The ESAT Move Register Preview

This is a preview of the book: 7 of its 407 pages, chosen to show how it is laid out and how an example is worked. It is not the whole book, and every page in it is marked as a preview.

Pages included, by their number in the book: 1, 5, 16, 17, 18, 343, 344.

The complete book is available from lucasrey.com.

The ESAT Move Register

Eleven shortcuts for answering ESAT questions in the time allowed

Eleven ways to reach the answer in fewer steps, each shown on questions from mathematics, physics, chemistry and biology, with the point at which it stops working.

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Contents

Move 1: Work out where the answer must lie, then rule out the options outside it

What you do: Before calculating, decide what the answer has to be like: its sign, roughly how big it is, whether it can be larger than a quantity in the question. Then cross out every option that breaks those limits.

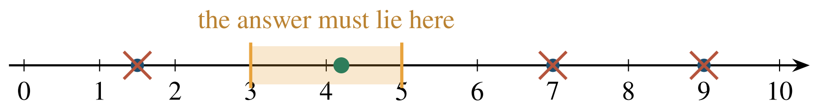

Figure 4: Options outside the range the answer must lie in are crossed out without calculating.

Tip: A part cannot exceed the whole it is taken from, and a probability cannot leave the interval from 0 to 1.

Tip: For a greatest value, check first whether the largest option can actually be reached. If it can, it is the answer.

Tip: For the largest possible quotient of rounded values, divide the top of one band by the bottom of the other: 240 g over 30 cm^3, both to the nearest 10, gives at most $\frac{245}{25} = 9.8\ g\,cm^{-3}$.

The examples in this section: 9 examples: Biology (4), Chemistry (1), Mathematics 1 (1), Mathematics 2 (2), Physics (1).

Example 1 · Biology · B1.1

Surface area to volume ratio across a cell membrane

QUESTION aim for 90 seconds; with the move, about 31 seconds

A biologist represents an animal cell by a rectangular box standing 25 μm tall on a base 60 μm by 12 μm. There is no cell wall, so the membrane is the entire boundary between the cytoplasm and the surroundings, and the cytoplasm fills the box completely. Taking the membrane to be smooth and unfolded, calculate the area of membrane per unit volume of the cell.

A $0.24\,\mu\text{m}^{-1}$ **B** $0.1\,\mu\text{m}^{-1}$ **C** $0.08\,\mu\text{m}^{-1}$ **D** $0.28\,\mu\text{m}^{-1}$

E $0.2\,\mu\text{m}^{-1}$ **F** $0.56\,\mu\text{m}^{-1}$

✓ **Answer: D**, $0.28\,\mu\text{m}^{-1}$

THE SAME PROBLEM IN OTHER WORDS

Stated plainly: Three edge lengths, 60 μm, 12 μm and 25 μm, fix a cuboid whose bounding surface comes to 5040 μm^2 and whose interior comes to 18000 μm^3. Dividing the first of those totals by the second gives $0.28\,\mu\text{m}^{-1}$, and the reciprocal sum $\frac{2}{60} + \frac{2}{12} + \frac{2}{25}$ returns that same $0.28\,\mu\text{m}^{-1}$ with neither total formed at all.

In an everyday setting: A whiteboard in the practical room carries a brick shape drawn in perspective, its longest edge labelled 60 μm, the short edge of the same face 12 μm and the upright 25 μm, with one line of shading round the outside annotated $0.28\,\mu\text{m}^{-1}$. Two boxed figures below the sketch give that annotation its value, 5040 μm^2 for the shaded skin and 18000 μm^3 for everything the skin holds in, the first divided by the second.

THE MOVE

The cancelled form of this ratio is $\frac{2}{60} + \frac{2}{12} + \frac{2}{25}$, and the smallest edge, 12 μm, fixes both ends of it: every term is positive and none exceeds $\frac{2}{12}$, so the value sits above $\frac{2}{12} \approx 0.167\,\mu\text{m}^{-1}$ and below $\frac{6}{12} = 0.5\,\mu\text{m}^{-1}$. That one reciprocal, taken before any face area is worked out, puts 0.08, 0.1 and 0.56 on the wrong side of the bound. Carrying the second reciprocal as well lifts the floor to $\frac{2}{12} + \frac{2}{25} \approx 0.247$, which drops 0.2 and 0.24 below it and leaves $0.28\,\mu\text{m}^{-1}$ standing alone.

THE USUAL METHOD, STEP BY STEP

1. **Total the membrane area:** With no cell wall the membrane is the complete outer boundary, so its area is the surface area of the cuboid. The faces come in three pairs: $60 \times 12 = 720$, $60 \times 25 = 1500$ and $12 \times 25 = 300$, all in μm^2, giving $2 \times (720 + 1500 + 300) = 5040\,\mu m^2$.
2. **Find the volume enclosed:** The cytoplasm fills the box, so the volume is $60 \times 12 \times 25 = 18000\,\mu m^3$.
3. **Divide, or cancel first:** $5040 \div 18000$ gives $0.28\,\mu m^{-1}$. The same figure comes from the cancelled form $\frac{2}{60} + \frac{2}{12} + \frac{2}{25}$, which needs neither total.

WHY THE OTHER OPTIONS ARE WRONG

A $0.24\,\mu m^{-1}$: *Six faces the size of the base*; works out the base, $60 \times 12 = 720\,\mu m^2$, and multiplies it by six as though every face matched it: $6 \times 720 = 4320\,\mu m^2$, and dividing by $18000\,\mu m^3$ gives $0.24\,\mu m^{-1}$. Only two of the six faces are the size of the base; the other four measure 60 by 25 and 12 by 25.

