outside*)

- **E** ($2q + 3p + \frac{1}{2}$): Applies the product law to a quotient, writing $\log_a \left(\frac{9\sqrt{a}}{8} \right) = \log_a 9 + \log_a \sqrt{a} + \log_a 8 = 2q + \frac{1}{2} + 3p$. Dividing inside a logarithm subtracts outside it, so the 8 contributes $-3p$. Testing $a = 2$, where $p = 1$, this reads $2q + \frac{7}{2}$ against the true $2q - \frac{5}{2}$, an error of 6, which is exactly $2 \log_a 8$. (*Division inside turned into addition*)
- **G** ($2q - 3p + 1$): Treats $\sqrt{a}$ as though it were a itself, so the term contributes $\log_a a = 1$ and the total becomes $2q - 3p + 1$. Since $\sqrt{a} = a^{1/2}$, the index $\frac{1}{2}$ comes down as a factor and the term is worth $\frac{1}{2}$, so this exceeds the correct value by exactly $\frac{1}{2}$. (*Root of the base mishandled*)
- **H** ($3p - 2q - \frac{1}{2}$): Subtracts the numerator's logarithm from the denominator's, writing $\log_a 8 - \log_a (9\sqrt{a}) = 3p - \left(2q + \frac{1}{2} \right) = 3p - 2q - \frac{1}{2}$. That is the logarithm of $\frac{8}{9\sqrt{a}}$, the reciprocal of the argument asked for, so every term has the wrong sign. The quotient law reads top minus bottom. (*Subtraction taken in the wrong order*)

Common mistake. Turning the division inside the logarithm into a division of logarithms, writing $\log_a \frac{9}{8}$ as $\frac{\log_a 9}{\log_a 8}$. The law says the logarithm of a quotient is the difference of the logarithms, so the two terms are subtracted, never divided.

Takeaway. The Cheat Code: Rewrite every number inside the logarithm as a power of something you already have a letter for, then let each index step outside as a multiplier. Roots are just indices too, and any root of the base itself contributes only its fractional index.

Time-saving tip. Speed Heuristic: Deal with $\sqrt{a}$ first, because a root of the base always contributes its index alone, here $\frac{1}{2}$, whatever the base is. That fixes the constant immediately, and the only remaining question is which of 2 and 3 multiplies which letter: $9 = 3^2$ gives $2q$, and $8 = 2^3$ gives $3p$.

Question 10

MM4.5 · Trigonometry · ★★★★★ · 90 s

For an acute angle θ , $\frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} = \frac{25}{12}$. Find the exact value of $\tan \theta$.

- A. $\frac{3}{4}$
- B. $\frac{24}{25}$
- C. $\frac{4}{3}$
- D. $\frac{24}{7}$
- E. $\frac{7}{24}$

Answer D. Key idea

The Pythagorean identity is what converts information about one trigonometric ratio into information about the others, since $\sin^2 \theta + \cos^2 \theta = 1$ ties them together for every angle. That means an expression built from sine and cosine can often be collapsed to a single ratio before any value is extracted, which is

much cheaper than solving for both at once. Once one ratio is pinned down, the quadrant fixes the sign of the other, and $\tan \theta = \frac{\sin \theta}{\cos \theta}$ converts the pair into the tangent that was asked for.

Worked solution

1. Put the two fractions over a common denominator

For an acute angle neither $\sin \theta$ nor $1 + \cos \theta$ is zero, so the sum may be combined:

$$\frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} = \frac{\sin^2 \theta + (1 + \cos \theta)^2}{\sin \theta (1 + \cos \theta)}$$

2. Collapse the numerator with the Pythagorean identity

Expanding the square and using $\sin^2 \theta + \cos^2 \theta = 1$:

$$\sin^2 \theta + 1 + 2 \cos \theta + \cos^2 \theta = 1 + 1 + 2 \cos \theta = 2(1 + \cos \theta)$$

3. Cancel and read off the sine

The factor $1 + \cos \theta$ appears above and below, so the whole left side is

$$\frac{2(1 + \cos \theta)}{\sin \theta (1 + \cos \theta)} = \frac{2}{\sin \theta}$$

Hence $\frac{2}{\sin \theta} = \frac{25}{12}$, giving $\sin \theta = \frac{24}{25}$.

4. Recover the cosine, then the tangent

$$\cos^2 \theta = 1 - \left(\frac{24}{25}\right)^2 = \frac{625 - 576}{625} = \frac{49}{625}$$

The angle is acute, so $\cos \theta = +\frac{7}{25}$ and

$$\tan \theta = \frac{24/25}{7/25} = \frac{24}{7}$$

Sanity check: with these values the first fraction is $\frac{24/25}{32/25} = \frac{3}{4}$ and the second is its reciprocal $\frac{4}{3}$, and $\frac{3}{4} + \frac{4}{3} = \frac{9 + 16}{12} = \frac{25}{12}$, the value given. The triple 7, 24, 25 also satisfies $49 + 576 = 625$, so the identity holds exactly.

Matches Option D.

Why the other options are wrong

- **A** ($\frac{3}{4}$): Evaluates the first fraction once $\sin \theta = \frac{24}{25}$ and $\cos \theta = \frac{7}{25}$ are known: $\frac{24/25}{1 + 7/25} = \frac{24/25}{32/25} = \frac{3}{4}$, and reports that. It is a genuine value in the problem, but it is $\frac{\sin \theta}{1 + \cos \theta}$, not $\frac{\sin \theta}{\cos \theta}$. (*Intermediate fraction quoted as the answer*)
- **B** ($\frac{24}{25}$): Solves $\frac{2}{\sin \theta} = \frac{25}{12}$ to get $\sin \theta = \frac{24}{25}$ and stops there. The sine is only the halfway point: the cosine still has to come from $\cos^2 \theta = 1 - \frac{576}{625} = \frac{49}{625}$ before the ratio $\frac{24}{7}$ can be formed. (*Answers the wrong quantity*)

- **C** ($\frac{4}{3}$): Uses $1 - \cos \theta$ in place of $\cos \theta$: $\frac{24/25}{1 - 7/25} = \frac{24/25}{18/25} = \frac{4}{3}$. The tangent divides by the cosine itself, $\frac{7}{25}$, which gives $\frac{24}{7}$. (*Wrong denominator used for the tangent*)
- **E** ($\frac{7}{24}$): Reads the collapsed left side as $\frac{2}{\cos \theta}$, so $\cos \theta = \frac{24}{25}$ and $\sin \theta = \frac{7}{25}$, giving $\tan \theta = \frac{7}{24}$. The factor that cancels is $1 + \cos \theta$, leaving $\sin \theta$ alone in the denominator, so it is the sine that equals $\frac{24}{25}$. (*Sine and cosine interchanged*)

Common mistake. Expanding $(1 + \cos \theta)^2$ as $1 + \cos^2 \theta$ and losing the cross term $2 \cos \theta$. Without it the numerator becomes the constant 2, nothing cancels, and the equation turns into a messy one in two unknowns instead of collapsing to $\frac{2}{\sin \theta}$.

Takeaway. The Cheat Code: When a sine and cosine expression is a fraction plus its own reciprocal, combine before you substitute anything. The cross term from the square is exactly what $\sin^2 \theta + \cos^2 \theta = 1$ needs to produce a repeated factor, and that factor cancels the whole problem down to one ratio.

Time-saving tip. Speed Heuristic: The second fraction is the reciprocal of the first, and combining always leaves $\frac{2}{\sin \theta}$, so $\sin \theta = \frac{24}{25}$ can be written down in one line. Then recognise 7, 24, 25 as a Pythagorean triple rather than taking a square root, and the tangent is immediate.

END OF SOLUTIONS

Questions 11 to 27 are in the full paper

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EXAMPLE GRAMMAR SCHOOL

ESAT Mock Examination

Physics

Paper SMP-2026-09 · Version 1.0

NAME

TEACHING GROUP

DATE

Instructions

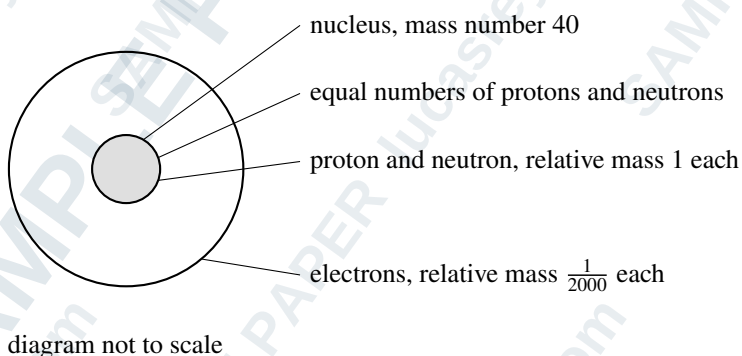
- Time allowed: **15 minutes**.
- There are **10 multiple-choice questions**. Each is worth 1 mark.
- There is no negative marking. Answer every question, even if you have to guess.
- Calculators are not permitted.
- Record your answers on the separate answer sheet. Only the answer sheet is marked.
- Formulae are not provided. The ESAT does not supply a formula sheet.

Note: the real ESAT is sat on screen at a Pearson test centre. This paper trains the mathematics and the pacing, not the on-screen interface.

Written for Example Grammar School by Lucas Rey, Engineering Science, University of Oxford; MRes, University of Cambridge.

Question 1

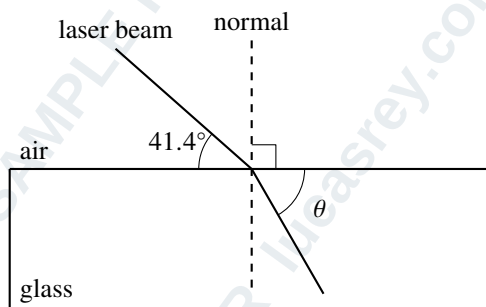
A textbook claims that virtually all of an atom's mass sits in its tiny central nucleus, and a student decides to check that claim numerically for the single neutral atom shown in the diagram. Treat the atom's mass as the sum of the masses of its individual particles. To a good approximation, what fraction of the atom's total mass is carried by its electrons?



- A. $\frac{1}{2000}$
- B. $\frac{1}{8000}$
- C. $\frac{1}{6000}$
- D. $\frac{1}{400}$
- E. $\frac{1}{100}$
- F. $\frac{1}{4000}$
- G. $\frac{1}{40000}$
- H. $\frac{1}{80000}$

Question 2

An optical technician sets a rectangular crown glass block on a bench with its top face horizontal. The block is known to bend a beam arriving at 30° to the normal into a path 19.5° from the normal inside the glass. She now aims her laser at the same face as shown, and the refracted beam inside the glass makes an angle θ with the face. Take $\sin 19.5^\circ = 0.33$ and $\sin 48.6^\circ = 0.75$. What is θ ?



- A. 70.5°
- B. 60.0°
- C. 57.6°
- D. 41.4°
- E. 30.0°

Question 3

A soft drink is made by stirring sugar syrup of density 1200 kg m^{-3} into water of density 1000 kg m^{-3} . A sealed carton holds 1.05 kg of the finished drink, and the drink's density is 1050 kg m^{-3} . Assume the two liquids mix without any change in total volume and that the carton itself has negligible mass. What volume of syrup does the carton contain?

- A. 350 cm^3
- B. 250 cm^3
- C. 750 cm^3
- D. 1000 cm^3
- E. 300 cm^3
- F. 500 cm^3
- G. 50 cm^3
- H. 875 cm^3

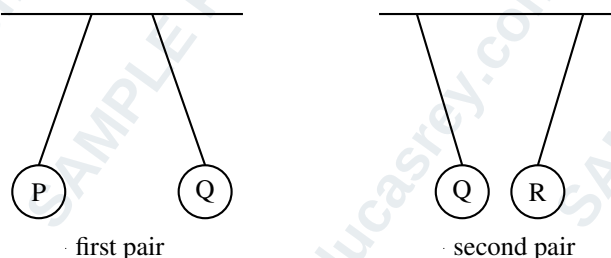
Question 4

An object of mass 5 kg is dropped from rest from a height of 45 m above the ground. Assuming negligible air resistance and $g = 10 \text{ N kg}^{-1}$, what is its speed just before hitting the ground?

- A. 15 m s^{-1}
- B. 150 m s^{-1}
- C. 21 m s^{-1}
- D. 3 m s^{-1}
- E. 900 m s^{-1}
- F. 90 m s^{-1}
- G. 450 m s^{-1}
- H. 30 m s^{-1}

Question 5

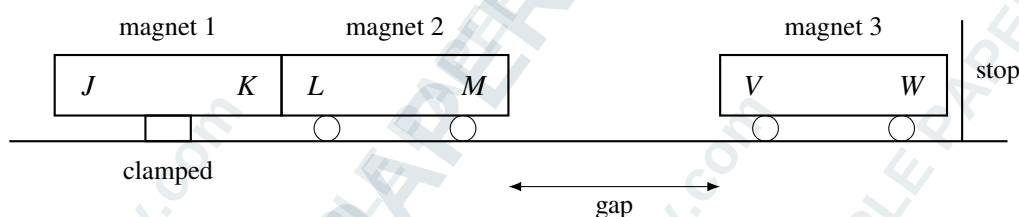
Three light insulating spheres hang on nylon threads in a dry laboratory. Sphere Q was charged by stroking it with a silk cloth, and the cloth was afterwards found to be negatively charged. Spheres P and R had each been given a charge of their own beforehand. The spheres are then brought close in pairs, and they hang as shown. Assume no charge leaks away to the air or along the threads during the tests. Which statement about sphere R is correct?



- A. R is positively charged, having lost electrons when it was charged.
- B. R is negatively charged, having gained electrons when it was charged.
- C. R is negatively charged, having lost electrons when it was charged.
- D. R is positively charged, having gained electrons when it was charged.
- E. R is uncharged, and is pulled towards Q only because Q is charged.

Question 6

Three identical bar magnets sit on low-friction trolleys on a straight horizontal rail, as shown. The left-hand magnet is clamped, and the other two are free to slide. When they are released, magnets 1 and 2 close up until they touch, while magnet 3 runs away from magnet 2 and rests against an end stop. A compass test shows that end J is a north pole. Which two lettered ends are both north poles?



- A. K and M
- B. J and M
- C. L and V
- D. J and L
- E. J and K

Question 7

A wire of length 0.1 m carries a current of 3 A perpendicular to a magnetic field of flux density 1.2 T. Find the force on the wire.

- A. 0.25 N
- B. 4.30 N
- C. 0.51 N
- D. 0.36 N
- E. 0.12 N
- F. 3.60 N
- G. 0.30 N
- H. 36.00 N

Question 8

A polymer belt runs over a rubber drum, both insulators and both initially uncharged. Friction leaves the belt carrying -7.2 nC . With the electron charge $1.6 \times 10^{-19} \text{ C}$, how many electrons has the drum lost?

- A. 7.2×10^{10}
- B. 4.5×10^{13}
- C. 9.0×10^{10}
- D. 4.5×10^{19}
- E. 4.5×10^{10}
- F. 2.2×10^{-11}
- G. 2.25×10^{10}
- H. 4.5×10^{28}

Question 9

A camping stove's electric plate draws 1200 W from a generator, and 360 W of this is lost to the pan and the surroundings. The pan holds 2.0 litres of water at 20°C . Water has density 1000 kg m^{-3} and specific heat capacity $4200 \text{ J kg}^{-1} \text{ K}^{-1}$. Assume the plate reaches its steady output immediately and the loss stays constant. How long does the water take to reach 80°C ?

- A. 600 s
- B. 1000 s
- C. 200 s
- D. 800 s
- E. 1050 s
- F. 294 s
- G. 1400 s
- H. 420 s

Question 10

A paraffin lamp burns inside a tall glass chimney that is open at the top and pierced by a ring of small holes around its base; these openings are the only ways in or out. The lamp stands in a draught-free room whose air stays at a steady 20°C , while the flame keeps the air inside the chimney much hotter. Which statement correctly describes how air moves through the chimney, and why?

- A.** Air enters at the top and leaves through the base holes, because the cooler, denser room air sinks into the chimney and pushes the warm air out below it.
- B.** Air enters at the top and leaves through the base holes, because warming the air raises its density, so the hot air sinks and spills out through the holes.
- C.** No air passes through either opening, because the difference in density between the hot and the cool air holds the two bodies of air apart.
- D.** Air enters through the base holes and leaves at the top, because warming the air lowers its density, so the denser room air displaces it upwards.
- E.** Air enters through the base holes and leaves at the top, because hot gas always rises, whatever the density of the air surrounding it.

END OF PAPER

Questions 11 to 27 are in the full paper

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EXAMPLE GRAMMAR SCHOOL

ESAT Mock: Answer Sheet

Physics

Paper SMP-2026-09 • Version 1.0

NAME

TEACHING GROUP

DATE

Fill in one bubble per question. If you change your mind, cross out the old mark clearly and fill the new bubble. Unfilled or doubly filled rows score zero.

Q	A	B	C	D	E	F	G	H
1	☐	☐	☐	☐	☐	☐	☐	☐
2	☐	☐	☐	☐	☐			
3	☐	☐	☐	☐	☐	☐	☐	☐
4	☐	☐	☐	☐	☐	☐	☐	☐
5	☐	☐	☐	☐	☐			
6	☐	☐	☐	☐	☐			
7	☐	☐	☐	☐	☐	☐	☐	☐
8	☐	☐	☐	☐	☐	☐	☐	☐
9	☐	☐	☐	☐	☐	☐	☐	☐
10	☐	☐	☐	☐	☐			

MARKER INITIALS

TOTAL OUT OF 10

EXAMPLE GRAMMAR SCHOOL

ESAT Mock: Mark Scheme

Physics

Paper SMP-2026-09 • Version 1.0

Answer grid

Q	Ans	Q	Ans	Q	Ans
1	F	10	D		
2	B				
3	B				
4	H				
5	B				
6	D				
7	D				
8	E				
9	A				

Totals

10 marks available, 1 mark per question. There is no negative marking, so a wrong answer scores zero and never a negative. Only the answer sheet is marked.

Diagnostic table

Use this to see which topics the cohort is losing marks on.

Q	Ans	Node	Group	Topic	Difficulty
1	F	P7.1	P7	Radioactivity	★★★★☆
2	B	P6.3	P6	Waves	★★★★☆
3	B	P5.4	P5	Matter	★★★★☆
4	H	P3.7	P3	Energy	★★★☆☆
5	B	P1.1	P1	Electricity	★★★☆☆
6	D	P2.1	P2	Magnetism and electromagnetism	★★★☆☆
7	D	P2.3	P2	The motor effect	★★★☆☆
8	E	P1.1	P1	Electricity	★★★☆☆
9	A	P4.4	P4	Thermal physics	★★★★☆
10	D	P4.2	P4	Thermal physics	★★★★☆

Marks available by topic group

Group	Topic	Marks
P1	Electricity	2
P2	Magnetism and electromagnetism	2
P3	Mechanics	1
P4	Thermal physics	2
P5	Matter	1
P6	Waves	1
P7	Radioactivity	1
Total		10

Worked solutions for every question are in the separate worked solutions document.

EXAMPLE GRAMMAR SCHOOL

ESAT Mock: Worked Solutions

Physics

Paper SMP-2026-09 • Version 1.0

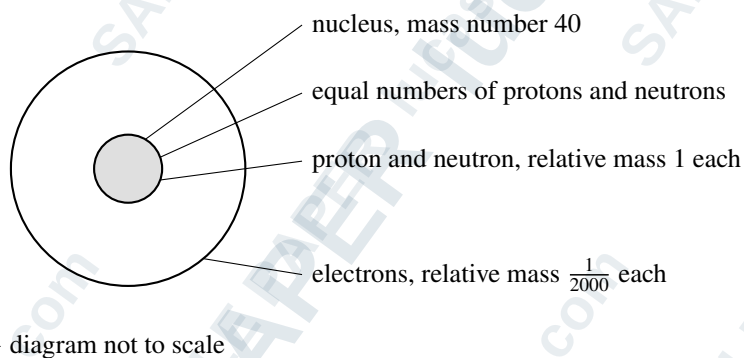
Staff copy

This document contains the answers. Keep it away from candidates until the paper has been sat. Marking guidance is in the separate mark scheme.

Question 1

P7.1 • Radioactivity • ★★★☆☆ • 90 s

A textbook claims that virtually all of an atom's mass sits in its tiny central nucleus, and a student decides to check that claim numerically for the single neutral atom shown in the diagram. Treat the atom's mass as the sum of the masses of its individual particles. To a good approximation, what fraction of the atom's total mass is carried by its electrons?