B $0.1\,\mu m^{-1}$: *Cube shortcut applied to the first edge*; uses the cube result $6s^2/s^3 = 6/s$ on the edge quoted first: $6 \div 60$ gives $0.1\,\mu m^{-1}$. This cell is not a cube. Its edges are $60\,\mu m$, $12\,\mu m$ and $25\,\mu m$, and only when all three are equal does one edge fix the ratio on its own.

C $0.08\,\mu m^{-1}$: *Only the base and the top counted*; takes the membrane as the base and the top alone, $2 \times 60 \times 12 = 1440\,\mu m^2$, which gives $0.08\,\mu m^{-1}$. The four faces around the side carry $3600\,\mu m^2$ of membrane and belong in the total too.

E $0.2\,\mu m^{-1}$: *Base and top left out*; counts only the four faces around the side of the box, $2 \times 60 \times 25 + 2 \times 12 \times 25 = 3600\,\mu m^2$, and divides that by $18000\,\mu m^3$ to get $0.2\,\mu m^{-1}$. The base and the top are membrane as well, and they add $1440\,\mu m^2$ between them.

F $0.56\,\mu m^{-1}$: *Each face counted twice*; treats the inner and the outer surface of the membrane as two separate areas, doubling the total to $10080\,\mu m^2$ and giving $0.56\,\mu m^{-1}$. One face carries one area of membrane, whichever side of it is looked at.

⚠ WHEN THE MOVE FAILS

What changes: Drop the condition that the surface is flat over the box and let the outer surface be thrown into folds carrying three times the area of the six plain faces.

The changed problem: *The same cell measures $60\,\mu m$ by $12\,\mu m$ by $25\,\mu m$ and holds the same $18000\,\mu m^3$, but its outer surface is folded, so the membrane laid flat would cover three times the area of the six plain faces, and the quantity wanted is still membrane area divided by enclosed volume.*

New answer: $0.84\,\mu m^{-1}$, from $3 \times 5040 = 15120\,\mu m^2$ of membrane over the unchanged $18000\,\mu m^3$, in place of the $0.28\,\mu m^{-1}$ of the flat case.

Why the move fails here: The ceiling the move leans on, $\frac{6}{12} = 0.5\,\mu m^{-1}$, counts six flat faces of the smallest edge, and a folded surface carries more area than the footprint it stands on. Applied unchanged here, the bound strikes $0.84\,\mu m^{-1}$ as too large and removes the true value.

Practice 3.1 · Physics · P1.2

Resistance, length and cross-sectional area

QUESTION aim for 90 seconds

A cylindrical metal wire has a length of 49 m and a cross-sectional area of 0.5 mm^2. If a 1.0 m sample of the same metal with cross-sectional area 1.0 mm^2 has a resistance of 0.1 Ω, what is the total resistance of the wire?

- **A** 9.80 Ω
- **B** 24 Ω
- **C** 4.90 Ω
- **D** 19.60 Ω
- **E** 78.22 Ω

YOUR WORKING

The worked solution is on the next page.

Solution to practice 3.1 · Physics

Resistance, length and cross-sectional area

✓ **Answer: A**, 9.80 Ω

THE KEY IDEA

For wires of the same metal, resistance is proportional to length and inversely proportional to cross-sectional area, $R = \dfrac{\rho L}{A}$. Scaling the sample by the two ratios gives the answer without finding the resistivity.

THE ROUTE

Question → Same resistivity → Scale by length → Scale by area → Combine the ratios → Answer

THE WORKING, STEP BY STEP

1. **Write resistance as a proportion:** Both wires are the same metal, so ρ is the same and $R \propto \dfrac{L}{A}$. The sample is the reference: $R_1 = 0.1\ \Omega$ for $L_1 = 1.0$ m and $A_1 = 1.0\ \text{mm}^2$.
2. **Scale for the length:** The wire is $\dfrac{49\ \text{m}}{1.0\ \text{m}} = 49$ times longer, so on length alone its resistance would be $49 \times 0.1\ \Omega = 4.9\ \Omega$.
3. **Scale for the area:** The wire's area is $\dfrac{0.5\ \text{mm}^2}{1.0\ \text{mm}^2} = 0.5$ of the sample's. Halving the area doubles the resistance, so $R = 4.9\ \Omega \times 2 = 9.8\ \Omega$.
4. **Combine in one line:** $R_2 = R_1 \times \dfrac{L_2}{L_1} \times \dfrac{A_1}{A_2} = 0.1 \times \dfrac{49}{1.0} \times \dfrac{1.0}{0.5} = 9.8\ \Omega$. Both areas are in mm^2, so the units cancel and no conversion is needed.

The wire has a resistance of 9.8 Ω, so the answer is A, 9.80 Ω.

WHY THE OTHER OPTIONS ARE WRONG

- **B** 24 Ω: *No single slip gives this*; the nearest route is $49 \times 0.5 = 24.5$, which drops the sample's 0.1 Ω and multiplies by the area. Even that gives 24.5 rather than 24.
- **C** 4.90 Ω: *Ignored the change in area*; $0.1 \times 49 = 4.9\ \Omega$ scales for length only. The wire has half the sample's area, which doubles this figure.
- **D** 19.60 Ω: *Squared the area ratio*; $0.1 \times 49 \times \left(\dfrac{1.0}{0.5}\right)^2 = 19.6\ \Omega$ treats 0.5 as a ratio of diameters. The areas are given directly, so their ratio is used once.
- **E** 78.22 Ω: *No single slip gives this*; 78.22 Ω is about eight times the correct value. It would need an extra factor of roughly 8 that nothing in the data supplies.

✓ **Check:** The wire is 49 times longer with half the area, so its resistance is about $49 \times 2 \approx 100$ times the sample's 0.1 Ω, which is close to 10 Ω.

Remember: Resistance rises with length and falls with area, so scale a reference wire by the length ratio and by the inverse of the area ratio.