- A. $\frac{1}{2000}$
- B. $\frac{1}{8000}$
- C. $\frac{1}{6000}$
- D. $\frac{1}{400}$
- E. $\frac{1}{100}$
- F. $\frac{1}{4000}$
- G. $\frac{1}{40000}$
- H. $\frac{1}{80000}$

Answer F. Key idea

On the relative mass scale a proton and a neutron each count as 1, so the mass number is also the relative mass of the nucleus. An electron is lighter than either by a factor of about two thousand, so although a neutral atom has as many electrons as protons, those electrons contribute a share of the mass smaller by that same factor. Comparing the two totals means counting the particles of each type first, weighting each count by its relative mass, and only then forming the ratio.

Worked solution

1. Count the three kinds of particle

The mass number counts nucleons only, and the protons and neutrons are equal in number, so

$$p = n = \frac{40}{2} = 20$$

The atom is neutral, so it carries one electron for every proton, giving 20 electrons.

2. Add up the mass held in the nucleus

A proton and a neutron each have relative mass 1, so the 40 nucleons contribute

$$20 \times 1 + 20 \times 1 = 40$$

3. Add up the mass held by the electrons

Each electron has relative mass $\frac{1}{2000}$, so the twenty of them contribute

$$20 \times \frac{1}{2000} = \frac{1}{100} = 0.01$$

4. Form the fraction and check it

$$\frac{0.01}{40 + 0.01} = \frac{0.01}{40.01} = \frac{1}{4001} \approx \frac{1}{4000}$$

Sanity check: the nucleons outnumber the electrons only two to one, while each electron is two thousand times lighter than a nucleon, so the answer should be about $\frac{1}{2} \times \frac{1}{2000}$. Whether the 0.01 is kept in the denominator or dropped makes no difference at this precision, which is the point of the claim being tested: the electrons are a rounding error in the mass of an atom.

Matches Option F.

Why the other options are wrong

- **A** ($\frac{1}{2000}$): The neutrons were left out of the atom's mass, so the nucleus was given a relative mass of 20 rather than 40, and the fraction came out as $\frac{0.01}{20} = \frac{1}{2000}$. Neutrons carry no charge but they carry the same relative mass as protons, and here they supply half the 40 mass units. (*Neutron mass omitted from the nucleus*)
- **B** ($\frac{1}{8000}$): Halved 40 to get 20 protons and 20 neutrons, then halved again on the way to the electrons and used 10 of them: $10 \times \frac{1}{2000} = \frac{1}{200}$, and $\frac{1}{200} \div 40 = \frac{1}{8000}$. A neutral atom carries one electron for every proton, and there are 20 protons here, so the electron count is 20. (*Mass number halved a second time*)
- **C** ($\frac{1}{6000}$): The electron mass $\frac{1}{100}$ was divided by the number of particles in the atom, $20 + 20 + 20 = 60$, giving $\frac{1}{6000}$. The denominator must be a relative mass, not a head count: the twenty electrons add only 0.01 to it, not twenty units. (*Particle count used in place of relative mass*)
- **D** ($\frac{1}{400}$): Reached $\frac{0.01}{40}$ correctly and then divided by 4 rather than by 40, which is the same as placing the decimal point one column late: $0.01 \div 40 = 0.00025 = \frac{1}{4000}$, whereas $0.0025 = \frac{1}{400}$. The atom's relative mass is 40, not 4. (*Decimal place lost in the final division*)
- **E** ($\frac{1}{100}$): The total relative mass of the electrons, $20 \times \frac{1}{2000} = \frac{1}{100}$, was quoted as the answer without ever being divided by the mass of the whole atom. That figure is a mass on the relative scale, not a share of the atom's mass, and dividing it by 40 is the step that turns it into one. (*Electron mass not expressed as a fraction*)
- **G** ($\frac{1}{40000}$): Cancelled $20 \times \frac{1}{2000}$ to $\frac{1}{100}$ instead of $\frac{1}{1000}$, losing a factor of ten, then divided by the atom's relative mass: $\frac{1}{100} \div 40 = \frac{1}{4000}$. Since $2000 \div 20 = 100$, the twenty electrons together weigh $\frac{1}{100}$. (*Cancelling slip in the electron total*)

- **H** ($\frac{1}{80000}$): Only a single electron was weighed, giving $\frac{1}{2000} \div 40 = \frac{1}{80000}$. A neutral atom holds one electron for every proton, so there are twenty of them here and the numerator must be twenty times as large. (*One electron used instead of all twenty*)

Common mistake. Building the atom's mass out of the protons alone, dividing 0.01 by 20 and answering $\frac{1}{2000}$. The neutron has no charge, which makes it easy to forget, but on the relative mass scale it weighs the same as a proton, so half of the 40 mass units in this atom are neutrons. Charge and mass are separate properties and the neutron is only absent from one of them.

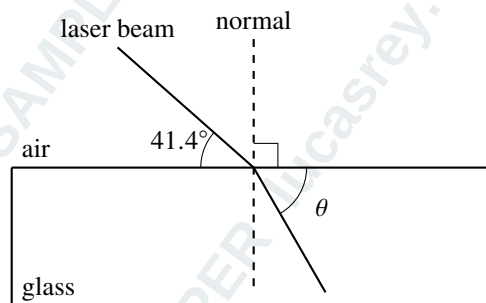
Takeaway. The Cheat Code: weight before you count. Write down how many protons, neutrons and electrons there are, multiply each count by its relative mass (1, 1 and $\frac{1}{2000}$), and let the mass number stand in for the whole nucleus rather than adding nucleons one at a time.

Time-saving tip. Speed Heuristic: with equal protons and neutrons the electron count is half the mass number, so the electron mass is $\frac{A}{2} \times \frac{1}{2000} = \frac{A}{4000}$ against a nuclear mass of A . The A cancels and the answer is $\frac{1}{4000}$ whatever the mass number is, so you never need the 40 at all.

Question 2

P6.3 · Waves · ★★☆☆ · 90 s

An optical technician sets a rectangular crown glass block on a bench with its top face horizontal. The block is known to bend a beam arriving at 30° to the normal into a path 19.5° from the normal inside the glass. She now aims her laser at the same face as shown, and the refracted beam inside the glass makes an angle θ with the face. Take $\sin 19.5^\circ = 0.33$ and $\sin 48.6^\circ = 0.75$. What is θ ?



- A. 70.5°
- B. 60.0°
- C. 57.6°
- D. 41.4°
- E. 30.0°

Answer B. Key idea

Refraction at a plane boundary obeys $n = \frac{\sin i}{\sin r}$, in which both angles are measured from the normal, the line drawn perpendicular to the surface. One measured pair of angles fixes n for that boundary, and the same n then sets the refracted direction for any other angle of incidence at that boundary. An angle given from the surface has to be turned into an angle from the normal by subtracting it from 90° , and turned back the same way when the answer is wanted from the surface.

Worked solution**Step 1: Read the angle of incidence from the diagram**

The beam is marked at 41.4° to the face, but refraction is worked from the normal, drawn perpendicular to that face. The angle of incidence is therefore $i = 90^\circ - 41.4^\circ = 48.6^\circ$.

Step 2: Get the refractive index from the check ray

The check ray crosses the same face at 30° to the normal and runs at 19.5° to the normal inside the glass, so $n = \frac{\sin 30^\circ}{\sin 19.5^\circ} = \frac{0.50}{0.33} = 1.5$ to two significant figures, a value typical of crown glass.

Step 3: Refract the laser beam

The same n governs every ray crossing this boundary, so $\sin r = \frac{\sin i}{n} = \frac{0.75}{1.5} = 0.50$, giving $r = 30^\circ$ measured from the normal.

Step 4: Convert to the angle asked for

θ is the angle to the face, not to the normal, so $\theta = 90^\circ - 30^\circ = 60.0^\circ$.

Sanity check: the beam arrives 48.6° from the normal and continues at 30° from it, so it has bent towards the normal on entering the glass, which is the direction expected for light passing from air into a denser medium. It is bent through 18.6° against the check ray's 10.5° , and the larger bend belongs to the larger angle of incidence.

Matches Option B.

Why the other options are wrong

- **A** (70.5°): This carries the check ray's path over to the laser beam, keeping $r = 19.5^\circ$ from the normal and converting it to the face: $90^\circ - 19.5^\circ = 70.5^\circ$. The refracted angle is not a fixed property of the glass; only n is, and here the beam arrives at 48.6° rather than 30° . (*Conceptual Error*)
- **C** (57.6°): This divides the angle instead of its sine: $48.6^\circ \div 1.5 = 32.4^\circ$, then $90^\circ - 32.4^\circ = 57.6^\circ$. Snell's law relates the sines, so the division to make is $\sin r = 0.75 \div 1.5$. (*Misapplied Formula*)
- **D** (41.4°): This treats the boundary as a mirror and applies angle of incidence = angle of reflection to the beam that crosses it, leaving the angle to the face unchanged at $90^\circ - 48.6^\circ = 41.4^\circ$. Equal angles hold for the part of the light reflected back into the air, not for the part refracted into the glass. (*Conceptual Error*)
- **E** (30.0°): This is r itself, measured from the normal: $\sin r = 0.75 \div 1.5 = 0.50$ gives $r = 30^\circ$. The diagram marks θ between the refracted beam and the face, which is $90^\circ - 30^\circ$. (*Misread the Question*)

Common mistake. Stopping at the angle measured from the normal. Snell's law returns $r = 30^\circ$, but the diagram marks θ between the refracted beam and the face, so the final $90^\circ - 30^\circ$ is still owed.

Takeaway. The Cheat Code: Every angle in $n = \sin i / \sin r$ is measured from the normal. When a ray diagram marks an angle to the surface instead, subtract it from 90° on the way in, and subtract from 90° again on the way out whenever the answer is wanted from the surface.

Time-saving tip. Speed Heuristic: Stay in sines and never write n as a decimal. Here $0.50 \div 0.33$ sits close to $3/2$, and 0.75 is exactly $3/2 \times 0.50$, so the refracted ray is at 30° to the normal and only $90 - 30$ is left to do.

Question 3

P5.4 · Matter · ★★☆☆ · 90 s

A soft drink is made by stirring sugar syrup of density 1200 kg m^{-3} into water of density 1000 kg m^{-3} . A sealed carton holds 1.05 kg of the finished drink, and the drink's density is 1050 kg m^{-3} . Assume the two liquids mix without any change in total volume and that the carton itself has negligible mass. What volume of syrup does the carton contain?

- A. 350 cm^3
- B. 250 cm^3
- C. 750 cm^3
- D. 1000 cm^3
- E. 300 cm^3
- F. 500 cm^3
- G. 50 cm^3
- H. 875 cm^3

Answer B. Key idea

Density is a mass divided by a volume, so a mixture can only be handled through its total mass and its total volume: the two ingredient densities cannot be combined directly. When the volumes simply add, every cubic centimetre of the mixture belongs to one liquid or the other, and the total mass is the sum of what each contributes. Comparing the mixture's mass with the mass the same volume of the lighter liquid alone would have shows how much extra mass the denser liquid brought, and dividing that surplus by the difference in densities gives its volume.

Worked solution

1. Volume of drink in the carton

$$\rho = \frac{m}{V}, \text{ so } V = \frac{m}{\rho} = \frac{1.05}{1050} = 1.0 \times 10^{-3} \text{ m}^3 = 1000 \text{ cm}^3.$$

2. Move to convenient units

$1200 \text{ kg m}^{-3} = 1.2 \text{ g cm}^{-3}$, $1000 \text{ kg m}^{-3} = 1.0 \text{ g cm}^{-3}$, and the drink is 1.05 g cm^{-3} . The carton therefore holds $1000 \times 1.05 = 1050 \text{ g}$ of drink.

3. Account for the extra mass

If the carton held nothing but water it would hold 1000 g . The syrup is responsible for the surplus of 50 g , and each 1 cm^3 of syrup that takes the place of 1 cm^3 of water adds $1.2 - 1.0 = 0.2 \text{ g}$. Hence

$$V_{\text{syrup}} = \frac{50}{0.2} = 250 \text{ cm}^3.$$

4. Rebuild the mixture as a check

250 cm^3 of syrup and 750 cm^3 of water have mass $250 \times 1.2 + 750 \times 1.0 = 300 + 750 = 1050 \text{ g}$ in 1000 cm^3 , a density of 1.05 g cm^{-3} as stated. Notice that 1050 sits a quarter of the way from 1000 to 1200 , and the syrup is a quarter of the volume.

Matches Option **B**.

Why the other options are wrong

- **A** (350 cm^3): Read the 1.05 kg as the mass of the water, giving 1050 cm^3 of it, then solved $\frac{1050 + 1.2V}{1050 + V} = 1.05$ for the syrup: $1050 + 1.2V = 1102.5 + 1.05V$, so $0.15V = 52.5$ and $V = 350 \text{ cm}^3$. The stated mass belongs to the finished drink, which fixes the carton at 1000 cm^3 before any splitting. (*Stated mass assigned to the water*)
- **C** (750 cm^3): Found the syrup correctly as 250 cm^3 and then quoted what was left over, $1000 - 250 = 750 \text{ cm}^3$, which is the volume of water rather than of syrup. (*Answered the complement*)
- **D** (1000 cm^3): Divided the 50 g surplus by $1.05 - 1.00 = 0.05 \text{ g cm}^{-3}$ instead of by $1.20 - 1.00 = 0.20 \text{ g cm}^{-3}$: $\frac{50}{0.05} = 1000 \text{ cm}^3$. The difference that matters is the one between the two liquids, since that is what each cubic centimetre of syrup gains over the water it displaces, and 1000 cm^3 is the whole carton with no water left in it. (*Wrong density difference used*)
- **E** (300 cm^3): Found the mass of syrup, $250 \times 1.2 = 300 \text{ g}$, and read that number straight off as a volume in cm^3 . That step is only valid for a liquid of density 1.0 g cm^{-3} , which the syrup is not. (*Mass read as a volume*)
- **F** (500 cm^3): Split the carton in half on the assumption that a mixture always lies midway between its ingredients. Half of each would give $\frac{1200 + 1000}{2} = 1100 \text{ kg m}^{-3}$, which is not the density the carton is stated to have. (*Assumed equal volumes*)
- **G** (50 cm^3): Divided the surplus 50 g by water's 1.0 g cm^{-3} to get 50 cm^3 , instead of by the 0.2 g cm^{-3} that each cubic centimetre of syrup actually adds when it displaces water. (*Wrong density used*)
- **H** (875 cm^3): Read the 1.05 kg as the mass of the syrup rather than of the finished drink, and converted it with the syrup's density: $\frac{1050}{1.2} = 875 \text{ cm}^3$. The carton holds 1050 g of drink altogether, of which only 300 g is syrup. (*Stated mass assigned to the syrup*)

Common mistake. Assuming a mixture always sits halfway between its ingredients and splitting the carton equally. Equal volumes would give $\frac{1200 + 1000}{2} = 1100 \text{ kg m}^{-3}$, not the 1050 kg m^{-3} stated: the drink is only a quarter of the way up from water, so only a quarter of it is syrup.

Takeaway. The Cheat Code: when volumes add, the mixture's density lands between the two ingredient densities in exactly the proportion of the volumes. Measure how far along the gap from the lighter liquid to the denser one the mixture sits, and that fraction is the fraction of the volume the denser liquid occupies.

Time-saving tip. Speed Heuristic: switch to g cm^{-3} and read the position on a scale rather than solving an equation. The gap from 1.00 to 1.20 is 0.20, the drink sits 0.05 along it, and $\frac{0.05}{0.20} = \frac{1}{4}$ of 1000 cm^3 is 250 cm^3 .

Question 4

P3.7 · Energy · ★★☆☆☆ · 25 s

An object of mass 5 kg is dropped from rest from a height of 45 m above the ground. Assuming negligible air resistance and $g = 10 \text{ N kg}^{-1}$, what is its speed just before hitting the ground?

- A. 15 m s^{-1}
- B. 150 m s^{-1}
- C. 21 m s^{-1}
- D. 3 m s^{-1}
- E. 900 m s^{-1}
- F. 90 m s^{-1}
- G. 450 m s^{-1}
- H. 30 m s^{-1}

Answer H. Key idea

Assuming no energy is lost to heat/friction, the total mechanical energy is conserved. Initial Gravitational Potential Energy (mgh) converts perfectly into Kinetic Energy ($\frac{1}{2}mv^2$) or Elastic Potential Energy.

Worked solution

1. Apply conservation of energy

$$\text{Initial GPE} = \text{Final KE } mgh = \frac{1}{2}mv^2$$

2. Cancel mass and solve for v

$$gh = \frac{1}{2}v^2 \implies v^2 = 2gh \implies v = \sqrt{2gh}$$

3. Substitute values

$$v = \sqrt{2 \times 10 \times 45} = \sqrt{900} = 30 \text{ m s}^{-1}$$

Matches Option **H**.

Why the other options are wrong

- **A** (15 m s^{-1}): Moved the half across the energy equation as a halving instead of a doubling: from $mgh = \frac{1}{2}mv^2$ wrote $v^2 = \frac{1}{2}gh = \frac{1}{2} \times 10 \times 45 = 225$, so $v = \sqrt{225} = 15$. The half multiplies v^2 , so it crosses over as $v^2 = 2gh = 900$ and $v = 30$. The same value comes from quoting the average speed of the fall, $h \div t = 45 \div 3 = 15$, which is half the final speed for a body starting from rest. (*Factor of two on the wrong side*)
- **B** (150 m s^{-1}): Arises from a common inversion, algebraic sign error, or omitting an intermediate conversion factor. (*Calculation / Conceptual Error*)
- **C** (21 m s^{-1}): Arises from a common inversion, algebraic sign error, or omitting an intermediate conversion factor. (*Calculation / Conceptual Error*)

- **D** (3 m s^{-1}): Stopped one line early and reported the time of flight as though it were the speed: $45 = \frac{1}{2} \times 10 \times t^2$ gives $t^2 = 9$ and $t = 3$, which is the number of seconds the fall takes. The speed still has to be found from $v = gt = 10 \times 3 = 30$. (*Fall time quoted as the speed*)
- **E** (900 m s^{-1}): Arises from a common inversion, algebraic sign error, or omitting an intermediate conversion factor. (*Calculation / Conceptual Error*)
- **F** (90 m s^{-1}): Took the fall time from $h = \frac{1}{2}gt^2$ but never rooted it: wrote $t = 2h/g = 90 \div 10 = 9 \text{ s}$ instead of $t = \sqrt{2h/g} = \sqrt{9} = 3 \text{ s}$, then $v = gt = 10 \times 9 = 90$. With the correct $t = 3 \text{ s}$ the speed is $10 \times 3 = 30$. (*Square root dropped when finding the fall time*)
- **G** (450 m s^{-1}): Arises from a common inversion, algebraic sign error, or omitting an intermediate conversion factor. (*Calculation / Conceptual Error*)

Common mistake. Multiplying by the mass or forgetting to square root $2gh$.

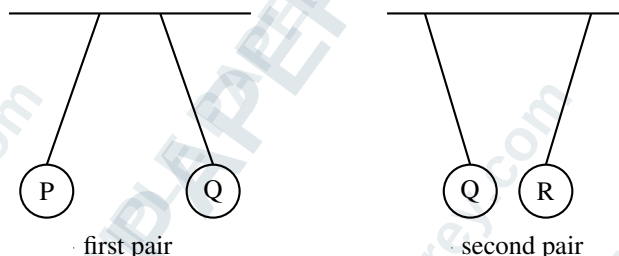
Takeaway. The Cheat Code: In GPE to KE conversions, the mass (m) always cancels out ($mgh = \frac{1}{2}mv^2 \rightarrow v = \sqrt{2gh}$). If mass is given in the prompt, it's a distractor designed to waste your calculation time. Ignore it.

Time-saving tip. Speed Heuristic: Look at the relative magnitudes and units in the multiple-choice options to eliminate extreme or impossible values immediately.

Question 5

P1.1 · Electricity · ★★☆☆☆ · 85 s

Three light insulating spheres hang on nylon threads in a dry laboratory. Sphere Q was charged by stroking it with a silk cloth, and the cloth was afterwards found to be negatively charged. Spheres P and R had each been given a charge of their own beforehand. The spheres are then brought close in pairs, and they hang as shown. Assume no charge leaks away to the air or along the threads during the tests. Which statement about sphere R is correct?



- A. R is positively charged, having lost electrons when it was charged.
- B. R is negatively charged, having gained electrons when it was charged.
- C. R is negatively charged, having lost electrons when it was charged.
- D. R is positively charged, having gained electrons when it was charged.
- E. R is uncharged, and is pulled towards Q only because Q is charged.

Answer B. Key idea

Rubbing two insulators together can move electrons and nothing else, so the material that ends up holding the extra electrons is the negative one and its partner is left positive. Once spheres carry charge, the way they hang shows their signs, because like charges repel and unlike charges attract.

Worked solution

1. Fix the sign of sphere Q

Charging by rubbing moves electrons only. The silk cloth finished negatively charged, so the silk gained electrons, and those electrons came off sphere Q. Sphere Q is therefore short of electrons and carries a positive charge.

2. Read the second pair, then count electrons

In the second pair the threads lean inwards, so Q and R are pulling towards each other: this is attraction. Both spheres carry a charge, and unlike charges attract, so R must carry the opposite sign to Q. Q is positive, so R is negative, and a negative sphere is one holding more electrons than protons. R gained electrons when it was charged.

Sanity check: the first pair hangs apart, which is repulsion, so P carries the same sign as Q, positive. That makes the three spheres positive, positive and negative, and exactly one of the two pairs tested holds unlike charges, which is the one drawn leaning together.

Matches Option B.

Why the other options are wrong

- **A** (R is positively charged, having lost electrons when it was charged.): Treats the inward lean of Q and R as evidence that their charges match, which makes R positive like Q, and then correctly pairs a positive charge with lost electrons. The lean is inwards, so that pair attracts and its charges are unlike. (*Interaction rule inverted*)
- **C** (R is negatively charged, having lost electrons when it was charged.): Gets the sign right, R negative, but books it as a loss of electrons. Losing electrons leaves a shortage of negative charge and so gives a positive sphere; a negative sphere is one that has gained electrons. (*Sign attached to the wrong electron move*)
- **D** (R is positively charged, having gained electrons when it was charged.): Makes both slips at once: takes the inward lean as repulsion, so R is called positive like Q, and then attaches that positive charge to a gain of electrons. Gaining electrons gives a negative sphere, so the two halves of the statement contradict each other. (*Interaction rule and electron count both inverted*)
- **E** (R is uncharged, and is pulled towards Q only because Q is charged.): Leans on the fact that a charged object can pull an uncharged one towards it, so the attraction is read as needing no charge on R. The stem states that R had already been given a charge of its own, so this is an attraction between two charged spheres and R cannot be neutral. (*Attraction assumed to show an uncharged sphere*)

Common mistake. Reading the inward lean of the second pair as a sign that Q and R match. Threads that lean together show attraction, and attraction between two charged spheres means opposite signs; it is the pair hanging apart, P and Q, that shares a sign.

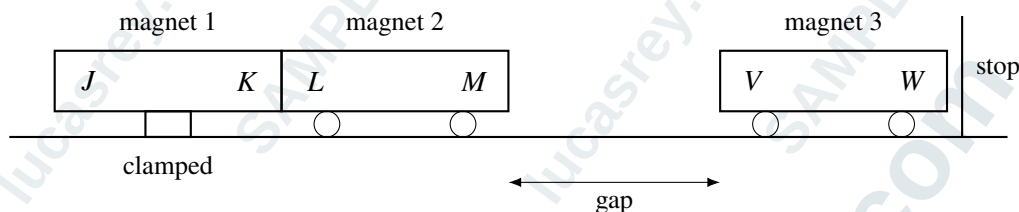
Takeaway. The Cheat Code: Only electrons move, so negative means extra electrons and positive means missing ones. Then let the threads speak: leaning together is unlike, hanging apart is like.

Time-saving tip. Speed Heuristic: settle the cloth first. Whatever the silk gained, Q lost, so Q is positive before you even look at the diagram, and the second pair then fixes R in a single move.

Question 6

P2.1 • Magnetism and electromagnetism • ★★☆☆☆ • 85 s

Three identical bar magnets sit on low-friction trolleys on a straight horizontal rail, as shown. The left-hand magnet is clamped, and the other two are free to slide. When they are released, magnets 1 and 2 close up until they touch, while magnet 3 runs away from magnet 2 and rests against an end stop. A compass test shows that end *J* is a north pole. Which two lettered ends are both north poles?



- A. *K* and *M*
- B. *J* and *M*
- C. *L* and *V*
- D. *J* and *L*
- E. *J* and *K*

Answer D. Key idea

Every bar magnet carries a north pole at one end and a south pole at the other, so fixing one end of a magnet fixes its other end automatically. Between two magnets the rule runs one way only: unlike poles attract and like poles repel, so watching what a free magnet does identifies the pair of faces nearest to each other. Chaining those two facts along a row lets one known pole propagate through every magnet in the line, because each observed movement settles a junction and each magnet then settles its own far end.

Worked solution

1. Work along the row as far as magnet 2

End *J* is a north pole, and the two ends of a single bar magnet are always opposite poles, so *K* is a south pole.

Magnets 1 and 2 have pulled themselves into contact, and attraction happens only between unlike poles. The end of magnet 2 facing *K* is *L*, so *L* must be unlike *K*: *L* is a north pole. Its far end *M* is therefore a south pole.

2. Cross the open gap to magnet 3

Magnet 3 has been pushed away and is held against the stop, and repulsion happens only between like poles. The end of magnet 3 facing *M* is *V*, so *V* is like *M*, making *V* a south pole and the far end *W* a north pole.

Reading the row from left to right gives N S, then N S, then S N. The north poles are *J*, *L* and *W*, and the only listed pair drawn entirely from those three is *J* and *L*.

Sanity check: with that labelling the contact between magnets 1 and 2 is a south face meeting a north face, which does attract, and the open gap is a south face meeting a south face, which does repel. Both observations are reproduced, so the labelling is consistent. Turning magnet 3 end for end would let it close the gap, which is the standard demonstration of the same rule.

Matches Option D.

Why the other options are wrong

- A (*K* and *M*): Attached the compass result to the wrong end of magnet 1, taking *K* as north so that *J* is south. The contact then makes *L* south and *M* north, and the repulsion makes *V* north and *W* south, leaving *K*, *M* and *V* as the north poles and *K* and *M* as the listed pair. (*Known pole placed on the wrong end*)

- **B** (J and M): Swapped the two rules, taking attraction as evidence of like poles and repulsion as evidence of unlike poles. Then K is south, the contact makes L south and M north, and the repulsion makes V south and W north, so the north poles come out as J , M and W . (*Attraction and repulsion rules exchanged*)
- **C** (L and V): Applied the rule correctly at the contact but read the open gap as attraction as well, on the idea that magnet 3 had simply not finished moving. That makes V unlike M , so V is north, and the north poles are taken to be J , L and V . (*Gap read as attraction*)
- **E** (J and K): Treated each magnet as though both of its ends were the same pole, so J north forces K north as well. The contact then makes both L and M south and the repulsion makes both V and W south, leaving J and K as the only north poles. (*Both ends of one magnet taken alike*)

Common mistake. Assuming that the magnets which ended up touching must have like poles facing each other, on the loose idea that similar things attract. The rule runs the other way round, and here it decides everything: the faces in contact are unlike, while the faces on either side of the open gap are alike.

Takeaway. The Cheat Code: one known pole unlocks a whole row. Alternate N and S across each magnet's own length, put unlike poles at every contact and like poles across every gap that has opened, and the six labels fall out in a single sweep from left to right.

Time-saving tip. Speed Heuristic: write the six letters in a line and fill them in one pass, flipping the pole within each magnet, keeping unlike poles where the magnets touch and like poles across the gap. The chain is finished by the time you reach W , so the options only have to be scanned once.

Question 7

P2.3 • The motor effect • ★★☆☆☆ • 45 s

A wire of length 0.1 m carries a current of 3 A perpendicular to a magnetic field of flux density 1.2 T. Find the force on the wire.

- A. 0.25 N
- B. 4.30 N
- C. 0.51 N
- D. 0.36 N
- E. 0.12 N
- F. 3.60 N
- G. 0.30 N
- H. 36.00 N

Answer D. Key idea

The motor effect force on a current-carrying wire perpendicular to a field: $F = BIL = 1.2 \times 3 \times 0.1 = 0.36$ N
Direction from Fleming's LEFT-HAND rule (thumb = force, first finger = field, second finger = current).
If the

Worked solution

The motor effect force on a current-carrying wire perpendicular to a field:

$$F = BIL = 1.2 \times 3 \times 0.1 = 0.36 \text{ N}$$

Direction from Fleming's LEFT-HAND rule (thumb = force, first finger = field, second finger = current).

If the wire runs PARALLEL to the field, $F = 0$, the formula needs the perpendicular arrangement (or $\sin \theta$).

Matches Option D.

Why the other options are wrong

- **A** (0.25 N): Rearranged the formula the wrong way and put the flux density underneath: $IL/B = (3 \times 0.1) \div 1.2 = 0.3 \div 1.2 = 0.25 \text{ N}$. B is a factor in $F = BIL$, not a divisor, so the product is $0.3 \times 1.2 = 0.36 \text{ N}$. (*Flux density divided instead of multiplied*)
- **B** (4.30 N): Added the quantities, the formula is a product. (*Conceptual Error*)
- **C** (0.51 N): Arithmetic slip. (*Arithmetic Error*)
- **E** (0.12 N): Left the current out of the product and multiplied only the flux density by the length: $BL = 1.2 \times 0.1 = 0.12 \text{ N}$. $F = BIL$ needs all three, so the 3 A must multiply in as well: $0.12 \times 3 = 0.36 \text{ N}$. (*Current omitted*)
- **F** (3.60 N): Forgot the length, $F = BIL$ needs all three. (*Arithmetic Error*)
- **G** (0.30 N): Omitted the flux density. (*Arithmetic Error*)
- **H** (36.00 N): Divided by the length instead of multiplying by it: $BI/L = (1.2 \times 3) \div 0.1 = 3.6 \div 0.1 = 36.00 \text{ N}$. Dividing by 0.1 multiplies by ten, so this is one hundred times the true force; multiplying by 0.1 gives $3.6 \times 0.1 = 0.36 \text{ N}$. (*Length inverted*)

Common mistake. Omitting a factor or adding the quantities.

Takeaway. The Cheat Code: $F = BIL$, 'BIL' rhymes with 'VIL'ence: force needs all three attackers. Left hand for motors, right hand for generators.

Time-saving tip. Write the ratio with the quantity you want on top. Set out that way, the substitution cannot go in upside down.

Question 8

P1.1 · Electricity · ★★☆☆☆ · 85 s

A polymer belt runs over a rubber drum, both insulators and both initially uncharged. Friction leaves the belt carrying -7.2 nC . With the electron charge $1.6 \times 10^{-19} \text{ C}$, how many electrons has the drum lost?

- A. 7.2×10^{10}
- B. 4.5×10^{13}
- C. 9.0×10^{10}
- D. 4.5×10^{19}
- E. 4.5×10^{10}
- F. 2.2×10^{-11}
- G. 2.25×10^{10}
- H. 4.5×10^{28}

Answer E. Key idea

Rubbing two insulators together charges them by moving electrons, and electrons are the only thing that moves: the protons stay locked in the nuclei of both materials. Nothing is created either, so if the two bodies begin uncharged, every electron one of them gains is an electron the other has lost, and the two final charges are equal in size and opposite in sign. Charge itself comes in whole electrons, so a measured charge divided by the charge on one electron gives the number that changed hands.

Worked solution

1. Turn the belt's charge into the drum's charge

The belt ends up negative, so the belt has gained electrons. Friction cannot create charge and cannot move protons, so those electrons came from the only other body in contact, the drum. Both started uncharged, which means the drum is left short of exactly the electrons the belt has taken:

$$Q_{\text{drum}} = +7.2 \text{ nC} = +7.2 \times 10^{-9} \text{ C}.$$

The number lost by the drum is therefore the same number the belt gained, not half of it and not twice it.

2. Divide by the charge on one electron

$$n = \frac{Q}{e} = \frac{7.2 \times 10^{-9}}{1.6 \times 10^{-19}}.$$

Split the division: $7.2 \div 1.6 = 4.5$, and $10^{-9} \div 10^{-19} = 10^{10}$, so

$$n = 4.5 \times 10^{10}.$$

Sanity check by multiplying back: $4.5 \times 10^{10} \times 1.6 \times 10^{-19} = 7.2 \times 10^{-9}$ C, the charge stated. A second check on the size: a few tens of thousands of millions of electrons sounds enormous, but a gram of matter holds of order 10^{23} electrons, so the drum has parted with a vanishing fraction of what it holds, which is why the belt and drum look completely unchanged.

Matches Option E.

Why the other options are wrong

- **A** (7.2×10^{10}): Divided by the power of ten alone and never divided by the 1.6: $7.2 \times 10^{-9} \div 10^{-19} = 7.2 \times 10^{10}$. The digits divide too, and $7.2 \div 1.6 = 4.5$, so the count is 4.5×10^{10} . (*Electron charge digits dropped*)
- **B** (4.5×10^{13}): Converted the prefix as though nano meant 10^{-6} : $Q = 7.2 \times 10^{-6}$ C, so $n = 7.2 \times 10^{-6} \div 1.6 \times 10^{-19} = 4.5 \times 10^{13}$. Nano is 10^{-9} , a thousand times smaller than micro, which pulls the count back to 4.5×10^{10} . (*Micro used for nano*)
- **C** (9.0×10^{10}): Counts the transfer twice, adding the electrons the belt gained to the electrons the drum lost: $2 \times 4.5 \times 10^{10} = 9.0 \times 10^{10}$. They are the same electrons. (*Double counting*)
- **D** (4.5×10^{19}): Drops the nano prefix and divides 7.2 C by the electron charge: $7.2 \div 1.6 \times 10^{19} = 4.5 \times 10^{19}$. One nanocoulomb is 10^{-9} C. (*Prefix ignored*)
- **F** (2.2×10^{-11}): Inverts the division, working out $1.6 \times 10^{-19} \div 7.2 \times 10^{-9} = 2.2 \times 10^{-11}$. A count of electrons cannot be a fraction of one. (*Inverted ratio*)
- **G** (2.25×10^{10}): Halves the charge first, on the idea that the 7.2 nC is shared between the belt and the drum: $3.6 \times 10^{-9} \div 1.6 \times 10^{-19} = 2.25 \times 10^{10}$. The two bodies each carry the full 7.2 nC, opposite in sign. (*Charge shared*)
- **H** (4.5×10^{28}): Wrote the nano prefix with a positive exponent, taking $Q = 7.2 \times 10^9$ C: $7.2 \times 10^9 \div 1.6 \times 10^{-19} = 4.5 \times 10^{28}$. A nanocoulomb is a billionth of a coulomb, so the exponent is -9 and $(-9) - (-19) = +10$. (*Prefix exponent sign*)

Common mistake. Assuming that a negative body and a positive body must have taken part in two separate exchanges, so that the electron count has to be shared out or doubled. One single transfer produced both charges at once, so the number the drum lost is exactly the number the belt gained.

Takeaway. The Cheat Code: Charging by friction is bookkeeping with one entry. Electrons move one way, so the negative body gained them, the positive body lost the same number, and the count is always the measured charge divided by 1.6×10^{-19} C.

Time-saving tip. Speed Heuristic: Do the digits and the powers of ten separately. $7.2 \div 1.6 = 4.5$ takes a moment, and -9 minus -19 gives $+10$, so the answer is 4.5×10^{10} before you have written the fraction out.

Question 9

P4.4 · Thermal physics · ★★☆☆ · 90 s

A camping stove's electric plate draws 1200 W from a generator, and 360 W of this is lost to the pan and the surroundings. The pan holds 2.0 litres of water at 20 °C. Water has density 1000 kg m^{-3} and specific heat capacity $4200 \text{ J kg}^{-1} \text{ K}^{-1}$. Assume the plate reaches its steady output immediately and the loss stays constant. How long does the water take to reach 80 °C?

- A. 600 s
- B. 1000 s
- C. 200 s
- D. 800 s
- E. 1050 s
- F. 294 s
- G. 1400 s
- H. 420 s

Answer A. Key idea

The energy a body of water must absorb is fixed by its mass, its specific heat capacity and its temperature rise, through $E = mc\Delta T$, and nothing about the heater changes that figure. The heater decides only how fast the energy is handed over. So the calculation splits cleanly in two: find the energy from the water alone, then find the rate at which energy actually arrives in the water, which is the supplied power minus whatever is lost, and divide the first by the second.

Worked solution**1. Mass of water in the pan**

2.0 litres is $2.0 \times 10^{-3} \text{ m}^3$, so

$$m = \rho V = 1000 \times 2.0 \times 10^{-3} = 2.0 \text{ kg}.$$

2. Energy the water must absorb

The temperature rise is $\Delta T = 80 - 20 = 60 \text{ K}$, and a rise of one kelvin is the same size as a rise of one degree Celsius, so

$$E = mc\Delta T = 2.0 \times 4200 \times 60 = 504000 \text{ J} = 504 \text{ kJ}.$$

3. Power that actually reaches the water

Of the 1200 W drawn from the generator, 360 W never gets into the water, so

$$P_{\text{useful}} = 1200 - 360 = 840 \text{ W}.$$

4. Time to deliver that energy

$$t = \frac{E}{P_{\text{useful}}} = \frac{504000}{840} = 600 \text{ s}.$$

Sanity check: $840 \times 600 = 504000 \text{ J}$, exactly the energy required. Ten minutes to take two litres from 20 °C to 80 °C on a plate of roughly one kilowatt is the sort of time a camping stove really needs.

Matches Option A.

Why the other options are wrong

- **B** (1000 s): Added the two temperatures rather than subtracting them, using $\Delta T = 80 + 20 = 100$ K: $E = 2.0 \times 4200 \times 100 = 840000$ J, then $840000 \div 840 = 1000$ s. The water only has to climb the 60 K between the two readings. (*Temperatures added*)
- **C** (200 s): Took the 20 °C the water starts at as the temperature change: $E = 2.0 \times 4200 \times 20 = 168000$ J, then $168000 \div 840 = 200$ s. ΔT is the difference between the two readings, $80 - 20 = 60$ K. (*Starting temperature used as ΔT*)
- **D** (800 s): Used the final temperature as the rise instead of the change: $E = 2.0 \times 4200 \times 80 = 672000$ J, then $672000 \div 840 = 800$ s. (*Final temperature used as ΔT*)
- **E** (1050 s): Subtracted the loss twice, once from the plate's supply and again from the power reaching the water: $1200 - 360 - 360 = 480$ W, giving $504000 \div 480 = 1050$ s. (*Loss double counted*)
- **F** (294 s): Noticed that 360 W of 1200 W is 30%, then took 30% off the time found from the full power instead of dividing by the 70% that arrives: $504000 \div 1200 = 420$ s, then $420 \times 0.70 = 294$ s. Losing power makes the heating slower, so the time is $420 \div 0.70 = 600$ s. (*Efficiency multiplied instead of divided*)
- **G** (1400 s): Divided by the wasted power rather than the delivered power, $504000 \div 360 = 1400$ s, which is the time the losses alone would take to carry away that much energy. (*Wrong power used*)
- **H** (420 s): Divided by the full power drawn from the generator, $504000 \div 1200 = 420$ s, treating every watt as reaching the water and ignoring the stated 360 W loss. (*Loss ignored*)

Common mistake. Dividing by the 1200 W the plate draws rather than the 840 W that reaches the water. The wasted 360 W is 30% of the supply, so this makes the heating look 180 s quicker than it really is.

Takeaway. The Cheat Code: energy first, power second. Work out $mc\Delta T$ for the water on its own, then decide separately how many watts are actually delivering it, and divide only at the very end. Losses never change the energy the water needs, only the time taken to hand it over.

Time-saving tip. Speed Heuristic: cancel before multiplying and 504000 never has to be written. $4200 \times 60 = 252000$, and $252000 \div 840 = 300$, so the time is $2 \times 300 = 600$ s. Spotting that 840 is 2×420 and 420 divides 4200 ten times makes the whole thing mental.

Question 10

P4.2 · Thermal physics · ★★☆☆ · 90 s

A paraffin lamp burns inside a tall glass chimney that is open at the top and pierced by a ring of small holes around its base; these openings are the only ways in or out. The lamp stands in a draught-free room whose air stays at a steady 20 °C, while the flame keeps the air inside the chimney much hotter. Which statement correctly describes how air moves through the chimney, and why?

- A.** Air enters at the top and leaves through the base holes, because the cooler, denser room air sinks into the chimney and pushes the warm air out below it.
- B.** Air enters at the top and leaves through the base holes, because warming the air raises its density, so the hot air sinks and spills out through the holes.
- C.** No air passes through either opening, because the difference in density between the hot and the cool air holds the two bodies of air apart.
- D.** Air enters through the base holes and leaves at the top, because warming the air lowers its density, so the denser room air displaces it upwards.
- E.** Air enters through the base holes and leaves at the top, because hot gas always rises, whatever the density of the air surrounding it.

Answer D. Key idea

Heating a fluid makes it expand, so the same mass fills a larger volume and its density falls. Flow then follows from a comparison rather than from the temperature on its own: a less dense body of fluid is

displaced upwards by the denser fluid around it, and whatever leaves at one opening has to be replaced through another, which is what turns a single rising column into a continuous circulation.

Worked solution

1. What the flame does to the density of the air

The flame warms the air inside the chimney. A warmed gas expands, so the same mass of air now occupies a larger volume, and density is mass divided by volume. The air in the chimney is therefore less dense than the 20°C air filling the room.

2. What a difference in density does

A body of fluid that is less dense than the fluid surrounding it does not stay where it is. The denser fluid, heavier for the same volume, settles underneath it and displaces it upwards. Here the room air is the denser of the two, so it pushes the warm column of chimney air up and out through the open top. The rising is not something the hot gas does of its own accord: it is what the surrounding air does to it.

3. Close the loop

Air that leaves the top has to be replaced, and the ring of holes at the base is the only other opening, so room air is drawn in there. The result is a steady upward draught: in at the bottom, up past the flame, out at the top, lasting as long as the temperature difference does.

Sanity check: cover the base holes and the replacement air cannot get in, so the draught stops. That is exactly why lamps of this design carry their openings at the bottom rather than at the top.

Matches Option D.

Why the other options are wrong

- **A** (Air enters at the top and leaves through the base holes, because the cooler, denser room air sinks into the chimney and pushes the warm air out below it.): Gets the density comparison right and the geometry wrong. The room air is indeed the denser of the two, but denser air entering at the top would have to drive the less dense warm air downwards past the flame and out at the base, which is the wrong way round for the lighter fluid. The cool air does move in, at the bottom. (*Conceptual, flow direction reversed*)
- **B** (Air enters at the top and leaves through the base holes, because warming the air raises its density, so the hot air sinks and spills out through the holes.): Inverts the effect of temperature on density. Warming a gas expands it, so the same mass fills a larger volume and the density falls rather than rises. Sinking and an outflow at the base then follow logically from a false first step. (*Conceptual, temperature and density inverted*)
- **C** (No air passes through either opening, because the difference in density between the hot and the cool air holds the two bodies of air apart.): Treats a density difference as a barrier instead of as the cause of the flow. Two bodies of air of different density in contact do not sit still: the denser one settles beneath the lighter one, and that movement is exactly the convection current the chimney relies on. (*Conceptual, density difference read as static*)
- **E** (Air enters through the base holes and leaves at the top, because hot gas always rises, whatever the density of the air surrounding it.): Gets the direction of the draught right for a reason that does not hold. Hot gas does not rise of its own accord, it rises when the fluid around it is denser. Surround the same hot gas with air that is hotter still, at the same pressure, and it would sink, so the comparison of densities, not the temperature by itself, is what drives the flow. (*Conceptual, right direction from the wrong mechanism*)

Common mistake. Treating "hot air rises" as a property that hot air carries around with it. Rising is a comparison: the air goes up only because the air around it is denser at that moment. Put the same warm air into surroundings that are hotter still, at the same pressure, and it would sink instead.

Takeaway. The Cheat Code: In any convection question, name the two bodies of fluid and ask which is denser right now. The denser one moves in underneath, the less dense one is pushed up, and the opening the fluid leaves by fixes the opening it must enter by.

Time-saving tip. Speed Heuristic: split the options by flow direction before reading any of the reasons. The hot, expanded air is the least dense thing in the chimney, so it has to leave at the top, which kills the two options that push air out at the base and the one that says nothing moves. Of the two survivors, keep the reason that names a density difference rather than a rule about hot gas.

END OF SOLUTIONS

Questions 11 to 27 are in the full paper

This is an extract. The 10 questions before this page are real questions from a commissioned worked solutions, shown in full and with nothing withheld, so that the standard can be judged rather than described.

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EXAMPLE GRAMMAR SCHOOL

ESAT Mock Examination

Chemistry

Paper SMP-2026-09 · Version 1.0

NAME

TEACHING GROUP

DATE

Instructions

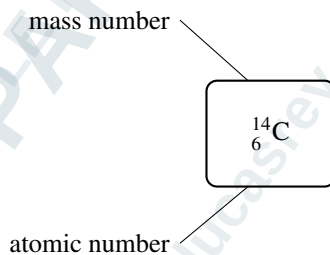
- Time allowed: **15 minutes**.
- There are **10 multiple-choice questions**. Each is worth 1 mark.
- There is no negative marking. Answer every question, even if you have to guess.
- Calculators are not permitted.
- Record your answers on the separate answer sheet. Only the answer sheet is marked.
- Formulae are not provided. The ESAT does not supply a formula sheet.

Note: the real ESAT is sat on screen at a Pearson test centre. This paper trains the mathematics and the pacing, not the on-screen interface.

Written for Example Grammar School by Lucas Rey, Engineering Science, University of Oxford; MRes, University of Cambridge.

Question 1

An atom of a carbon isotope is represented by the standard notation shown. How many neutrons does it contain?



- A. 9
- B. 7
- C. 20
- D. 14
- E. 8
- F. 6

Question 2

A student sets three changes going in identical open glass beakers on a bench in a laboratory held at 19 °C. In the first, magnesium ribbon is dropped into dilute hydrochloric acid and the thermometer climbs to 34 °C. In the second, citric acid solution is stirred into sodium hydrogencarbonate and the thermometer falls to 12 °C. In the third, two solids are ground together with a rod; no thermometer is used, but within a minute a film of ice forms on the outside of that beaker from moisture in the room air. Which statement about the three enthalpy changes is correct?

- A. All three have a negative ΔH .
- B. The first and second have a negative ΔH ; the third is positive.
- C. The first has a positive ΔH ; the second and third are negative.
- D. The first has a negative ΔH ; the second and third are positive.
- E. All three have a positive ΔH .
- F. The first and third have a negative ΔH ; the second is positive.

Question 3

A fertiliser is sold as pure ammonium sulfate, $(\text{NH}_4)_2\text{SO}_4$. Using A_r values $H = 1$, $N = 14$, $O = 16$, $S = 32$, calculate its relative formula mass.

- A. 132
- B. 118
- C. 96
- D. 130
- E. 128
- F. 228
- G. 84
- H. 114

Question 4

Element Q lies in the same Period as chlorine and two Groups to the left of it. A chlorine atom has the electron configuration 2, 8, 7. Which configuration belongs to an atom of Q?

- A. 2, 8, 8
- B. 2, 5
- C. 2, 8, 2
- D. 2, 8, 3
- E. 2, 8, 4
- F. 2, 8, 6
- G. 2, 8, 5
- H. 2, 8, 8, 1

Question 5

A technician cuts a cube of sodium from a lump kept under oil and drops it into a trough of cold water. The cube floats, and within a second or two it melts into a ball that darts across the surface, leaving the water alkaline. Why does the cube melt?

- A. The oil clinging to the freshly cut surface catches fire on the water, and its flame melts the metal.
- B. Friction with the water, as the cube is driven across the surface, heats it up to its melting point.
- C. The hydrogen given off burns as it escapes, and that flame supplies the heat needed to melt the metal.
- D. Sodium's metallic bonding is weak, so the cold water breaks the lattice apart without any heating.
- E. Sodium's melting point is low, and its reaction with the water releases heat fast enough to reach that temperature.
- F. The sodium hydroxide formed dissolves into the metal and lowers its melting point below room temperature.

Question 6

Calculate the volume of O_2 gas produced at room temperature and pressure (RTP) when 50.0 g of potassium chlorate decomposes according to the equation: $2KClO_3(s) \rightarrow 2KCl(s) + 3O_2(g)$ (Take molar gas volume at RTP = $24.0 \text{ dm}^3 \text{ mol}^{-1}$, $M_r(\text{potassium chlorate}) = 122.5$).

- A. 29.39 dm^3
- B. 14.69 dm^3
- C. 7.35 dm^3
- D. 13.71 dm^3
- E. 1200.0 dm^3
- F. 9.80 dm^3
- G. 6.53 dm^3
- H. 146.9 dm^3

Question 7

In a blast furnace, iron(III) oxide (Fe_2O_3) is reduced by carbon monoxide to iron and carbon dioxide. Balanced with the smallest whole numbers, what is the sum of all the coefficients in this equation?

- A. 7
- B. 18
- C. 8
- D. 12
- E. 11
- F. 5
- G. 4
- H. 9

Question 8

A technician stirs dry sand, which does not dissolve, into a blue solution of copper(II) sulfate in water. Dry crystals of copper(II) sulfate must be recovered from the mixture, with no sand in them. Which pair of steps, carried out in this order, achieves that?

- A. Heat the whole mixture until every drop of water has boiled away, then collect the solid left in the dish.
- B. Boil the whole mixture and condense the vapour, then keep the clear liquid that collects in the receiver.
- C. Cool the whole mixture in ice until crystals appear, then pour it onto filter paper and dry what is caught.
- D. Pour the mixture onto filter paper, then heat the liquid that runs through until it is saturated and cool it slowly.
- E. Pour the mixture onto filter paper, then wash the solid caught in the paper with warm water and dry it.

Question 9

Calculate the relative molar mass, M_r , of magnesium hydroxide, $Mg(OH)_2$. ($A_r(Mg) = 24$, $A_r(O) = 16$, $A_r(H) = 1$.)

- A. 58
- B. 42
- C. 57
- D. 82
- E. 41
- F. 26
- G. 56
- H. 29

Question 10

Tellurium has a higher relative atomic mass than iodine, 127.6 against 126.9, yet every modern Periodic Table places tellurium first, with iodine falling in the halogen column beneath bromine. Which rule does the modern table order its elements by?

- A. Elements are ordered by increasing number of neutrons, and tellurium's atoms hold fewer neutrons than iodine's atoms.
- B. Elements are ordered by increasing relative atomic mass, with awkward pairs swapped to keep chemical families together.
- C. Elements are ordered by decreasing atomic radius, and tellurium's atoms are larger than iodine's atoms.
- D. Elements are ordered by increasing proton number, and tellurium has 52 protons against iodine's 53.
- E. Elements are ordered by increasing number of outer-shell electrons, six for tellurium and seven for iodine.
- F. Elements are ordered by increasing number of occupied electron shells, which is greater for iodine than for tellurium.

END OF PAPER

Questions 11 to 27 are in the full paper

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EXAMPLE GRAMMAR SCHOOL

ESAT Mock: Answer Sheet

Chemistry

Paper SMP-2026-09 • Version 1.0

NAME

TEACHING GROUP

DATE

Fill in one bubble per question. If you change your mind, cross out the old mark clearly and fill the new bubble. Unfilled or doubly filled rows score zero.

Q	A	B	C	D	E	F	G	H
1	☐	☐	☐	☐	☐	☐		
2	☐	☐	☐	☐	☐	☐		
3	☐	☐	☐	☐	☐	☐	☐	☐
4	☐	☐	☐	☐	☐	☐	☐	☐
5	☐	☐	☐	☐	☐	☐		
6	☐	☐	☐	☐	☐	☐	☐	☐
7	☐	☐	☐	☐	☐	☐	☐	☐
8	☐	☐	☐	☐	☐			
9	☐	☐	☐	☐	☐	☐	☐	☐
10	☐	☐	☐	☐	☐	☐		

MARKER INITIALS

TOTAL OUT OF 10

EXAMPLE GRAMMAR SCHOOL

ESAT Mock: Mark Scheme

Chemistry

Paper SMP-2026-09 • Version 1.0

Answer grid

Q	Ans	Q	Ans	Q	Ans
1	E	10	D		
2	D				
3	A				
4	G				
5	E				
6	B				
7	H				
8	D				
9	A				

Totals

10 marks available, 1 mark per question. There is no negative marking, so a wrong answer scores zero and never a negative. Only the answer sheet is marked.

Diagnostic table

Use this to see which topics the cohort is losing marks on.

Q	Ans	Node	Group	Topic	Difficulty
1	E	C1.3	C1	Know and be able to use the terms atomic number and mass number, together with standard notation (e.g. $^{12}_6\text{C}$), and so be able to calculate the number of protons, neutrons and electrons in any atom	★★☆☆☆
2	D	C11.1	C11	Energetics	★★☆☆☆
3	A	C4.1	C4	Quantitative chemistry	★★☆☆☆
4	G	C2.4	C2	The Periodic Table	★★☆☆☆
5	E	C7.1	C7	Groups 1, 17 and 18	★★☆☆☆
6	B	C4.8	C4	Quantitative chemistry	★★☆☆☆
7	H	C3.4	C3	Chemical reactions and equations	★★☆☆☆
8	D	C8.2	C8	Separation techniques	★★☆☆☆
9	A	C4.1	C4	Quantitative chemistry	★★☆☆☆
10	D	C2.2	C2	The Periodic Table	★★☆☆☆

Marks available by topic group

Group	Topic	Marks
C1	Atomic structure	1
C2	The Periodic Table	2
C3	Chemical reactions and equations	1
C4	Quantitative chemistry	3
C7	Groups 1, 17 and 18	1
C8	Separation techniques	1
C11	Energetics	1
Total		10

Worked solutions for every question are in the separate worked solutions document.

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EXAMPLE GRAMMAR SCHOOL

ESAT Mock: Worked Solutions

Chemistry

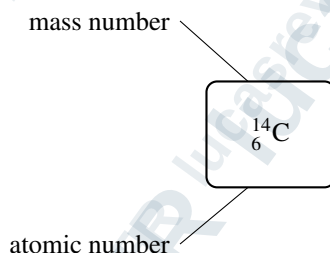
Paper SMP-2026-09 · Version 1.0

Staff copy

This document contains the answers. Keep it away from candidates until the paper has been sat. Marking guidance is in the separate mark scheme.

Question 1 C1.3 · Know and be able to use the terms atomic number and mass number, together with standard notation (e.g. ${}^{12}_6\text{C}$), and so be able to calculate the number of protons, neutrons and electrons in any atom or ion. ★★☆☆☆ · 45 s

An atom of a carbon isotope is represented by the standard notation shown. How many neutrons does it contain?



- A. 9
- B. 7
- C. 20
- D. 14
- E. 8
- F. 6

Answer E. Key idea

Standard notation ${}^A_Z\text{X}$: Z (lower) is the atomic/proton number, which defines the element and equals the electron count in a neutral atom, while A (upper) is the mass number, protons plus neutrons, so neutrons = $A - Z$. For carbon-14 (${}^{14}_6\text{C}$): 6 protons, $14 - 6 = 8$ neutrons and 6 electrons.

Worked solution

Standard notation ${}^A_Z\text{X}$:

- Z (lower) = ATOMIC/proton number = number of protons (defines the element; = electrons if neutral).
- A (upper) = MASS number = protons + neutrons.
- Neutrons = $A - Z$.

For carbon-14 (${}^{14}_6\text{C}$): 6 protons, 8 neutrons, 6 electrons (neutral).

Matches Option E.

Why the other options are wrong

- **A** (9): Off by one. (*Arithmetic Error*)
- **B** (7): Assumed a nucleus holds equal numbers of protons and neutrons and halved the mass number: $14 / 2 = 7$. That works for carbon-12, not for this isotope; neutrons = $A - Z = 14 - 6 = 8$. (*Conceptual Error*)
- **C** (20): Added them; subtract instead. (*Arithmetic Error*)
- **D** (14): That's the mass number. (*Conceptual Error*)
- **F** (6): That's the proton number. (*Conceptual Error*)

Common mistake. Swapping A and Z in the notation.

Takeaway. The Cheat Code: 'An on top adds, Z at the bottom identifies': $A = p + n$, $Z = \text{identity}$. Neutrons = $A - Z$. Always.

Time-saving tip. An adds ($p+n$); Z identifies.

Question 2

C11.1 • Energetics • ★★☆☆☆ • 90 s

A student sets three changes going in identical open glass beakers on a bench in a laboratory held at $19\text{ }^{\circ}\text{C}$. In the first, magnesium ribbon is dropped into dilute hydrochloric acid and the thermometer climbs to $34\text{ }^{\circ}\text{C}$. In the second, citric acid solution is stirred into sodium hydrogencarbonate and the thermometer falls to $12\text{ }^{\circ}\text{C}$. In the third, two solids are ground together with a rod; no thermometer is used, but within a minute a film of ice forms on the outside of that beaker from moisture in the room air. Which statement about the three enthalpy changes is correct?

- A.** All three have a negative ΔH .
- B.** The first and second have a negative ΔH ; the third is positive.
- C.** The first has a positive ΔH ; the second and third are negative.
- D.** The first has a negative ΔH ; the second and third are positive.
- E.** All three have a positive ΔH .
- F.** The first and third have a negative ΔH ; the second is positive.

Answer D. Key idea

The sign of an enthalpy change records the direction energy travelled, not how large the change was. A reaction that pushes energy out into its surroundings leaves the chemicals lower in energy than they started, so ΔH is negative and the surroundings warm. A reaction that pulls energy in from its surroundings leaves the chemicals higher in energy, so ΔH is positive and the surroundings cool. A thermometer is only one way of seeing which way the energy went; anything showing the mixture has dropped below room temperature is the same evidence.

Worked solution

1. The first beaker

The thermometer climbs from the room's $19\text{ }^{\circ}\text{C}$ to $34\text{ }^{\circ}\text{C}$. Energy has been pushed out of the reacting chemicals into the solution, the glass and the air, which is what exothermic means. The chemicals themselves end lower in energy than they began, so ΔH is negative.

2. The second beaker

Here the thermometer falls from $19\text{ }^{\circ}\text{C}$ to $12\text{ }^{\circ}\text{C}$. Energy has been drawn out of the surroundings and into the reacting chemicals, so the change is endothermic and ΔH is positive.

3. The third beaker

There is no reading, but ice forms on the outside of the glass. Water vapour from the room can only freeze there if the outside of that beaker has been pulled below 0°C , far below the room's 19°C . Energy is therefore flowing from the room into the beaker, exactly as in the second one, so this change is endothermic too and ΔH is positive.

Sanity check: one beaker ended warmer than the room and two ended colder, so one ΔH is negative and two are positive, and every beaker has been classified from the direction energy crossed its wall rather than from what was put in it.

Matches Option D.

Why the other options are wrong

- **A** (All three have a negative ΔH .): Assumes any change that runs on its own must be giving energy out, so all three are called negative. The second beaker's thermometer drops seven degrees, which cannot happen while the chemicals in it are warming their surroundings. (*All changes assumed exothermic*)
- **B** (The first and second have a negative ΔH ; the third is positive.): Classifies the second beaker from what is in it rather than from the thermometer: an acid meeting a hydrogencarbonate is read as a neutralisation, and neutralisation is remembered as exothermic, so its ΔH is called negative. The reading is the evidence and it falls from 19°C to 12°C , which is energy drawn into the chemicals and a positive ΔH . The third beaker, with no reaction to recognise, is then read correctly from its frost, so the same candidate treats two beakers that both cool the room in opposite ways. (*Reaction type recalled in place of the reading*)
- **C** (The first has a positive ΔH ; the second and third are negative.): Applies the convention the wrong way round throughout, calling the beaker that warmed positive and the two that cooled negative. The sign belongs to the chemicals, whose energy falls when they heat their surroundings. (*Sign convention inverted*)
- **E** (All three have a positive ΔH .): Treats every temperature change as a positive ΔH , reading the size of the change rather than its direction. The first beaker finished 15°C above the room, which is energy leaving the chemicals, and that carries a negative sign. (*Direction of energy flow ignored*)
- **F** (The first and third have a negative ΔH ; the second is positive.): Correct on the two beakers with thermometers, but credits the freezing of the condensed water to the mixture inside the third. The ice forms because that beaker is colder than the room, which is the signature of energy going in, not out. (*Frost misread as energy released*)

Common mistake. Reading the ice on the third beaker as evidence that energy was given out, on the grounds that freezing releases energy. Freezing does release energy, but the water that froze belongs to the room: it gave energy up because the mixture inside was taking energy from it.

Takeaway. The Cheat Code: Ask which way energy crossed the glass. Out of the chemicals means the surroundings warm and ΔH is negative; into the chemicals means the surroundings cool, frost included, and ΔH is positive.

Time-saving tip. Speed Heuristic: Sort the beakers by whether each ended warmer or colder than the 19°C room, ignoring what is in them. One warmer and two colder means one negative and two positive, and only one option carries that split.

Question 3

C4.1 · Quantitative chemistry · ★★☆☆☆ · 85 s

A fertiliser is sold as pure ammonium sulfate, $(\text{NH}_4)_2\text{SO}_4$. Using A_r values $H = 1$, $N = 14$, $O = 16$, $S = 32$, calculate its relative formula mass.

- A. 132
- B. 118
- C. 96
- D. 130
- E. 128
- F. 228
- G. 84
- H. 114

Answer A. Key idea

A relative formula mass is the sum of the relative atomic masses of every atom in the formula, each counted as often as it appears. A subscript written outside a bracket multiplies the whole group inside it, every element and every subscript within, so the safe route is to evaluate each bracketed group once as a block, multiply that block by its subscript, and only then add the remaining atoms. Relative masses have no units, being ratios against carbon-12.

Worked solution**1. Price the ammonium block and apply the bracket**

One ammonium group is one nitrogen and four hydrogens:

$$M_r(\text{NH}_4) = 14 + (4 \times 1) = 18$$

The subscript 2 outside the bracket multiplies everything inside it, the nitrogen and all four hydrogens alike, so the two ammonium groups contribute

$$2 \times 18 = 36$$

2. Price the sulfate block and add

$$M_r(\text{SO}_4) = 32 + (4 \times 16) = 32 + 64 = 96$$

$$M_r((\text{NH}_4)_2\text{SO}_4) = 36 + 96 = 132$$

Cross-check atom by atom rather than by block: 2 nitrogen = 28, 8 hydrogen = 8, 1 sulfur = 32, 4 oxygen = 64, and $28 + 8 + 32 + 64 = 132$. The two routes agree, which confirms the bracket was expanded correctly.

Sanity check: the salt must weigh more than the sulfate part alone, 96, and the extra can only be the two light ammonium groups, worth 36 between them. A value below 100 would mean the ammonium had been lost, and a value near 200 would mean something had been double counted.

Matches Option A.

Why the other options are wrong

- **B** (118): Applied the outside subscript to the hydrogens and not to the nitrogen, doubling four hydrogens to eight but leaving one nitrogen, so $(NH_4)_2$ was priced as $14 + 8 = 22$ and then $22 + 96 = 118$. The subscript multiplies every atom inside the bracket, giving two nitrogens as well as eight hydrogens, so the block is worth 36. (*Bracket applied to hydrogen only*)
- **C** (96): Quoted the sulfate part alone, $32 + 64 = 96$, leaving out both ammonium groups. Those two groups add a further 36 to the formula mass. (*Cation omitted*)
- **D** (130): Priced the cation as ammonia rather than ammonium, $NH_3 = 14 + 3 = 17$, so the two groups came to $2 \times 17 = 34$ and the total to $34 + 96 = 130$. The group in this salt is NH_4 , worth $14 + 4 = 18$, so the pair is worth 36. (*Ammonia used in place of ammonium*)
- **E** (128): Applied the subscript to the nitrogen only and left the hydrogens single, pricing $(NH_4)_2$ as $N_2H_4 = 28 + 4 = 32$, then $32 + 96 = 128$. The 2 outside the bracket multiplies the four hydrogens as well. (*Bracket partially expanded*)
- **F** (228): Read the subscript 2 as multiplying the entire formula instead of the bracketed group alone, so one unit was priced at $18 + 96 = 114$ and then doubled: $2 \times 114 = 228$. Only the ammonium group stands inside the bracket; the sulfate is outside it and is counted once. (*Bracket subscript applied to the whole formula*)
- **G** (84): Dropped the subscript on oxygen and priced the anion as SO : $36 + 32 + 16 = 84$. Sulfate carries four oxygens, worth 64, not one. (*Subscript on oxygen ignored*)
- **H** (114): Left the outside subscript unused and counted a single ammonium group: $18 + 96 = 114$. The bracket subscript doubles the whole group, so 36 is needed, not 18. (*Bracket subscript not applied*)

Common mistake. Multiplying only the first atom inside the bracket by the subscript, so $(NH_4)_2$ is treated as N_2H_4 and priced at 32 instead of 36. The subscript outside a bracket applies to every atom within it, giving two nitrogens and eight hydrogens.

Takeaway. The Cheat Code: work in blocks. Find the M_r of each bracketed group first, multiply that whole block by its subscript, then add the rest. A bracket is never distributed to one atom and not the others.

Time-saving tip. Speed Heuristic: memorise the common ions as single numbers and reuse them. Ammonium is 18 and sulfate is 96, so this salt is $96 + 2 \times 18 = 132$ in one line, and those two block values reappear in every ammonium and every sulfate question on the paper.

Question 4

C2.4 • The Periodic Table • ★★☆☆☆ • 85 s

Element Q lies in the same Period as chlorine and two Groups to the left of it. A chlorine atom has the electron configuration 2, 8, 7. Which configuration belongs to an atom of Q?

- A. 2, 8, 8
- B. 2, 5
- C. 2, 8, 2
- D. 2, 8, 3
- E. 2, 8, 4
- F. 2, 8, 6
- G. 2, 8, 5
- H. 2, 8, 8, 1

Answer G. Key idea

Position in the Periodic Table and electron configuration carry the same information twice over. The number of electrons in the outermost shell is the Group number and the number of occupied shells is the Period, so a move across a Period changes only the last figure of the configuration.

Worked solution

1. Read chlorine's position out of its configuration

The last number is the outer shell. Chlorine has 7 electrons there, so it sits in Group 7. The count of numbers is the number of occupied shells: 2, then 8, then 7 is three shells, so chlorine sits in Period 3.

2. Move two Groups to the left and rebuild the configuration

Q is in the same Period, so it still has three occupied shells, and the two inner ones stay full at 2 and 8. Two Groups to the left of Group 7 is $7 - 2 = 5$, so Q has 5 electrons in its outer shell.

Putting the three shells together:

2, 8, 5

Check: $2 + 8 + 5 = 15$ electrons, so Q has atomic number 15. That is phosphorus, which does sit in Period 3 two columns to the left of chlorine, with sulfur between the two, and its five outer electrons read straight back as Group 5.

Matches Option G.

Why the other options are wrong

- **A** (2, 8, 8): Gave the configuration Q reaches once it has reacted rather than the configuration of the atom. A Group 5 atom gains three electrons to fill its outer shell, $5 + 3 = 8$, so Q^{3-} is 2, 8, 8, the same arrangement as argon. The stem asks for an atom of Q, which keeps its own 15 electrons and so is 2, 8, 5. (*Ion given instead of the atom*)
- **B** (2, 5): Counts chlorine's Period as 2 by counting only its full shells, the 2 and the 8, then puts Q's five outer electrons in a second shell. Three shells are occupied in 2, 8, 7, so Q keeps three: 2, 8, 5. (*Period read from the full shells only*)
- **C** (2, 8, 2): Takes the words 'two Groups' as the Group number itself, placing Q in Group 2 of Period 3, which is magnesium, 2, 8, 2. Two Groups to the left of Group 7 is Group 5. (*Group number taken from the wrong words*)
- **D** (2, 8, 3): Carried out the same instruction twice over, once to fix the Group and once again on the electron count: $7 - 2 = 5$, then $5 - 2 = 3$, which is aluminium, 2, 8, 3. The move across is made once, and Group 5 already means five electrons in the outer shell. (*Shift applied twice*)
- **E** (2, 8, 4): Read 'two Groups to the left' as leaving two Groups in between, so sulfur and phosphorus were both stepped over and Q was placed at silicon: $7 - 3 = 4$ outer electrons, 2, 8, 4. Two Groups to the left means two columns of movement, which lands on phosphorus with sulfur alone in between. (*Counted the Groups in between*)
- **F** (2, 8, 6): Moves one column instead of two, landing on Group 6, which is sulfur, 2, 8, 6. The move is $7 - 2 = 5$, so the outer shell holds five electrons, not six. (*Moved one column too few*)
- **H** (2, 8, 8, 1): Moves two places to the right instead of the left, passing argon, 2, 8, 8, and arriving at potassium, 2, 8, 8, 1. That atom has four occupied shells, so it is not even in chlorine's Period. (*Moved the wrong way*)

Common mistake. Reading 'two Groups to the left' as 'Group 2' and answering 2, 8, 2. The phrase counts two columns across from chlorine's own Group 7, which lands on Group 5, not on Group 2.

Takeaway. The Cheat Code: The last figure of a configuration is the Group and the count of figures is the Period. Stepping along a Period moves only that last figure, by one for each column crossed.

Time-saving tip. Speed Heuristic: Only the final figure changes. Chlorine ends in 7, so two columns to the left ends in 5, and the leading 2, 8 is untouched, which disposes of A at once; the answer must also keep exactly three shells, which disposes of E.

Question 5

C7.1 · Groups 1, 17 and 18 · ★★☆☆☆ · 90 s

A technician cuts a cube of sodium from a lump kept under oil and drops it into a trough of cold water. The cube floats, and within a second or two it melts into a ball that darts across the surface, leaving the water alkaline. Why does the cube melt?

- A. The oil clinging to the freshly cut surface catches fire on the water, and its flame melts the metal.
- B. Friction with the water, as the cube is driven across the surface, heats it up to its melting point.
- C. The hydrogen given off burns as it escapes, and that flame supplies the heat needed to melt the metal.
- D. Sodium's metallic bonding is weak, so the cold water breaks the lattice apart without any heating.
- E. Sodium's melting point is low, and its reaction with the water releases heat fast enough to reach that temperature.
- F. The sodium hydroxide formed dissolves into the metal and lowers its melting point below room temperature.

Answer E. Key idea

Two properties of the alkali metals meet in this demonstration. The chemical one is that a Group 1 metal reacts with water to give hydrogen and a dissolved hydroxide, releasing a great deal of energy in a very short time. The physical one is that these metals melt at temperatures far below those of ordinary metals. A reaction hot enough and a melting point low enough together explain a solid metal turning liquid in a trough of cold water.

Worked solution**1. Identify the reaction and where its energy goes**

Sodium is an alkali metal, so with water it gives hydrogen and a hydroxide:



The sodium hydroxide dissolving in the trough is why the water is left alkaline. The reaction is strongly exothermic, and it takes place at the surface of the cube, so the energy is delivered straight into a small piece of metal instead of being spread through the whole trough.

2. Bring in the physical property that makes melting possible

Alkali metals are soft and low melting, and sodium melts a little below 100 °C, which is extraordinarily low for a metal. A cube weighing a fraction of a gram needs very little energy to reach that temperature, so a reaction running fast at its surface carries it past the melting point in a second or two, and surface tension then pulls the liquid into a ball.

3. Rule out the other suggested sources of heat

Hydrogen escaping from sodium on cold water often does not ignite at all, and the ball forms whether it does or not, so no flame is needed. The oil is carried off by the knife and by the water. Friction is not the source either: a piece trapped on a floating scrap of filter paper stays where it is put and still melts. And the hydroxide formed is soluble, so it passes into the trough rather than into the metal, and cannot change the metal's melting point.

Sanity check: sodium warmed to 100 °C in a dry tube melts with no water and no flame anywhere near it, which shows the melting point is doing the work.

Matches Option E.

Why the other options are wrong

- **A** (The oil clinging to the freshly cut surface catches fire on the water, and its flame melts the metal.): Blames the storage oil. Most of it is wiped away by the cut and the rest floats off on the water, and a film thin enough to cling could not release the energy needed to melt the metal beneath it. (*Heat attributed to the storage oil*)
- **B** (Friction with the water, as the cube is driven across the surface, heats it up to its melting point.): Puts the heat into friction. A piece of sodium trapped on a floating scrap of filter paper stays where it is put and melts just the same, so the darting about cannot be the source; it is a consequence of the escaping gas. (*Heat attributed to friction*)
- **C** (The hydrogen given off burns as it escapes, and that flame supplies the heat needed to melt the metal.): Puts the heat in a flame rather than in the reaction. Hydrogen from sodium on cold water frequently does not ignite, and the ball still forms, so the melting cannot depend on the gas burning. (*Heat attributed to burning hydrogen*)
- **D** (Sodium's metallic bonding is weak, so the cold water breaks the lattice apart without any heating.): Explains the ball as a lattice coming apart in water instead of as metal that has been warmed past its melting point. Weak metallic bonding is the reason sodium's melting point is low, not a way of reaching it: the cube enters the water solid and rounds only after a second or two of reaction, and the same cube melts in a dry tube warmed to about 100 °C with no water present at all. (*Melting attributed to weak bonding rather than to heating*)
- **F** (The sodium hydroxide formed dissolves into the metal and lowers its melting point below room temperature.): Treats the product as acting on the metal. Sodium hydroxide is soluble and passes into the trough, which is why the water is left alkaline, and sodium's melting point is close to 100 °C, nowhere near room temperature. (*Product credited with lowering the melting point*)

Common mistake. Crediting the burning hydrogen with the melting. Sodium on cold water melts before any flame appears, and very often no flame appears at all. The heat comes from the reaction itself, released at the metal's surface faster than the cold water can carry it away.

Takeaway. The Cheat Code: when a Group 1 metal melts on water, two facts are working together: the reaction is strongly exothermic and the metal's melting point is unusually low. Neither fact on its own explains it, and no outside flame is needed.

Time-saving tip. Speed Heuristic: ask what each explanation needs before it can work. A flame, a burning oil film, friction and a hydroxide inside the metal are all extra assumptions, while a low melting point and an exothermic reaction are properties sodium has already.

Question 6

C4.8 · Quantitative chemistry · ★★☆☆ · 40 s

Calculate the volume of O_2 gas produced at room temperature and pressure (RTP) when 50.0 g of potassium chlorate decomposes according to the equation: $2KClO_3(s) \rightarrow 2KCl(s) + 3O_2(g)$ (Take molar gas volume at RTP = $24.0 \text{ dm}^3 \text{ mol}^{-1}$, $M_r(\text{potassiumchlorate}) = 122.5$).

- A. 29.39 dm^3
- B. 14.69 dm^3
- C. 7.35 dm^3
- D. 13.71 dm^3
- E. 1200.0 dm^3
- F. 9.80 dm^3
- G. 6.53 dm^3
- H. 146.9 dm^3

Answer B. Key idea

Apply fundamental principles of Quantitative chemistry. Isolate the unknown variable systematically and check dimensional and physical consistency.

Worked solution

1. Calculate moles of reactant

$$n = \frac{\text{mass}}{M_r} = \frac{50.0}{122.5} = 0.4082 \text{ mol}$$

2. Apply mole ratio from balanced equation

$$\text{Moles of } O_2 = 0.4082 \times 1.5 = 0.6122 \text{ mol}$$

3. Calculate gas volume at RTP

$$V = n \times 24.0 \text{ dm}^3 \text{ mol}^{-1} = 0.6122 \times 24.0 = 14.69 \text{ dm}^3$$

Matches Option B.

Why the other options are wrong

- **A** (29.39 dm^3): Multiplies by the 3 in front of O_2 but forgets the 2 in front of $KClO_3$: $0.4082 \times 3 = 1.2245 \text{ mol}$, then $1.2245 \times 24.0 = 29.39 \text{ dm}^3$. Using 3 rather than $\frac{3}{2}$ doubles the answer exactly. (*Divisor of the ratio dropped*)
- **C** (7.35 dm^3): Arises from a common inversion, algebraic sign error, or omitting an intermediate conversion factor. (*Calculation / Conceptual Error*)
- **D** (13.71 dm^3): Arises from a common inversion, algebraic sign error, or omitting an intermediate conversion factor. (*Calculation / Conceptual Error*)
- **E** (1200.0 dm^3): Arises from a common inversion, algebraic sign error, or omitting an intermediate conversion factor. (*Calculation / Conceptual Error*)
- **F** (9.80 dm^3): Converts the mass to moles correctly but reads the equation as one to one, taking one mole of O_2 per mole of chlorate: $50.0/122.5 = 0.4082 \text{ mol}$, then $0.4082 \times 24.0 = 9.80 \text{ dm}^3$. The equation gives three O_2 for every two $KClO_3$, so the moles of gas are 1.5 times the moles of solid. (*Mole ratio ignored*)
- **G** (6.53 dm^3): Applies the ratio upside down, $0.4082 \times \frac{2}{3} = 0.2721 \text{ mol}$, then $0.2721 \times 24.0 = 6.53 \text{ dm}^3$. The 2 belongs to $KClO_3$ and the 3 to O_2 , so moving from chlorate to oxygen multiplies by $\frac{3}{2}$, not $\frac{2}{3}$. (*Mole ratio inverted*)
- **H** (146.9 dm^3): Arises from a common inversion, algebraic sign error, or omitting an intermediate conversion factor. (*Calculation / Conceptual Error*)

Common mistake. Multiplying the mass directly by 24 without converting to moles first.

Takeaway. The Cheat Code: Always inspect units and algebraic structure before calculating. Simplifying fractions and factoring terms early reduces computation time by more than 50%.

Time-saving tip. Speed Heuristic: Look at the relative magnitudes and units in the multiple-choice options to eliminate extreme or impossible values immediately.

Question 7

C3.4 · Chemical reactions and equations · ★★☆☆☆ · 85 s

In a blast furnace, iron(III) oxide (Fe_2O_3) is reduced by carbon monoxide to iron and carbon dioxide. Balanced with the smallest whole numbers, what is the sum of all the coefficients in this equation?

- A. 7
- B. 18
- C. 8
- D. 12
- E. 11
- F. 5
- G. 4
- H. 9

Answer H. Key idea

A balanced equation conserves every element, and once the coefficients are reduced to the smallest whole numbers there is only one set that does it. A formula written with no number in front of it still stands for one formula unit, so it counts when the coefficients are totalled.

Worked solution**1. Balance the equation**

The skeleton is $Fe_2O_3 + CO \rightarrow Fe + CO_2$.

Iron first: two Fe atoms on the left need $2Fe$ on the right.

Carbon and oxygen travel together, so let the CO and the CO_2 coefficients both be x . Oxygen on the left is $3 + x$, three from the oxide and one from each CO, and on the right it is $2x$. Setting $3 + x = 2x$ gives $x = 3$.

Balanced: $Fe_2O_3 + 3CO \rightarrow 2Fe + 3CO_2$. Check the atoms: Fe $2 = 2$, C $3 = 3$, O $3 + 3 = 6$ against $3 \times 2 = 6$.

2. Add the coefficients, including the unwritten one

Fe_2O_3 carries no number in front of it, which means one formula unit, not none. The four coefficients are 1, 3, 2 and 3:

$$1 + 3 + 2 + 3 = 9$$

Sanity check: no smaller whole numbers balance this equation, and doubling every coefficient would give $2 + 6 + 4 + 6 = 18$, so 9 is the smallest sum available.

Matches Option **H**.

Why the other options are wrong

- **A (7)**: Sets both carbon species to 2 so that they match the two iron atoms, $Fe_2O_3 + 2CO \rightarrow 2Fe + 2CO_2$, giving $1 + 2 + 2 + 2 = 7$. Oxygen is $3 + 2 = 5$ on the left and 4 on the right, so it is still unbalanced. (*Coefficients copied from the iron*)
- **B (18)**: Doubles everything to keep the oxygen count even, $2Fe_2O_3 + 6CO \rightarrow 4Fe + 6CO_2$, and totals $2 + 6 + 4 + 6 = 18$. Every atom balances, Fe $4 = 4$, C $6 = 6$, O $6 + 6 = 12$ against 12, but the question asks for the smallest whole numbers and every coefficient here still divides by two. (*Coefficients not reduced*)
- **C (8)**: Balances the equation correctly but adds only the coefficients that appear in print, $3 + 2 + 3 = 8$, missing the unwritten 1 in front of Fe_2O_3 . (*Invisible coefficient dropped*)

- **D (12):** Reaches for the other blast furnace equation, reduction by carbon rather than by carbon monoxide: $2Fe_2O_3 + 3C \rightarrow 4Fe + 3CO_2$ totals $2 + 3 + 4 + 3 = 12$. That equation is balanced, but the stem fixes carbon monoxide as the reducing agent, so CO stands on the left and CO_2 on the right. (*Coke equation recalled instead*)
- **E (11):** Totals the atoms rather than the numbers written in front of the formulae. On the left Fe_2O_3 contributes 5 atoms and $3CO$ contributes $3 \times 2 = 6$, giving 11, and since the equation balances the right hand side returns the same, $2 + 3 \times 3 = 11$. A coefficient is the multiplier in front of a formula, not the number of atoms that formula stands for. (*Atoms counted instead of coefficients*)
- **F (5):** Balances the iron and stops there, $Fe_2O_3 + CO \rightarrow 2Fe + CO_2$, giving $1 + 1 + 2 + 1 = 5$. Oxygen is then $3 + 1 = 4$ on the left against 2 on the right. (*Stopped after the metal*)
- **G (4):** Treats the skeleton $Fe_2O_3 + CO \rightarrow Fe + CO_2$ as already balanced and adds four invisible ones, $1 + 1 + 1 + 1 = 4$. It is not balanced: two Fe atoms on the left face one on the right. (*Skeleton equation counted*)

Common mistake. Adding only the numbers that are actually printed. Fe_2O_3 carries an invisible coefficient of 1, so totalling $3 + 2 + 3$ reports 8 for an equation that contains four species.

Takeaway. The Cheat Code: Balance the metal first, then the element that appears in only one compound on each side, and leave oxygen until last when it is split between two compounds. Then count every formula, including the ones whose coefficient is an unwritten 1.

Time-saving tip. Speed Heuristic: Carbon and oxygen move together here, so the CO and CO_2 coefficients must be equal, and the three spare oxygen atoms in Fe_2O_3 force both of them to be 3 with no trial and error at all.

Question 8

C8.2 · Separation techniques · ★★☆☆☆ · 90 s

A technician stirs dry sand, which does not dissolve, into a blue solution of copper(II) sulfate in water. Dry crystals of copper(II) sulfate must be recovered from the mixture, with no sand in them. Which pair of steps, carried out in this order, achieves that?

- A. Heat the whole mixture until every drop of water has boiled away, then collect the solid left in the dish.
- B. Boil the whole mixture and condense the vapour, then keep the clear liquid that collects in the receiver.
- C. Cool the whole mixture in ice until crystals appear, then pour it onto filter paper and dry what is caught.
- D. Pour the mixture onto filter paper, then heat the liquid that runs through until it is saturated and cool it slowly.
- E. Pour the mixture onto filter paper, then wash the solid caught in the paper with warm water and dry it.

Answer D. Key idea

A mixture is taken apart by physical processes chosen to match how each component is held in it. A solid that never dissolved is still a separate phase and can be strained out on filter paper, while a solid that has dissolved is spread through the solvent and can only be recovered by removing solvent until the solution is saturated and then cooling it so crystals grow. The order matters, because anything still undissolved when crystals form is trapped inside them.

Worked solution

1. Sort the mixture into its parts

Three things are present: water, which is the solvent; copper(II) sulfate, a solid that has dissolved in that water; and sand, a solid that has not dissolved at all. The wanted product is the dissolved solid, as dry crystals, so the water is a discard and the sand is an impurity to be removed.

2. Take out the insoluble solid first

Sand grains are far larger than the dissolved copper(II) sulfate, so filter paper holds the sand back and lets the blue solution pass. Every gram of the dissolved salt is carried through in that liquid, and the paper keeps the sand. Doing this first is what guarantees a clean product: solid left in the beaker while crystals grow is trapped inside them.

3. Recover the dissolved solid from the clear liquid

Heat the blue filtrate to drive off part of the water until the solution is saturated, which is when a drop on a cold rod crystallises. Then let it cool slowly. Solubility falls as the temperature falls, so the excess salt comes out of solution as crystals, which are filtered off and dried.

Sanity check: the sand never dissolved, so it can only leave on filter paper, and the salt was dissolved, so it can only return as a solid by concentrating and cooling. Boiling everything to dryness would leave a powder still mixed with sand.

Matches Option D.

Why the other options are wrong

- **A** (Heat the whole mixture until every drop of water has boiled away, then collect the solid left in the dish.): Boiling every drop away does bring the salt out, but sand cannot evaporate: the dish is left holding copper(II) sulfate powder mixed with the sand, which the question forbids. It also gives a powder rather than the crystals asked for. (*Conceptual Error*)
- **B** (Boil the whole mixture and condense the vapour, then keep the clear liquid that collects in the receiver.): Condensing the vapour keeps the water and throws away the target. The copper(II) sulfate is not volatile, so it stays in the flask with the sand, and the liquid collected in the receiver is pure water. (*Wrong component kept*)
- **C** (Cool the whole mixture in ice until crystals appear, then pour it onto filter paper and dry what is caught.): Cooling does grow crystals, but the sand is still suspended in the liquid, so the paper catches sand and crystals together. With no water removed first the solution is not saturated either, so the crop is tiny. (*Correct steps, wrong order*)
- **E** (Pour the mixture onto filter paper, then wash the solid caught in the paper with warm water and dry it.): This keeps the wrong fraction. The paper traps the sand, while the copper(II) sulfate is in the liquid that runs through, and washing the paper with warm water only dissolves any salt still clinging to the sand. (*Residue and filtrate confused*)

Common mistake. Reaching for the heat first. Boiling the whole mixture to dryness does drive off the water, but the sand is still sitting in the dish, so the solid recovered is a mixture rather than pure copper(II) sulfate, and evaporating to dryness gives a fine powder rather than crystals.

Takeaway. The Cheat Code: Take the insoluble solid out before you take the water out. Filter first, then concentrate and cool. Filtering after the crystals have grown just collects the impurity along with them.

Time-saving tip. Speed Heuristic: Ask which half you are keeping. Here it is the dissolved solid, so the water must leave and the liquid running through the paper is the valuable half. That single question kills every route that keeps the condensed vapour or the residue on the paper.

Question 9

C4.1 · Quantitative chemistry · ★★☆☆☆ · 45 s

Calculate the relative molar mass, M_r , of magnesium hydroxide, $Mg(OH)_2$. ($A_r(Mg) = 24$, $A_r(O) = 16$, $A_r(H) = 1$.)

- A. 58
- B. 42
- C. 57
- D. 82
- E. 41
- F. 26
- G. 56
- H. 29

Answer A. Key idea

M_r is the sum of the A_r values, each multiplied by its subscript: $24 + (2 \times 16) + (2 \times 1) = 58$.

Worked solution**1. Read the subscripts off the formula**

$Mg(OH)_2$ contains $1 \times Mg$, $2 \times O$, $2 \times H$.

2. Multiply each A_r by how many of that atom there are

$$M_r = 24 + (2 \times 16) + (2 \times 1)$$

3. Add the contributions

$$M_r = 58$$

M_r is a *relative* mass, so it carries no units.

Matches Option A.

Why the other options are wrong

- **B (42)**: Reads the 2 as belonging to the H it stands next to, giving $24 + 16 + (2 \times 1) = 42$. The bracket closes after the H, so the 2 also applies to the O, which contributes $2 \times 16 = 32$. (*Bracket applied to the hydrogen only*)
- **C (57)**: Doubles the O but leaves the H single: $24 + (2 \times 16) + 1 = 57$. The subscript sits outside the bracket, so it multiplies everything inside it and the hydrogen contributes $2 \times 1 = 2$. (*Bracket applied to the oxygen only*)
- **D (82)**: Mg was counted twice. Each element in the formula contributes exactly once, multiplied by its subscript. (*Double counting*)
- **E (41)**: Every subscript was read as 1, so the repeated atoms were counted only once. Each subscript multiplies the A_r of the atom it follows. (*Ignored subscripts*)
- **F (26)**: The contribution of O was left out altogether. Work through the formula atom by atom so nothing is missed. (*Omitted a term*)
- **G (56)**: Drops the hydrogen because $A_r(H) = 1$ looks too small to matter: $24 + (2 \times 16) = 56$. Every atom in the formula contributes, and the two hydrogen atoms add 2. (*Hydrogen discarded as negligible*)
- **H (29)**: Roughly half the true value, usually the result of halving somewhere, or of dropping a large multiple. M_r is a plain sum, with no division anywhere. (*Spurious division*)

Common mistake. Ignoring a subscript that sits outside a bracket, such as the 2 in $\text{Mg}(\text{OH})_2$, which applies to *both* the O and the H.

Takeaway. The Cheat Code: work left to right through the formula and write the running total as you go. Bracketed groups multiply everything inside them, expand the bracket before you add anything.

Time-saving tip. Read what the question actually asks for before you calculate: a large share of lost marks here come from producing a correct intermediate value and stopping one step early.

Question 10

C2.2 · The Periodic Table · ★★☆☆ · 85 s

Tellurium has a higher relative atomic mass than iodine, 127.6 against 126.9, yet every modern Periodic Table places tellurium first, with iodine falling in the halogen column beneath bromine. Which rule does the modern table order its elements by?

- A. Elements are ordered by increasing number of neutrons, and tellurium's atoms hold fewer neutrons than iodine's atoms.
- B. Elements are ordered by increasing relative atomic mass, with awkward pairs swapped to keep chemical families together.
- C. Elements are ordered by decreasing atomic radius, and tellurium's atoms are larger than iodine's atoms.
- D. Elements are ordered by increasing proton number, and tellurium has 52 protons against iodine's 53.
- E. Elements are ordered by increasing number of outer-shell electrons, six for tellurium and seven for iodine.
- F. Elements are ordered by increasing number of occupied electron shells, which is greater for iodine than for tellurium.

Answer D. Key idea

The modern Periodic Table is ordered by proton number, and every rival ordering is either a consequence of that rule or an approximation to it. Proton number is fixed for an element, since all of its isotopes share it, so it hands each element exactly one position and increases by exactly one from each element to the next. Relative atomic mass is an average taken over an isotope mixture, so it can run backwards between neighbours, which is why the tellurium and iodine pair is the standard test of which rule the table actually obeys.

Worked solution

1. Test the relative atomic mass rule against what the table does

If relative atomic mass set the order, the lighter element would come first. Iodine at 126.9 is the lighter of the pair, so a mass ordered table would read iodine and then tellurium. Every modern table reads tellurium and then iodine, so the observed order contradicts the mass rule outright. The chemistry contradicts it as well: mass ordering would drop iodine into the column headed by oxygen and sulfur, where it would be expected to form compounds of the type H_2X and to have no halogen behaviour, whereas iodine in fact forms HI , NaI and I_2 exactly as chlorine and bromine do.

2. Apply the proton number rule and check both facts at once

Tellurium has 52 protons and iodine has 53. Counting up in proton number puts tellurium at position 52 and iodine at position 53, which is the order the table uses, with no exception made and nothing moved by hand. The same rule lands iodine in Group 7 beneath bromine, which is where its chemistry says it belongs.

Sanity check: every isotope of one element shares its proton number but differs in mass, so only proton number gives each element a single unambiguous place in the sequence. Relative atomic mass cannot, and the tellurium and iodine pair is precisely where that failure shows.

Matches Option D.

Why the other options are wrong

- **A** (Elements are ordered by increasing number of neutrons, and tellurium's atoms hold fewer neutrons than iodine's atoms.): Substitutes the neutron count for the proton count, and states the fact backwards as well. Iodine's only stable isotope holds $127 - 53 = 74$ neutrons, while tellurium's common isotopes hold $128 - 52 = 76$ and $130 - 52 = 78$, so tellurium carries more neutrons rather than fewer, and those extra neutrons are exactly why it is the heavier element despite having one proton less. Neutron count also differs from one isotope of an element to the next, so it cannot fix a single place for each element. (*Neutron count used in place of proton count*)
- **B** (Elements are ordered by increasing relative atomic mass, with awkward pairs swapped to keep chemical families together.): This is Mendeleev's rule with a patch stitched on. Mass ordering gives iodine at 126.9 before tellurium at 127.6, so the pair has to be swapped deliberately to keep iodine with the halogens. The modern table needs no such patch, because proton number already gives the observed order. (*Superseded rule kept with an exception*)
- **C** (Elements are ordered by decreasing atomic radius, and tellurium's atoms are larger than iodine's atoms.): Uses a trend in place of a rule. Atomic radius does fall from tellurium to iodine across Period 5, but it jumps back up at the start of the next Period, so it cannot generate one continuously increasing sequence for the whole table. (*Periodic trend mistaken for the ordering*)
- **E** (Elements are ordered by increasing number of outer-shell electrons, six for tellurium and seven for iodine.): Works inside a single Period and fails everywhere else. Tellurium has six outer electrons and iodine seven, which happens to give the right order for this pair, but the count drops back to one at the start of the next Period, so the sequence breaks at every row boundary. (*Local pattern generalised*)
- **F** (Elements are ordered by increasing number of occupied electron shells, which is greater for iodine than for tellurium.): Occupied shells label the Periods, not the sequence within one. Tellurium and iodine both lie in Period 5 and so have the same number of occupied shells, which cannot separate them at all, let alone place tellurium first. (*Period label used as the ordering rule*)

Common mistake. Accepting relative atomic mass as the ordering rule and treating tellurium and iodine as an exception that was moved by hand to tidy the columns. Nothing was moved: proton number produces that order automatically, and it is mass ordering, not the table, that fails.

Takeaway. The Cheat Code: when a table position and a relative atomic mass disagree, back the proton number. Mass is an average over isotopes and can invert between neighbours, while proton number counts up by one every single time.

Time-saving tip. Speed Heuristic: an ordering rule has to give a whole number that rises by exactly one from each element to the next, right across the table. Radius rises and falls, outer electrons reset at the start of every Period, shell count is constant along a Period, and mass inverts here. Only proton number survives that filter.

END OF SOLUTIONS

Questions 11 to 27 are in the full paper

This is an extract. The 10 questions before this page are real questions from a commissioned worked solutions, shown in full and with nothing withheld, so that the standard can be judged rather than described.

The rest of the paper is not hidden behind this page. It was never in this file. Each commissioned paper is written for one school, recorded against it, and issued to no other, so the questions in it are not published anywhere, including here.

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EXAMPLE GRAMMAR SCHOOL

ESAT Mock Examination

Biology

Paper SMP-2026-09 · Version 1.0

NAME

TEACHING GROUP

DATE

Instructions

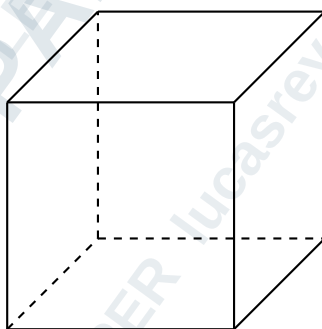
- Time allowed: **15 minutes**.
- There are **10 multiple-choice questions**. Each is worth 1 mark.
- There is no negative marking. Answer every question, even if you have to guess.
- Calculators are not permitted.
- Record your answers on the separate answer sheet. Only the answer sheet is marked.
- Formulae are not provided. The ESAT does not supply a formula sheet.

Note: the real ESAT is sat on screen at a Pearson test centre. This paper trains the mathematics and the pacing, not the on-screen interface.

Written for Example Grammar School by Lucas Rey, Engineering Science, University of Oxford; MRes, University of Cambridge.

Question 1

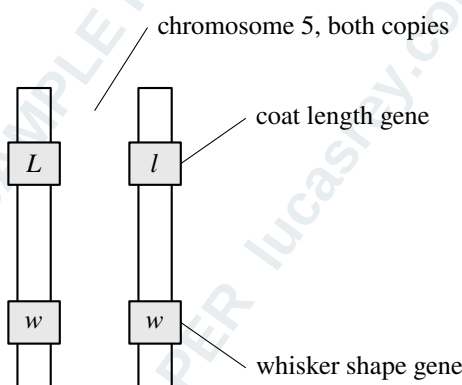
The diagram shows a cubic cell, with the length of one side marked. What is its surface area to volume ratio in its simplest integer form?



- A. 6 : 1
- B. 1 : 10
- C. 3 : 5
- D. 5 : 3
- E. 3 : 50

Question 2

In domestic cats, one gene on chromosome 5 controls coat length, with the short-fur allele L completely dominant over the long-fur allele l . A second gene on the same chromosome controls whisker shape, with the straight allele W completely dominant over the curled allele w . The diagram shows the alleles carried on one cat's two copies of chromosome 5. Which statement describes this cat correctly?



- A. It is homozygous for coat length and heterozygous for whisker shape, so it has short fur and straight whiskers.
- B. It is homozygous for coat length and homozygous for whisker shape, so it has short fur and curled whiskers.
- C. It is heterozygous for coat length and heterozygous for whisker shape, so it has short fur and straight whiskers.
- D. It is heterozygous for coat length and homozygous for whisker shape, so it has short fur and curled whiskers.
- E. It is heterozygous for coat length and homozygous for whisker shape, so it has long fur and curled whiskers.
- F. It is homozygous for coat length and homozygous for whisker shape, so it has long fur and curled whiskers.

Question 3

Gene expression is controlled from the nucleus by

- A. selecting which genes are transcribed to mRNA.
- B. deleting unused genes permanently.
- C. moving genes to the ribosomes.
- D. choosing which proteins are eaten.
- E. dissolving the nuclear membrane.

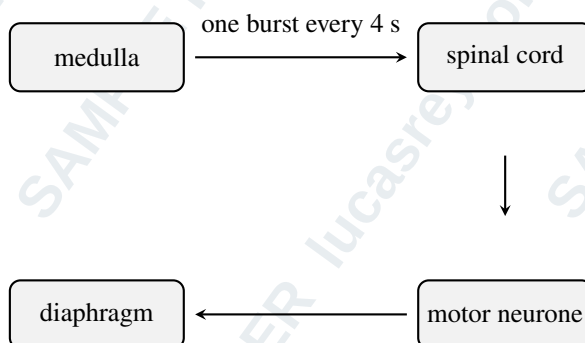
Question 4

A sperm carrying a Y chromosome fertilises an X egg. The child will be

- A. female (XX).
- B. female (X).
- C. male (YY).
- D. male (XX).
- E. male (XXY).
- F. male (XY).
- G. female (XY).
- H. cannot be determined.

Question 5

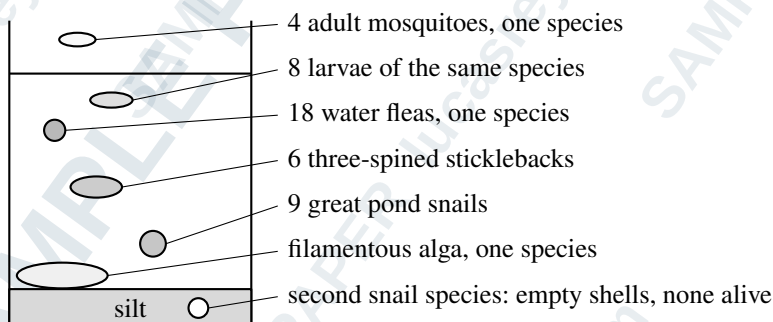
The diagram shows the pathway and the timing of the bursts of impulses that drive each inspiration in a resting adult. Every burst contains 30 impulses, and each whole burst produces one contraction of the diaphragm, drawing 450 cm^3 of air into the lungs. Breathing stays quiet and regular. Calculate the volume of air drawn into the lungs each minute.



- A. $6750 \text{ dm}^3 \text{ min}^{-1}$
- B. $13.5 \text{ dm}^3 \text{ min}^{-1}$
- C. $27.0 \text{ dm}^3 \text{ min}^{-1}$
- D. $0.45 \text{ dm}^3 \text{ min}^{-1}$
- E. $1.8 \text{ dm}^3 \text{ min}^{-1}$
- F. $202.5 \text{ dm}^3 \text{ min}^{-1}$
- G. $67.5 \text{ dm}^3 \text{ min}^{-1}$
- H. $6.75 \text{ dm}^3 \text{ min}^{-1}$

Question 6

A drainage ditch is surveyed as part of a study of freshwater ecology. The diagram shows the ditch in cross-section, and what the survey recorded in it. The surveyors also record the temperature of the water, the depth of the silt and the concentration of dissolved oxygen. Assume the survey found every species that is present as a living organism, and that the ditch is isolated from other water bodies. How many populations make up the community in this ditch?



- A. 5
- B. 4
- C. 6
- D. 7
- E. 45

Question 7

A single cell undergoes 4 consecutive rounds of mitosis. Assuming all daughter cells survive and divide, how many cells will be present at the end?

- A. 8
- B. 488
- C. 16
- D. 831
- E. 547

Question 8

Which list goes from simplest to most complex?

- A. muscle cell → muscle tissue → heart → circulatory system.
- B. circulatory system → heart → muscle tissue → muscle cell.
- C. muscle tissue → muscle cell → circulatory system → heart.
- D. muscle cell → heart → muscle tissue → circulatory system.
- E. heart → muscle cell → muscle tissue → circulatory system.
- F. muscle cell → muscle tissue → circulatory system → heart.

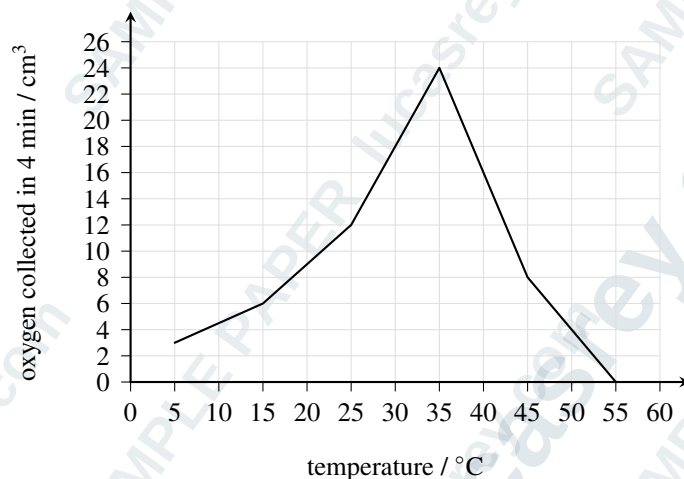
Question 9

A gardener takes a single healthy potato plant and grows 200 new plants from its tubers, planting no seed. She then crosses two different potato varieties and raises 200 seedlings from the seed produced. Assuming no mutations occur, which statement about the two groups of 200 plants is correct?

- A. The tuber-grown plants are all genetically identical; the seedlings are not.
- B. The seedlings are all genetically identical; the tuber-grown plants are not.
- C. All 400 plants are genetically identical to one another.
- D. Neither group contains any two plants that are genetically identical.
- E. Each seedling is genetically identical to one tuber-grown plant, though neither group is uniform.

Question 10

A student investigates catalase, the enzyme in potato tissue that breaks hydrogen peroxide down into water and oxygen. Identical potato discs are dropped into hydrogen peroxide held in a water bath, and the volume of oxygen collected during the first four minutes is recorded at six temperatures. The graph shows the results. Below its optimum temperature this enzyme obeys a temperature coefficient $Q_{10} = 2.0$, meaning that the rate is multiplied by 2.0 for every 10°C rise. Assume hydrogen peroxide is in excess throughout and that all the gas is collected at room temperature and pressure. Using the 15°C reading, calculate the rate of oxygen production at 45°C that this relationship predicts.



- A. $9 \text{ cm}^3 \text{ min}^{-1}$
- B. $24 \text{ cm}^3 \text{ min}^{-1}$
- C. $3 \text{ cm}^3 \text{ min}^{-1}$
- D. $48 \text{ cm}^3 \text{ min}^{-1}$
- E. $6 \text{ cm}^3 \text{ min}^{-1}$
- F. $2 \text{ cm}^3 \text{ min}^{-1}$
- G. $8 \text{ cm}^3 \text{ min}^{-1}$
- H. $12 \text{ cm}^3 \text{ min}^{-1}$

END OF PAPER

Questions 11 to 27 are in the full paper

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EXAMPLE GRAMMAR SCHOOL

ESAT Mock: Answer Sheet

Biology

Paper SMP-2026-09 • Version 1.0

NAME

TEACHING GROUP

DATE

Fill in one bubble per question. If you change your mind, cross out the old mark clearly and fill the new bubble. Unfilled or doubly filled rows score zero.

Q	A	B	C	D	E	F	G	H
1	☐	☐	☐	☐	☐			
2	☐	☐	☐	☐	☐	☐		
3	☐	☐	☐	☐	☐			
4	☐	☐	☐	☐	☐	☐	☐	☐
5	☐	☐	☐	☐	☐	☐	☐	☐
6	☐	☐	☐	☐	☐			
7	☐	☐	☐	☐	☐			
8	☐	☐	☐	☐	☐	☐		
9	☐	☐	☐	☐	☐			
10	☐	☐	☐	☐	☐	☐	☐	☐

MARKER INITIALS

TOTAL OUT OF 10

EXAMPLE GRAMMAR SCHOOL

ESAT Mock: Mark Scheme

Biology

Paper SMP-2026-09 • Version 1.0

Answer grid

Q	Ans	Q	Ans	Q	Ans
1	C	10	H		
2	D				
3	A				
4	F				
5	H				
6	A				
7	C				
8	A				
9	A				

Totals

10 marks available, 1 mark per question. There is no negative marking, so a wrong answer scores zero and never a negative. Only the answer sheet is marked.

Diagnostic table

Use this to see which topics the cohort is losing marks on.

Q	Ans	Node	Group	Topic	Difficulty
1	C	B2.1	B2	Cells	★★☆☆☆
2	D	B4.2	B4	Genetics	★★☆☆☆
3	A	B4.1	B4	Know and understand the nucleus as a site of genetic material in eukaryotic cells	★★☆☆☆
4	F	B3.4	B3	Sex determination	★★☆☆☆
5	H	B9.2	B9	Human physiology	★★☆☆☆
6	A	B10.1	B10	Ecology	★★☆☆☆
7	C	B3.1	B3	Cell division	★☆☆☆☆
8	A	B1.3	B1	Know and understand the levels of organisation within organisms as	★★☆☆☆
9	A	B3.3	B3	Cell division and reproduction	★★☆☆☆
10	H	B8.3	B8	Enzymes and digestion	★★☆☆☆

Marks available by topic group

Group	Topic	Marks
B1	Cells and organisation	1
B2	Transport across membranes	1
B3	Cell division and reproduction	3
B4	Genetics	2
B8	Enzymes and digestion	1
B9	Human physiology	1
B10	Ecology	1
Total		10

Worked solutions for every question are in the separate worked solutions document.

Questions 11 to 27 are in the full paper

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EXAMPLE GRAMMAR SCHOOL

ESAT Mock: Worked Solutions

Biology

Paper SMP-2026-09 • Version 1.0

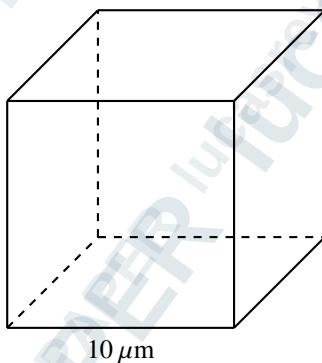
Staff copy

This document contains the answers. Keep it away from candidates until the paper has been sat. Marking guidance is in the separate mark scheme.

Question 1

B2.1 • Cells • ★★☆☆☆ • 40 s

The diagram shows a cubic cell, with the length of one side marked. What is its surface area to volume ratio in its simplest integer form?



- A. 6 : 1
- B. 1 : 10
- C. 3 : 5
- D. 5 : 3
- E. 3 : 50

Answer C. Key idea

Surface area grows with the square of a length while volume grows with the cube, so the ratio of the two is not a fixed property of a shape: it depends on size. For a cube of side L the surface area is $6L^2$ and the volume is L^3 , so the ratio is $6L^2 : L^3$, which simplifies to $6 : L$. The larger the cell, the smaller its surface area to volume ratio, which is why a large cell cannot supply its interior by diffusion across its surface alone.

Worked solution

1. Calculate Surface Area

$$SA = 6 \times L^2 = 6 \times (10)^2 = 6 \times 100 = 600 \mu\text{m}^2$$

2. Calculate Volume

$$V = L^3 = (10)^3 = 1000 \mu\text{m}^3$$

3. Simplify ratio

Ratio = 600 : 1000 Divide both sides by their greatest common divisor (200): 3 : 5

Matches Option C.

Why the other options are wrong

- **A** (6 : 1): Squared the side instead of cubing it when finding the volume, so $V = 100$ rather than 1000. That gives 600 : 100, which simplifies to 6 : 1. (*Calculation Error*)
- **B** (1 : 10): Counted the area of one face only, taking $SA = 10 \times 10 = 100$ instead of $6 \times 100 = 600$. That gives 100 : 1000, which simplifies to 1 : 10. (*Conceptual Error*)
- **D** (5 : 3): Reversed the ratio to Volume : Surface Area, giving 1000 : 600, which simplifies to 5 : 3. (*Reading Error*)
- **E** (3 : 50): Forgot to square the side in the surface area, taking $SA = 6 \times 10 = 60$ instead of $6 \times 100 = 600$. That gives 60 : 1000, which simplifies to 3 : 50. (*Calculation Error*)

Common mistake. Reversing the ratio to Volume:Surface Area.

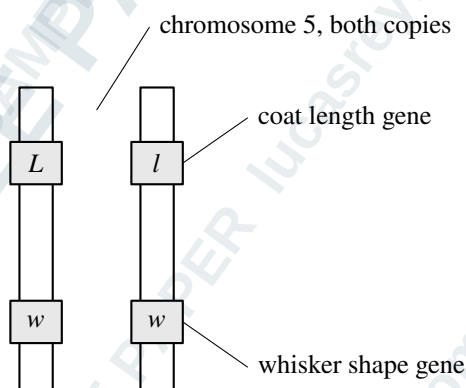
Takeaway. The Cheat Code: for a cube of side L the ratio is always $6 : L$ before simplifying. Compute the two quantities separately, then cancel: never quote the ratio the other way round, because surface area comes first in the name.

Time-saving tip. Speed Heuristic: Look at the relative magnitudes and units in the multiple-choice options to eliminate extreme or impossible values immediately.

Question 2

B4.2 · Genetics · ★★☆☆☆ · 85 s

In domestic cats, one gene on chromosome 5 controls coat length, with the short-fur allele L completely dominant over the long-fur allele l . A second gene on the same chromosome controls whisker shape, with the straight allele W completely dominant over the curled allele w . The diagram shows the alleles carried on one cat's two copies of chromosome 5. Which statement describes this cat correctly?



- It is homozygous for coat length and heterozygous for whisker shape, so it has short fur and straight whiskers.
- It is homozygous for coat length and homozygous for whisker shape, so it has short fur and curled whiskers.
- It is heterozygous for coat length and heterozygous for whisker shape, so it has short fur and straight whiskers.
- It is heterozygous for coat length and homozygous for whisker shape, so it has short fur and curled whiskers.
- It is heterozygous for coat length and homozygous for whisker shape, so it has long fur and curled whiskers.
- It is homozygous for coat length and homozygous for whisker shape, so it has long fur and curled whiskers.

Answer D. Key idea

Homozygous and heterozygous describe one pair of alleles, not an organism as a whole, so the same cat can be homozygous at one gene and heterozygous at another. Whether the two alleles at that gene are identical is the only thing that decides the term. Which characteristic then shows is a separate question, settled by dominance: a completely dominant allele shows whenever it is present, so the recessive characteristic appears only where both alleles at that gene are the recessive one.

Worked solution**1. Compare the two alleles at each gene**

Take the genes one at a time, reading across the pair of chromosomes.

At the coat-length gene the cat carries L on one copy and l on the other. Two different versions of one gene means heterozygous.

At the whisker gene it carries w on one copy and w on the other. Two identical versions means homozygous, and here homozygous for the recessive allele.

2. Apply dominance to get the characteristics

L is completely dominant, so it shows whenever it is present: the Ll cat has short fur. The whisker pair is ww , with no W anywhere on either chromosome, so there is nothing to mask the recessive allele and the whiskers are curled.

The cat is therefore heterozygous for coat length, homozygous for whisker shape, short-furred and curl-whiskered.

Sanity check: the two options offering straight whiskers both need a W that the diagram does not show, and the two options offering long fur both need the L on the left-hand copy to be masked by l , which contradicts L being completely dominant. Only one statement survives both tests.

Matches Option **D**.

Why the other options are wrong

- **A** (It is homozygous for coat length and heterozygous for whisker shape, so it has short fur and straight whiskers.): Swaps the two words, calling the mixed pair Ll homozygous and the identical pair ww heterozygous. Following that through, the supposedly heterozygous whisker pair is then given the dominant characteristic, so the whiskers come out straight. (*Terms Swapped*)
- **B** (It is homozygous for coat length and homozygous for whisker shape, so it has short fur and curled whiskers.): Reads both characteristics correctly, short fur because L is completely dominant in the Ll pair and curled whiskers because the ww pair contains no W to mask the recessive allele, but calls both genes homozygous on the grounds that each gene is present on both copies of chromosome 5. Homozygous asks whether the two alleles at one gene are the same letter, and at the coat-length gene they are L and l , so that pair is heterozygous. This differs from the reading that makes the same mistake with the terms and then also lets the recessive l show and reports long fur: here dominance is applied correctly and only the terminology is wrong, which is why it needs the definition to eliminate. (*Homozygous read as carrying two copies of the gene*)
- **C** (It is heterozygous for coat length and heterozygous for whisker shape, so it has short fur and straight whiskers.): Assumes an individual must be heterozygous at every gene because its two chromosome copies came from different parents. Both pairs are then called heterozygous, and a heterozygote shows the dominant characteristic, giving straight whiskers as well as short fur. (*Conceptual Error*)
- **E** (It is heterozygous for coat length and homozygous for whisker shape, so it has long fur and curled whiskers.): Reads both terms correctly but lets the lower-case l show in the Ll pair. That treats the long-fur allele as the one expressed in a heterozygote, which reverses the dominance stated, and gives long fur. (*Dominance Reversed*)

- **F** (It is homozygous for coat length and homozygous for whisker shape, so it has long fur and curled whiskers.): Takes homozygous to mean nothing more than carrying two alleles of one gene, which is true of every pair, so both genes are called homozygous. The recessive letter is then read off at each, giving long fur and curled whiskers. (*Conceptual Error*)

Common mistake. Calling the whole cat heterozygous because its two chromosomes came from different parents. The terms apply gene by gene, and at the whisker gene the two copies happen to carry the same allele.

Takeaway. The Cheat Code: Read each gene's pair on its own. Two letters the same is homozygous, two letters different is heterozygous. Then, with complete dominance, the small letter shows only where there is no capital in that pair.

Time-saving tip. Speed Heuristic: Go to the whisker gene first. Both copies carry *w*, so curled whiskers are certain and the two options promising straight whiskers go at once. Only the coat-length pair on the three survivors then has to be read.

Question 3

B4.1 · Know and understand the nucleus as a site of genetic material in eukaryotic cells · ★★☆☆☆ · 45 s

Gene expression is controlled from the nucleus by

- A. selecting which genes are transcribed to mRNA.
- B. deleting unused genes permanently.
- C. moving genes to the ribosomes.
- D. choosing which proteins are eaten.
- E. dissolving the nuclear membrane.

Answer A. Key idea

The nucleus controls gene expression by selecting which of its genes are transcribed into mRNA at any given time. The DNA stays inside the nucleus; only the mRNA copy leaves, so control is exercised over transcription rather than over the genes themselves.

Worked solution

The nucleus: stores chromosomes (DNA+protein), transcribes selected genes to mRNA, controls the cell's activities. Prokaryotes keep DNA free in the cytoplasm.

Applied: gene expression is controlled from the nucleus by... → selecting which genes are transcribed to mRNA.

Matches Option A.

Why the other options are wrong

- **B** (deleting unused genes permanently.): Cells regulate TRANSCRIPTION, not genome content. (*Conceptual Error*)
- **C** (moving genes to the ribosomes.): mRNA travels; genes stay. (*Conceptual Error*)
- **D** (choosing which proteins are eaten.): Nutrition is irrelevant to gene expression control. (*Conceptual Error*)
- **E** (dissolving the nuclear membrane.): That happens only in division. (*Conceptual Error*)

Common mistake. Nutrition is irrelevant to gene expression control.

Takeaway. Treat the nucleus as both library and head office: it stores the chromosomes intact and decides which genes get transcribed. Regulation changes which mRNA is made, never which genes the cell possesses.

Time-saving tip. nucleus = library + head office, apply and verify.

Question 4

B3.4 · Sex determination · ★★☆☆☆ · 45 s

A sperm carrying a Y chromosome fertilises an X egg. The child will be

- A. female (XX).
- B. female (X).
- C. male (YY).
- D. male (XX).
- E. male (XXY).
- F. male (XY).
- G. female (XY).
- H. cannot be determined.

Answer F. Key idea

Every egg carries an X, so the sperm's sex chromosome decides the outcome: an X sperm gives a female (XX), a Y sperm gives a male (XY). Here a Y sperm fertilises the X egg, so the zygote is XY and the child is male.

Worked solution

Females are XX (all eggs carry X). Males are XY (sperm split 50/50 X:Y). Punnett square of $XX \times XY$: 2 XX, 2 XY $\rightarrow$ 1:1 sex ratio, decided at fertilisation.

Applied: a sperm carrying a Y chromosome fertilises an X egg. The child will be... $\rightarrow$ male (XY).

Matches Option F.

Why the other options are wrong

- **A** (female (XX).): Two X's make female; this zygote has X and Y. (*Conceptual Error*)
- **B** (female (X).): Records only the mother's contribution, the single X of the egg, and calls the zygote female because no Y has been written down. The sperm's Y enters the zygote as well, giving XY. (*Conceptual Error*)
- **C** (male (YY).): Assumes the male pair matches, by analogy with the female XX: the sperm brought a Y, so the pair is written YY. The egg still contributes its X, so the zygote is XY. (*Conceptual Error*)
- **D** (male (XX).): XX is female genotype. (*Conceptual Error*)
- **E** (male (XXY).): Forgets that meiosis halves the chromosome number and treats the mother as passing both her X chromosomes: XX from the egg plus Y from the sperm gives XXY, called male because a Y is present. An egg carries one X, not two, so the zygote is XY. (*Conceptual Error*)
- **G** (female (XY).): XY determines male. (*Conceptual Error*)
- **H** (cannot be determined.): The chromosomes given settle it. (*Conceptual Error*)

Common mistake. Defaulting to 'female' because the egg contributes an X: the egg always carries X, so it is the sperm's chromosome (here Y) that decides the sex.

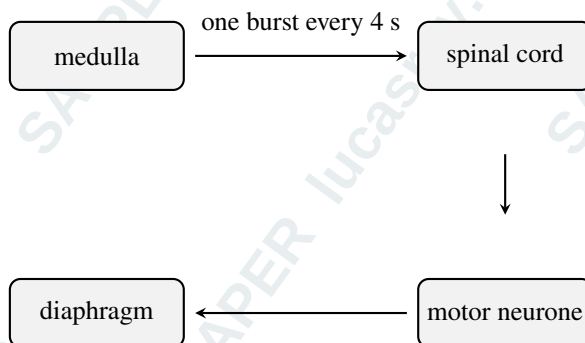
Takeaway. The Cheat Code: Mum can only give an X, so Dad's sperm flips the coin: an X sperm makes XX (female) and a Y sperm makes XY (male), which is why the ratio is 1:1.

Time-saving tip. Mum only has X; Dad's sperm flips the coin, apply and verify.

Question 5

B9.2 · Human physiology · ★★☆☆☆ · 90 s

The diagram shows the pathway and the timing of the bursts of impulses that drive each inspiration in a resting adult. Every burst contains 30 impulses, and each whole burst produces one contraction of the diaphragm, drawing 450 cm^3 of air into the lungs. Breathing stays quiet and regular. Calculate the volume of air drawn into the lungs each minute.



- A. $6750 \text{ dm}^3 \text{ min}^{-1}$
- B. $13.5 \text{ dm}^3 \text{ min}^{-1}$
- C. $27.0 \text{ dm}^3 \text{ min}^{-1}$
- D. $0.45 \text{ dm}^3 \text{ min}^{-1}$
- E. $1.8 \text{ dm}^3 \text{ min}^{-1}$
- F. $202.5 \text{ dm}^3 \text{ min}^{-1}$
- G. $67.5 \text{ dm}^3 \text{ min}^{-1}$
- H. $6.75 \text{ dm}^3 \text{ min}^{-1}$

Answer H. Key idea

A motor neurone carries impulses out from the central nervous system to an effector, and the whole burst that arrives drives one contraction of that effector, however many impulses the burst contains. The volume of air moved per minute is then the volume drawn in by one contraction multiplied by the number of contractions in a minute, with both quantities put into the units the answer is wanted in.

Worked solution**1. Turn the burst interval into a breathing rate**

One whole burst produces one contraction of the diaphragm, so one burst is one inspiration whatever the number of impulses inside it. A burst every 4 seconds gives

$$\frac{60}{4} = 15 \text{ inspirations per minute.}$$

The 30 impulses describe the traffic along the motor neurone that drives a single contraction; they are not a count of breaths.

2. Put the volume into the units of the answer

$$450 \text{ cm}^3 = \frac{450}{1000} = 0.45 \text{ dm}^3 \text{ drawn in per inspiration.}$$

3. Combine volume per breath with breaths per minute

$$0.45 \times 15 = 6.75 \text{ dm}^3 \text{ min}^{-1}.$$

Sanity check: a resting adult moves roughly half a cubic decimetre of air about fifteen times a minute, so an answer near $7 \text{ dm}^3 \text{ min}^{-1}$ is the size expected. An answer of $6750 \text{ dm}^3 \text{ min}^{-1}$ would be over six cubic metres of air every minute.

Matches Option H.

Why the other options are wrong

- **A** ($6750 \text{ dm}^3 \text{ min}^{-1}$): Multiplies 450 by 15 and labels the result in cubic decimetres: $450 \times 15 = 6750$. The tidal volume is measured in cm^3 and $1000 \text{ cm}^3 = 1 \text{ dm}^3$, so it must be divided by 1000 before the multiplication. (*Volume conversion omitted*)
- **B** ($13.5 \text{ dm}^3 \text{ min}^{-1}$): Takes the 30 impulses in a burst as thirty inspirations: $0.45 \times 30 = 13.5$. Every impulse in one burst serves the same single contraction, so the burst counts once, not thirty times. (*Impulses counted as breaths*)
- **C** ($27.0 \text{ dm}^3 \text{ min}^{-1}$): Assumes one inspiration each second, giving sixty per minute: $0.45 \times 60 = 27.0$. The recording places one burst every 4 seconds, so the sixty seconds must be divided by four first. (*Interval dropped from the conversion*)
- **D** ($0.45 \text{ dm}^3 \text{ min}^{-1}$): Converts the tidal volume and stops there: $450 \div 1000 = 0.45$. That is the volume drawn in by one contraction, and it still has to be multiplied by the 15 inspirations a minute contains. (*Volume per breath reported as volume per minute*)
- **E** ($1.8 \text{ dm}^3 \text{ min}^{-1}$): Reads 'a burst every 4 seconds' as four inspirations per minute: $0.45 \times 4 = 1.8$. An interval has to be inverted before it becomes a rate, and $60 \div 4$ gives 15 inspirations per minute. (*Interval used directly as a rate*)
- **F** ($202.5 \text{ dm}^3 \text{ min}^{-1}$): Gets the burst rate right but gives every impulse its own contraction: $30 \times 15 = 450$ contractions per minute, then $0.45 \times 450 = 202.5$. The stem states that one whole burst, all 30 impulses of it, produces a single contraction. (*Each impulse counted as a contraction*)
- **G** ($67.5 \text{ dm}^3 \text{ min}^{-1}$): Divides by 100 instead of 1000: $450 \div 100 = 4.5$, then $4.5 \times 15 = 67.5$. There are 1000 cm^3 in 1 dm^3 , so each inspiration is 0.45 dm^3 . (*Wrong factor in the volume conversion*)

Common mistake. Using the number of impulses in a burst as the breathing rate. The impulses are the signal that produces one contraction; how often breathing happens is set by how often a burst arrives, which here is once every 4 seconds.

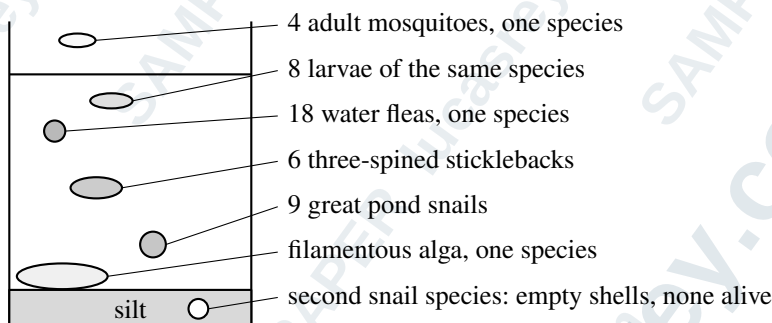
Takeaway. The Cheat Code: Turn an interval into a per-minute rate before anything else, and convert the volume into the units the options use at the same time. Any per-minute total is then volume per event multiplied by events per minute.

Time-saving tip. Speed Heuristic: $60 \div 4 = 15$, then split the multiplication: $0.45 \times 10 = 4.5$ and $0.45 \times 5 = 2.25$, which add to 6.75. The count of 30 impulses never enters the arithmetic.

Question 6

B10.1 · Ecology · ★★☆☆☆ · 90 s

A drainage ditch is surveyed as part of a study of freshwater ecology. The diagram shows the ditch in cross-section, and what the survey recorded in it. The surveyors also record the temperature of the water, the depth of the silt and the concentration of dissolved oxygen. Assume the survey found every species that is present as a living organism, and that the ditch is isolated from other water bodies. How many populations make up the community in this ditch?



- A. 5
- B. 4
- C. 6
- D. 7
- E. 45

Answer A. Key idea

The levels of organisation in an ecosystem are nested: an individual is one organism, a population is all the individuals of one species living in the same place, a community is all the populations living there together, and the ecosystem is that community together with the non-living surroundings it interacts with. Counting populations therefore means counting living species rather than individuals, and a larva and an adult of one species belong to a single population, while a species left only as dead remains has no living members and so no population in that place.

Worked solution**1. Separate the living organisms from everything else recorded**

A community is made up of living organisms only, so the temperature of the water, the depth of the silt and the concentration of dissolved oxygen are set aside straight away. They are the non-living part of the ecosystem, not members of the community.

2. Group the living individuals into species

Water fleas, great pond snails, three-spined sticklebacks, mosquitoes and the filamentous alga make five species. The alga is counted: a population is a population whether the organism photosynthesises or not, and the producers belong to the community as much as the consumers do. The 8 larvae and the 4 adult mosquitoes are one species at two stages of a single life cycle, so they are 12 individuals forming one population, not two populations.

3. Rule out the species with no living members

The second snail species appears only as empty shells in the silt. A population is the living individuals of a species in a place, so with none alive in the ditch that species contributes no population to this community. The community is therefore made up of 5 populations.

Sanity check: every number in the survey, 18, 9, 6, 8 and 4, counts individuals, and individuals sit one level below populations, so the answer must be far smaller than their total of 45.

Matches Option A.

Why the other options are wrong

- **B** (4): Took a community to mean the animals in it and left the filamentous alga out, counting only water fleas, snails, sticklebacks and mosquitoes for 4. A community is every population in the habitat, and the producers are part of it just as the consumers are. (*Producers excluded*)
- **C** (6): Added the second snail species on the strength of its empty shells, giving $5 + 1 = 6$. Shells are dead remains, and with no living individual of that species in the ditch there is no population of it to count. (*Dead remains counted*)
- **D** (7): Counted the empty shells as a population and also split the mosquitoes into a larval population and an adult population, giving $5 + 1 + 1 = 7$. The shells are not alive, and the larvae and adults of one species make one population. (*Dead remains counted and life stages split*)
- **E** (45): Added the individuals recorded, $18 + 9 + 6 + 8 + 4 = 45$, and gave that as the answer. Individuals are the level below populations: those 45 organisms are grouped into populations, and the question asks how many of those groups there are. (*Individuals counted instead of populations*)

Common mistake. Splitting one species across its life stages. The 8 mosquito larvae and the 4 adult mosquitoes look like two different kinds of animal and get counted as two populations, but they are the same species living in the same ditch, so they are 12 individuals of one population.

Takeaway. The Cheat Code: Go down the list once and ask two questions of every entry: is it alive, and is it a species already counted? Anything non-living belongs to the ecosystem rather than the community, and any second life stage, second sex or second colour form of a species already on the list adds nothing to the total.

Time-saving tip. Speed Heuristic: Count species names, not entries. Strike out the three physical measurements, merge the larvae with the adults, and delete the shells, and the survey collapses to five living species before a single number is used.

Question 7

B3.1 · Cell division · ★☆☆☆☆ · 20 s

A single cell undergoes 4 consecutive rounds of mitosis. Assuming all daughter cells survive and divide, how many cells will be present at the end?

- A. 8
- B. 488
- C. 16
- D. 831
- E. 547

Answer C. Key idea

Apply fundamental principles of Cell division. Isolate the unknown variable systematically and check dimensional and physical consistency.

Worked solution

1. Understand mitosis cell division

Each round of mitosis doubles the number of cells. The formula for the number of cells after n divisions is 2^n .

2. Calculate

$$2^4 = 16$$

Matches Option C.

Why the other options are wrong

- **A** (8): Multiplied by 2 instead of calculating 2^n . (*Mathematical Error*)
- **B** (488): Arises from a common inversion, algebraic sign error, or omitting an intermediate conversion factor. (*Calculation / Conceptual Error*)
- **D** (831): Arises from a common inversion, algebraic sign error, or omitting an intermediate conversion factor. (*Calculation / Conceptual Error*)
- **E** (547): Arises from a common inversion, algebraic sign error, or omitting an intermediate conversion factor. (*Calculation / Conceptual Error*)

Common mistake. Multiplying the number of divisions by 2 instead of taking 2 to the power of n .

Takeaway. The Cheat Code: Always inspect units and algebraic structure before calculating. Simplifying fractions and factoring terms early reduces computation time by more than 50%.

Time-saving tip. Speed Heuristic: Look at the relative magnitudes and units in the multiple-choice options to eliminate extreme or impossible values immediately.

Question 8

B1.3 · Know and understand the levels of organisation within organisms as · ★★☆☆☆ · 45 s

Which list goes from simplest to most complex?

- A.** muscle cell → muscle tissue → heart → circulatory system.
- B.** circulatory system → heart → muscle tissue → muscle cell.
- C.** muscle tissue → muscle cell → circulatory system → heart.
- D.** muscle cell → heart → muscle tissue → circulatory system.
- E.** heart → muscle cell → muscle tissue → circulatory system.
- F.** muscle cell → muscle tissue → circulatory system → heart.

Answer A. Key idea

muscle cell → muscle tissue → heart → circulatory system

Worked solution

The hierarchy: CELL (basic unit) → TISSUE (similar cells, one job) → ORGAN (different tissues, one structure) → ORGAN SYSTEM (organs cooperating) → ORGANISM.

Applied: which list goes from simplest to most complex. . . → muscle cell → muscle tissue → heart → circulatory system.

Matches Option A.

Why the other options are wrong

- **B** (circulatory system → heart → muscle tissue → muscle cell.): Reversed. (*Conceptual Error*)
- **C** (muscle tissue → muscle cell → circulatory system → heart.): Tissues precede systems, not organs. (*Conceptual Error*)
- **D** (muscle cell → heart → muscle tissue → circulatory system.): Tissues build organs, not the reverse. (*Conceptual Error*)

- **E** (heart → muscle cell → muscle tissue → circulatory system.): Swaps both pairs: it places muscle tissue before the muscle cell (cells build tissues) and the circulatory system before the heart (organs build organ systems). (*Conceptual Error*)
- **F** (muscle cell → muscle tissue → circulatory system → heart.): Gets the bottom of the hierarchy right, cell then tissue, then swaps the top two by placing the heart last as the largest single structure named. The circulatory system contains the heart together with the blood vessels, so the organ sits below the organ system: cell, tissue, organ, organ system. (*Organ ranked above the system containing it*)

Common mistake. Cells are the building blocks, they come first.

Takeaway. cells→tissues→organs→systems→organism

Time-saving tip. cells→tissues→organs→systems→organism, apply and verify.

Question 9

B3.3 · Cell division and reproduction · ★★☆☆☆ · 85 s

A gardener takes a single healthy potato plant and grows 200 new plants from its tubers, planting no seed. She then crosses two different potato varieties and raises 200 seedlings from the seed produced. Assuming no mutations occur, which statement about the two groups of 200 plants is correct?

- A. The tuber-grown plants are all genetically identical; the seedlings are not.
- B. The seedlings are all genetically identical; the tuber-grown plants are not.
- C. All 400 plants are genetically identical to one another.
- D. Neither group contains any two plants that are genetically identical.
- E. Each seedling is genetically identical to one tuber-grown plant, though neither group is uniform.

Answer A. Key idea

The number of parents decides whether offspring are copies. Asexual reproduction involves one parent and no fusion of gametes, so when no mutations occur the offspring carry exactly that parent's genes and are therefore genetically identical to the parent and to each other. Sexual reproduction involves two parents, and each offspring receives its own combination of genes drawn from both, so the offspring differ from their parents and from one another. Growing plants from the tubers of one plant is the first case; raising plants from seed made by crossing two varieties is the second.

Worked solution

1. Classify each of the gardener's two methods

Tubers are part of the parent plant itself. Growing 200 plants from the tubers of one potato plant involves that single plant and no other, and no gametes fuse, so this is asexual reproduction with one parent.

The seed was produced by crossing two different varieties. Two parents contributed and each seed carries a combination of genes drawn from both, so this is sexual reproduction with two parents.

2. Apply the genetic consequence to each group

Asexual reproduction copies one parent's genes, and the stem rules out mutations, so nothing at all introduces a difference. All 200 tuber-grown plants therefore carry the same genes as the original plant and so as each other: that group is uniform.

Sexual reproduction combines two different sets of genes, and every seed receives its own combination. The two varieties crossed were different varieties, so there is real variation to draw on, and the 200 seedlings differ from one another.

Putting the two together: the tuber-grown group is genetically identical throughout and the seedlings are not.

Sanity check: the phrase assuming no mutations occur is what makes the first group exactly uniform, since mutation is the only thing that could disturb a copy. It does nothing to the second group, because the seedlings' variation comes from combining two parents rather than from mutation.

Matches Option A.

Why the other options are wrong

- **B** (The seedlings are all genetically identical; the tuber-grown plants are not.): This swaps the two methods over. It follows from thinking that a seed, which is shed by one plant, must be a copy of it, while tubers dug up and replanted in separate places must give rise to different plants. The direction is backwards: the seed here carries genes from two varieties, and the tubers carry the genes of one plant only. (*Asexual and sexual routes reversed*)
- **C** (All 400 plants are genetically identical to one another.): This takes assuming no mutations occur to mean that nothing can differ anywhere, so all 400 plants are counted as copies. Removing mutation does fix the tuber-grown group as identical, but the seedlings' variation was never caused by mutation. It comes from combining the genes of two different varieties, which the assumption does not touch. (*No-mutation assumption over-extended*)
- **D** (Neither group contains any two plants that are genetically identical.): This applies the rule for sexual reproduction to both batches, assuming that reproduction always produces offspring that differ. It overlooks the defining feature of the one-parent route: with no gametes fusing and no mutations, there is nothing to make one tuber-grown plant differ from the next, so that group contains 200 plants that are identical. (*Variation assumed to be universal*)
- **E** (Each seedling is genetically identical to one tuber-grown plant, though neither group is uniform.): This assumes the two batches were drawn from one shared stock, so that the seedlings mirror the tuber plants one for one. Nothing in the scenario links them that way. The tuber plants copy a single original plant, while each seedling is a fresh combination from two different varieties, so pairing them off across the groups has no basis, and it also denies that the tuber group is uniform. (*Invents a pairing between the two groups*)

Common mistake. Reading assuming no mutations occur as a promise that nothing anywhere can differ, and concluding that all 400 plants must match. That assumption only removes the one source of difference that could disturb copies made from a single parent. Variation among the seedlings has an entirely different origin: two parents, each contributing a different set of genes.

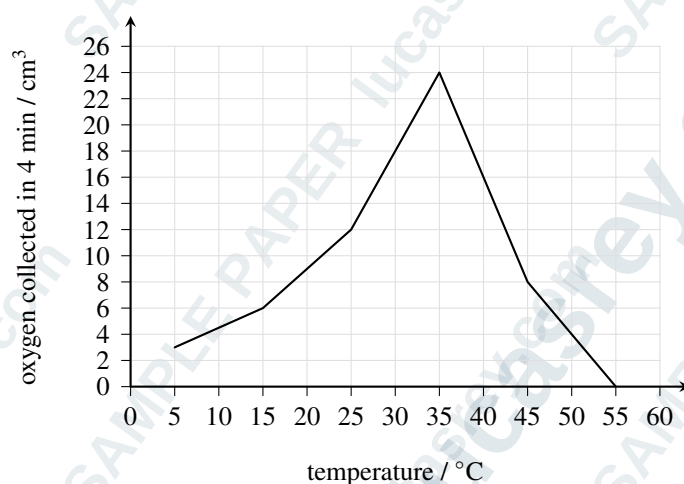
Takeaway. The Cheat Code: Count the parents before anything else. One parent with no mutations means the offspring are copies of it and of each other; two parents means every offspring is a fresh combination and no two need match. Tubers, runners, bulbs and cuttings are all the one-parent route, even though the plants they give end up growing separately.

Time-saving tip. Speed Heuristic: Label the two batches one parent and two parents before you read a single option, then look for the option that pairs identical with the one-parent batch. Four of the five can be dismissed without thinking about potatoes at all.

Question 10

B8.3 · Enzymes and digestion · ★★☆☆ · 90 s

A student investigates catalase, the enzyme in potato tissue that breaks hydrogen peroxide down into water and oxygen. Identical potato discs are dropped into hydrogen peroxide held in a water bath, and the volume of oxygen collected during the first four minutes is recorded at six temperatures. The graph shows the results. Below its optimum temperature this enzyme obeys a temperature coefficient $Q_{10} = 2.0$, meaning that the rate is multiplied by 2.0 for every 10°C rise. Assume hydrogen peroxide is in excess throughout and that all the gas is collected at room temperature and pressure. Using the 15°C reading, calculate the rate of oxygen production at 45°C that this relationship predicts.



- A. $9 \text{ cm}^3 \text{ min}^{-1}$
- B. $24 \text{ cm}^3 \text{ min}^{-1}$
- C. $3 \text{ cm}^3 \text{ min}^{-1}$
- D. $48 \text{ cm}^3 \text{ min}^{-1}$
- E. $6 \text{ cm}^3 \text{ min}^{-1}$
- F. $2 \text{ cm}^3 \text{ min}^{-1}$
- G. $8 \text{ cm}^3 \text{ min}^{-1}$
- H. $12 \text{ cm}^3 \text{ min}^{-1}$

Answer H. Key idea

Below the temperature at which it denatures, an enzyme works faster as the temperature rises, because substrate and enzyme molecules collide more often and with more energy. A temperature coefficient of 2.0 states that this speeding up is a doubling for every 10°C , so a rise made up of n ten-degree steps multiplies the rate by 2.0^n , not by $2.0n$. Above the optimum the relationship fails, because the active site is losing its shape and the measured rate falls. Any prediction of this kind must start from a rate, so a volume collected over a fixed interval has to be divided by that interval first.

Worked solution**1. Turn the graph reading into a rate**

At 15°C the apparatus collects 6 cm^3 of oxygen in four minutes. A rate is a volume per unit time, so this reading must be divided by the collection time before it can be scaled:

$$\text{rate at } 15^{\circ}\text{C} = \frac{6}{4} = 1.5 \text{ cm}^3 \text{ min}^{-1}$$

2. Find the multiplying factor for the temperature rise

From 15°C to 45°C is a rise of 30°C, which is three separate 10°C steps. Each step multiplies the rate by 2.0, so the steps compound:

$$\text{factor} = 2.0^3 = 8$$

3. Apply the factor

$$\text{predicted rate} = 1.5 \times 8 = 12 \text{ cm}^3 \text{ min}^{-1}$$

Sanity check: the plotted points below the optimum do double every ten degrees, running $3 \rightarrow 6 \rightarrow 12 \rightarrow 24 \text{ cm}^3$ per four minutes at 5, 15, 25 and 35°C. One further step gives 48 cm^3 per four minutes at 45°C, and dividing by four again returns $12 \text{ cm}^3 \text{ min}^{-1}$. The graph shows only 8 cm^3 per four minutes actually collected at 45°C, because above the optimum the enzyme is denaturing and the relationship no longer holds; the question asks for the predicted value, not the measured one.

Matches Option **H**.

Why the other options are wrong

- **A** ($9 \text{ cm}^3 \text{ min}^{-1}$): The number of steps is used as a multiplier rather than a power: $2.0 \times 3 = 6$, then $1.5 \times 6 = 9 \text{ cm}^3 \text{ min}^{-1}$. Repeated doubling compounds, so the correct factor is $2.0^3 = 8$. (*Multiplying by the coefficient instead of raising it to a power*)
- **B** ($24 \text{ cm}^3 \text{ min}^{-1}$): Counts the four temperatures 15, 25, 35 and 45°C as four steps rather than the three gaps between them: $2.0^4 = 16$, then $1.5 \times 16 = 24$. A rise of 30°C contains three steps of 10°C, so the factor is $2.0^3 = 8$. (*Ten-degree steps counted one too many*)
- **C** ($3 \text{ cm}^3 \text{ min}^{-1}$): The coefficient is applied once, whatever the size of the rise: $1.5 \times 2.0 = 3 \text{ cm}^3 \text{ min}^{-1}$. This treats Q_{10} as a single doubling rather than a doubling per 10°C, and it ignores two of the three steps between 15°C and 45°C. (*Coefficient applied once for the whole rise*)
- **D** ($48 \text{ cm}^3 \text{ min}^{-1}$): The four-minute volume is used as though it were already a rate: $6 \times 8 = 48$. That is the volume the apparatus would collect in four minutes at 45°C, so it is a factor of four too large for an answer in $\text{cm}^3 \text{ min}^{-1}$. (*Collection time never divided out*)
- **E** ($6 \text{ cm}^3 \text{ min}^{-1}$): Counts only the two readings lying between the start and the target, at 25 and 35°C, as the steps: $2.0^2 = 4$, then $1.5 \times 4 = 6$. The rise from 15°C to 45°C is three ten-degree steps, not two. (*Ten-degree steps counted one too few*)
- **F** ($2 \text{ cm}^3 \text{ min}^{-1}$): This is the measured point read straight off the graph at 45°C: 8 cm^3 in four minutes, so $8 \div 4 = 2 \text{ cm}^3 \text{ min}^{-1}$. It is what the enzyme really does, but the enzyme is denaturing there, so it is not what the Q_{10} relationship predicts. (*Reading the observed value instead of extrapolating*)
- **G** ($8 \text{ cm}^3 \text{ min}^{-1}$): Reads the plotted point at 45°C, 8 cm^3 collected in four minutes, and quotes the 8 as though it were already a rate; the multiplying factor $2.0^3 = 8$ lands on the same number and makes it look confirmed. Dividing by the four minutes gives $2 \text{ cm}^3 \text{ min}^{-1}$, and that point is the denatured measurement rather than the prediction asked for. (*Graph value quoted without dividing by the collection time*)

Common mistake. Multiplying the coefficient by the number of steps instead of raising it to that power: taking $2.0 \times 3 = 6$ and reporting $1.5 \times 6 = 9 \text{ cm}^3 \text{ min}^{-1}$. Three successive doublings give $2 \times 2 \times 2 = 8$, not 6.

Takeaway. The Cheat Code: A Q_{10} is a multiplier per 10°C, so count the whole ten-degree steps the temperature moves and make that count the power, never the multiplier. Then check what the axis of the graph actually measures: a volume collected over a fixed interval is not yet a rate, and one division separates the two.

Time-saving tip. Speed Heuristic: Do the division by four last. $6 \times 8 = 48$, then $48 \div 4 = 12$, and both steps are mental arithmetic. The intermediate 48 is itself printed as an option, so seeing your working land on a listed value is the signal that you have not yet divided by the collection time.

END OF SOLUTIONS

Questions 11 to 27 are in the full paper

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